8.4 Answers

8.4.1 Solution to 8.2 Example

(i)

$$f(x) = \frac{5}{2x} + 6$$

$$f(5) = \frac{5}{2 \times 5} + 6$$

$$= 6.5$$

[1 mark]

(ii)

$$f(x) = \frac{5}{2x} + 6$$

rename $f(x)$ as y

$$y = \frac{5}{2x} + 6$$

swap $x \Leftrightarrow y$

$$x = \frac{5}{2y} + 6$$

make y the subject

$$x - 6 = \frac{5}{2y}$$

$$\frac{1}{x - 6} = \frac{2y}{5}$$

$$2y = \frac{5}{x - 6}$$

$$y = \frac{5}{2x - 12}$$

rename y as $f^{-1}(x)$

$$f^{-1}(x) = \frac{5}{2x - 12}$$

[3 marks]

(iii)

$$f^{-1}(x) = \frac{5}{2x - 12}$$

$$f^{-1}(6.5) = \frac{5}{2 \times 6.5 - 12}$$

$$= 5 \text{ as it should}$$

[1 mark]

8.4.2 Solution to 8.3 Exercise

Answer 1

(i)

$$f(x) = 7 - 3x$$
$$y = 7 - 3x$$
$$\text{swap } x \Leftrightarrow y$$
$$x = 7 - 3y$$
$$3y = 7 - x$$
$$y = \frac{7 - x}{3}$$
$$f^{-1}(x) = \frac{7 - x}{3}$$

(ii)

$$g(x) = 8x + 3$$
$$y = 8x + 3$$
$$\text{swap } x \Leftrightarrow y$$
$$x = 8y + 3$$
$$8y = x - 3$$
$$y = \frac{x - 3}{8}$$
$$g^{-1}(x) = \frac{x - 3}{8}$$

(iii)

$$h(x) = \frac{1}{2}x + 5$$
$$y = \frac{1}{2}x + 5$$
$$\text{swap } x \Leftrightarrow y$$
$$x = \frac{1}{2}y + 5$$
$$\frac{1}{2}y = x - 5$$
$$y = 2x - 10$$
$$h^{-1}(x) = 2x - 10$$

(iv)

$$k(x) = \frac{x}{5} - 4$$
$$y = \frac{x}{5} - 4$$
$$\text{swap } x \Leftrightarrow y$$
$$x = \frac{y}{5} - 4$$
$$\frac{y}{5} = x + 4$$
$$y = 5x + 20$$
$$k^{-1}(x) = 5x + 20$$

[12 marks]

Answer 2

(i)

$$m(x) = 2(3 - 5x)$$

$$y = 2(3 - 5x)$$

swap $x \Leftrightarrow y$

$$x = 2(3 - 5y)$$

$$x = 6 - 10y$$

$$10y = 6 - x$$

$$y = \frac{6 - x}{10}$$

$$m^{-1}(x) = \frac{6 - x}{10}$$

(ii)

$$n(x) = \frac{x - 8}{3}$$

$$y = \frac{x - 8}{3}$$

swap $x \Leftrightarrow y$

$$x = \frac{y - 8}{3}$$

$$y - 8 = 3x$$

$$y = 3x + 8$$

$$n^{-1}(x) = 3x + 8$$

(iii)

$$p(x) = \frac{7x}{4}$$

$$y = \frac{7x}{4}$$

swap $x \Leftrightarrow y$

$$x = \frac{7y}{4}$$

$$7y = 4x$$

$$y = \frac{4x}{7}$$

$$p^{-1}(x) = \frac{4x}{7}$$

(iv)

$$q(x) = \frac{3x}{5} + 4$$

$$y = \frac{3x}{5} + 4$$

swap $x \Leftrightarrow y$

$$x = \frac{3y}{5} + 4$$

$$5x = 3y + 20$$

$$3y = 5x - 20$$

$$y = \frac{5x - 20}{3}$$

$$q^{-1}(x) = \frac{5x - 20}{3}$$

[12 marks]

Answer 3

(i)

$$r(x) = \frac{x}{11}$$

$$y = \frac{x}{11}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{y}{11}$$

$$y = 11x$$

$$r^{-1}(x) = 11x$$

(ii)

$$s(x) = \frac{1}{4x}$$

$$y = \frac{1}{4x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{1}{4y}$$

$$y = \frac{1}{4x}$$

$$s^{-1}(x) = \frac{1}{4x} \qquad x \neq 0$$

(iii)

$$t(x) = \frac{3}{2x}$$

$$y = \frac{3}{2x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{3}{2y}$$

$$y = \frac{3}{2x}$$

$$t^{-1}(x) = \frac{3}{2x} \qquad x \neq 0$$

(iv)

$$u(x) = \frac{1}{x} + 4$$

$$y = \frac{1}{x} + 4$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{1}{y} + 4$$

$$\frac{1}{y} = x - 4$$

$$y = \frac{1}{x - 4}$$

$$u^{-1}(x) = \frac{1}{x - 4} \qquad x \neq 4$$

[12 marks]

Answer 4

(i)

$$v(x) = 9 - \frac{1}{x}$$

$$y = 9 - \frac{1}{x}$$

swap $x \Leftrightarrow y$

$$x = 9 - \frac{1}{y}$$

$$\frac{1}{y} = 9 - x$$

$$y = \frac{1}{9 - x}$$

$$v^{-1}(x) = \frac{1}{9 - x} \quad x \neq 9$$

(ii)

$$w(x) = 5 - \frac{3}{x}$$

$$y = 5 - \frac{3}{x}$$

swap $x \Leftrightarrow y$

$$x = 5 - \frac{3}{y}$$

$$\frac{3}{y} = 5 - x$$

$$\frac{y}{3} = \frac{1}{5 - x}$$

$$y = \frac{3}{5 - x}$$

$$w^{-1}(x) = \frac{3}{5 - x} \quad x \neq 5$$

(iii)

$$z(x) = \frac{2}{x} + 5$$

$$y = \frac{2}{x} + 5$$

swap $x \Leftrightarrow y$

$$x = \frac{2}{y} + 5$$

$$\frac{2}{y} = x - 5$$

$$\frac{y}{2} = \frac{1}{x - 5}$$

$$y = \frac{2}{x - 5}$$

$$z^{-1}(x) = \frac{2}{x - 5} \quad x \neq 5$$

(iv)

$$a(x) = \frac{x + 4}{x - 3}$$

$$y = \frac{x + 4}{x - 3}$$

swap $x \Leftrightarrow y$

$$x = \frac{y + 4}{y - 3}$$

$$xy - 3x = y + 4$$

$$xy - y = 3x + 4$$

$$y(x - 1) = 3x + 4$$

$$y = \frac{3x + 4}{x - 1}$$

$$a^{-1}(x) = \frac{3x + 4}{x - 1} \quad x \neq 1$$

[12 marks]