

9.4 Answers

GCSE Mathematics Functions I

9.4.1 Solution to 9.2 Example (Teaching Video)

$$f(x) = \frac{5x - 1}{2x + 3}, \quad x \in \mathbb{R}, \quad x \neq -\frac{3}{2}$$

$$y = \frac{5x - 1}{2x + 3}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{5y - 1}{2y + 3}$$

$$x(2y + 3) = 5y - 1$$

$$2xy + 3x = 5y - 1$$

$$3x + 1 = 5y - 2xy$$

$$3x + 1 = y(5 - 2x)$$

$$y = \frac{3x + 1}{5 - 2x}$$

$$f^{-1}(x) = \frac{3x + 1}{5 - 2x}, \quad x \neq 1$$

$$\text{Note: } f(7) = 2 : f^{-1}(2) = 7$$

[4, 1 marks]

9.4.2 Solutions to 9.3 Exercise

Answer 1

$$(i) \quad b(x) = \frac{7 - x}{x}$$

$$y = \frac{7 - x}{x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{7 - y}{y}$$

$$xy = 7 - y$$

$$xy + y = 7$$

$$y(x + 1) = 7$$

$$y = \frac{7}{x + 1}$$

$$b^{-1}(x) = \frac{7}{x + 1}, \quad x \neq -1$$

$$(ii) \quad c(x) = \frac{5 - 3x}{x}$$

$$y = \frac{5 - 3x}{x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{5 - 3y}{y}$$

$$xy = 5 - 3y$$

$$xy + 3y = 5$$

$$y(x + 3) = 5$$

$$y = \frac{5}{x + 3}$$

$$c^{-1}(x) = \frac{5}{x + 3}, \quad x \neq -3$$

[4, 4 marks]

Answer 2**(i)**

$$d(x) = \frac{x+3}{x+2}$$

$$y = \frac{x+3}{x+2}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{y+3}{y+2}$$

$$xy + 2x = y + 3$$

$$xy - y = 3 - 2x$$

$$y(x-1) = 3-2x$$

$$y = \frac{3-2x}{x-1}$$

$$d^{-1}(x) = \frac{3-2x}{x-1}, \quad x \neq 1$$

(ii)

$$e(x) = \frac{x+1}{x-2}$$

$$y = \frac{x+1}{x-2}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{y+1}{y-2}$$

$$xy - 2x = y + 1$$

$$xy - y = 1 + 2x$$

$$y(x-1) = 2x+1$$

$$y = \frac{2x+1}{x-1}$$

$$e^{-1}(x) = \frac{2x+1}{x-1}, \quad x \neq 1$$

[4, 4 marks]**Answer 3****(i)**

$$f(x) = \frac{1}{2x} + \frac{1}{3x}$$

$$y = \frac{1}{2x} + \frac{1}{3x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{1}{2y} + \frac{1}{3y}$$

$$6xy = 3 + 2$$

$$6xy = 5$$

$$y = \frac{5}{6x}$$

$$f^{-1}(x) = \frac{5}{6x}, \quad x \neq 0$$

(ii)

$$g(x) = \frac{3}{5x} + \frac{1}{4x} + 2$$

$$y = \frac{3}{5x} + \frac{1}{4x} + 2$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{3}{5y} + \frac{1}{4y} + 2$$

$$20xy = 12 + 5 + 40y$$

$$20xy - 40y = 17$$

$$y(20x - 40) = 17$$

$$y = \frac{17}{20x - 40}$$

$$g^{-1}(x) = \frac{17}{20x - 40}, \quad x \neq 2$$

[4, 4 marks]

Answer 4**(i)**

$$h(x) = \frac{x^2 + 5x + 6}{x^2 + 6x + 8}$$

$$y = \frac{x^2 + 5x + 6}{x^2 + 6x + 8}$$

swap $x \Leftrightarrow y$

$$x = \frac{y^2 + 5y + 6}{y^2 + 6y + 8}$$

$$x = \frac{(y + 2)(y + 3)}{(y + 2)(y + 4)}$$

$$x = \frac{y + 3}{y + 4}$$

$$xy + 4x = y + 3$$

$$xy - y = 3 - 4x$$

$$y = \frac{3 - 4x}{x - 1}$$

$$h^{-1}(x) = \frac{3 - 4x}{x - 1}, \quad x \neq 1$$

(ii)

$$k(x) = \frac{x^2 - 4}{x^2 - 4x + 4}$$

$$y = \frac{x^2 - 4}{x^2 - 4x + 4}$$

swap $x \Leftrightarrow y$

$$x = \frac{y^2 - 4}{y^2 - 4y + 4}$$

$$x = \frac{(y - 2)(y + 2)}{(y - 2)(y - 2)}$$

$$x = \frac{y + 2}{y - 2}$$

$$xy - 2x = y + 2$$

$$xy - y = 2x + 2$$

$$y = \frac{2x + 2}{x - 1}$$

$$k^{-1}(x) = \frac{2x + 2}{x - 1}, \quad x \neq 1$$

[4, 4 marks]