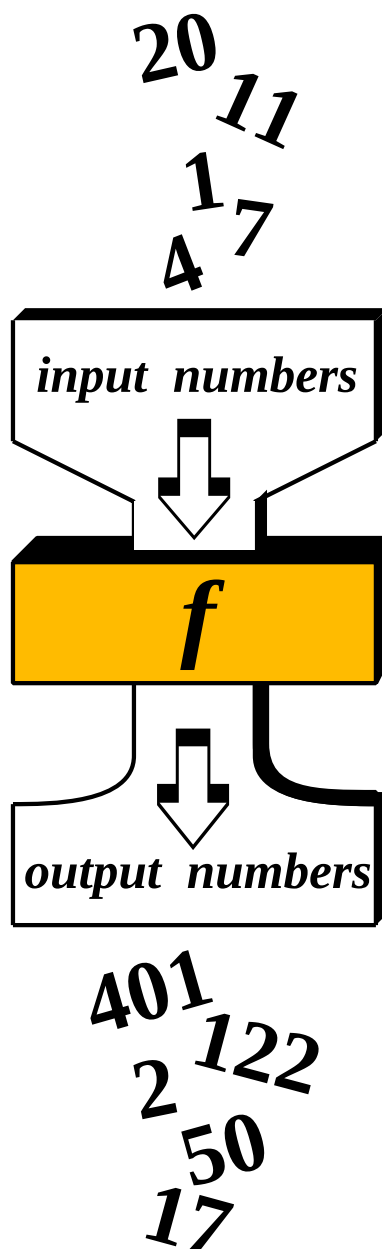


GCSE Mathematics

FUNCTIONS

I



Functions I

Lesson 1

GCSE Mathematics Functions I

1.1 What is a function ?

At a simple level, a function is a mathematical rule that “does things to numbers”.
If it is helpful, a function can be given a name.

Here is a description of the function called f .

$$f(x) = 1 + \sqrt{x}, \quad x \in \mathbb{Z}, \quad x \geq 0$$

Now, suppose I have the number 25.

What does function f do to the number 25 ?

Teaching Video : [http://www.NumberWonder.co.uk/Video/v9002\(1\).mp4](http://www.NumberWonder.co.uk/Video/v9002(1).mp4)



[2 marks]

1.2 For You To Do

Consider the function, $f(x) = x^2 + 1, \quad x \in \mathbb{R}$

Calculate,

(i) $f(7)$

(ii) $f(1)$

(iii) $f(11)$

(iv) $f(0)$

(v) $f(5)$

(vi) $f(-5)$

(vii) $f(-6)$

(viii) $f(\sqrt{3})$

[8 marks]

Now turn over the page to see if you got these questions correct.

1.3 Answers to For You To Do

In words the function $f(x) = x^2 + 1$ takes the number it is given and “squares it and adds one”

When given the number 7, this function it will square 7 and add 1

$$f(x) = x^2 + 1$$

$$f(7) = 7^2 + 1$$

$$= 49 + 1$$

$$= 50$$

Repeating this for all of the questions gives the following answers,

(i) $f(7) = 50$ (ii) $f(1) = 2$

(iii) $f(11) = 122$ (iv) $f(0) = 1$

(v) $f(5) = 26$ (vi) $f(-5) = 26$ because $(-5)(-5) = 25$

(vii) $f(-6) = 37$ (viii) $f(\sqrt{3}) = 4$ because $(\sqrt{3})(\sqrt{3}) = 3$

[8 marks]

Finally, notice the domain, $x \in \mathbb{R}$

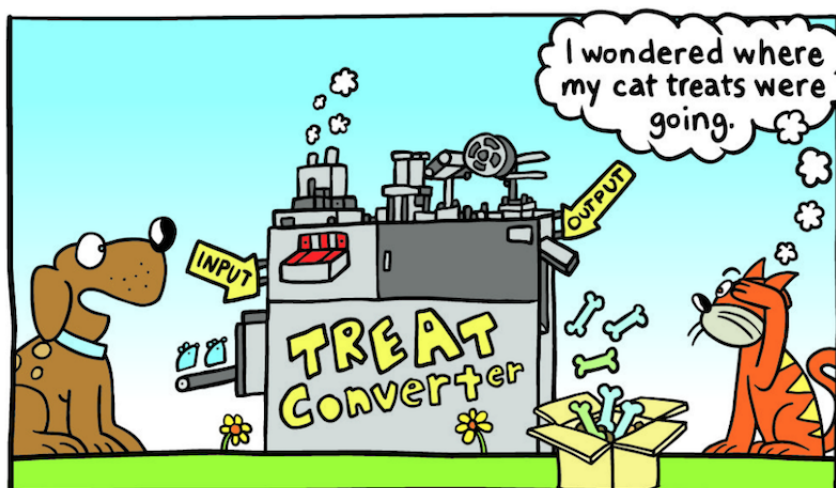
Any Real Number may be used as input into the “square and adds one” function.

The Real Numbers are “all the numbers you officially know about”.

This includes Integers, $\mathbb{Z}$, Rational Numbers, $\mathbb{Q}$, and Irrational Numbers.

There are lots of numbers that you “don't officially know about” !

Further A-Level Mathematicians, for example, know of Complex Numbers, $\mathbb{C}$.



“It is my treat-converter function machine. However many cat treats I input, the machine outputs TWICE that many dog biscuits. Isn't that cool?”

It's not just numbers that can go into functions !
(For example, later on we'll input algebraic expressions)

1.4 Exercise

Marks Available : 40

Question 1

Function f is described as,

$$f(x) = 4x + 13, \quad x \in \mathbb{Z}$$

Write down the value of,

(i) $f(1)$

(ii) $f(2)$

(iii) $f(3)$

(iv) $f(0)$

(v) $f(10)$

(vi) $f(-2)$

[3 marks]

What does the domain, $x \in \mathbb{Z}$, tell you about this function ?



[1 mark]

Question 2

Function g is described as,

$$g(x) = \frac{x^2}{2} \quad x \in \mathbb{Z}$$

Write down the value of,

(i) $g(1)$

(ii) $g(-1)$

(iii) $g(4)$

(iv) $g(10)$

(v) $g(0)$

(vi) $g(-3)$

[3 marks]

The range of this function, the numbers that can occur as answers, are all non negative. Why this is ?



[1 mark]

Question 3

Function h is the function described by, $h(x) = 7 + \sqrt{x}$, $x \in \mathbb{R}$, $x \geq 0$

Write down the value of,

(i) $h(100)$

(ii) $h(4)$

(iii) $h(81)$

(iv) $h(0)$

(v) $h(400)$

(vi) $h(121)$

[3 marks]

Explain why the restriction is placed upon the domain that only numbers greater than or equal to zero can be allowed into this function.



[1 mark]

Question 4

Given that, $m(x) = 14 + 3x$, $x \in \mathbb{Z}$, write down the value of,

(i) $m(1)$

(ii) $m(-1)$

(iii) $m(6)$

(iv) $m(9)$

(v) $m(-3)$

(vi) $m(-5)$

[3 marks]

Question 5

Given that, $n(x) = 10 - 2x$, $x \in \mathbb{R}$, write down the value of,

(i) $n(4)$

(ii) $n(0)$

(iii) $n(-2)$

(iv) $n(11)$

(v) $n(-8)$

(vi) $n\left(\frac{1}{2}\right)$

[3 marks]

Question 6

Consider the function, $d(x) = \frac{6}{x}$, $x \in \mathbb{R}$, $x \neq 0$

Write down the value of,

- (i) $d(3)$ (ii) $d(6)$ (iii) $d(12)$
- (iv) $d(-2)$ (v) $d(0.1)$ (vi) $d(-0.1)$

[3 marks]

Explain the restriction $x \neq 0$ on the domain.

In other words, why is the number zero is not allowed into this function ?



[1 mark]

Question 7

The function e is given by, $e(x) = \sqrt{4x + 1}$, $x \in \mathbb{R}$, $x \geq -0.25$

Write down the value of,

- (i) $e(0)$ (ii) $e(2)$ (iii) $e(6)$
- (iv) $e(12)$ (v) $e\left(\frac{3}{4}\right)$ (vi) $e\left(-\frac{1}{4}\right)$

[3 marks]

Question 8

The function k is given by, $k(x) = 4x^2 + 3$, $x \in \mathbb{R}$

Write down the value of,

- (i) $k(1)$ (ii) $k(-1)$ (iii) $k(2)$
- (iv) $k(-2)$ (v) $k(5)$ (iv) $k(-5)$

[3 marks]

Question 9

The quadratic function A is, $A(x) = 2x^2 + 3x - 1$, $x \in \mathbb{R}$

Write down the value of,

(i) $A(1)$

(ii) $A(0)$

(iii) $A(5)$

(iv) $A(10)$

(v) $A(100)$

(iv) $A(-1)$

[3 marks]

Question 10

The function H has the definition, $H(x) = (2x + 5)(3x + 4)$, $x \in \mathbb{Z}$

Write down the value of,

(i) $H(1)$

(ii) $H(0)$

(iii) $H(-2)$

(iv) $H(-1)$

(v) $H(2)$

(vi) $H(-3)$

[3 marks]

Question 11

This question is about a mystery function, M .

We can drop numbers into M , and see what it does to those numbers.

If I drop the numbers 0, 1, 2, 3, 4 and 5 into M then 1, 4, 7, 10, 13 and 16 come out.

In other words $M(0) = 1$

$$M(1) = 4$$

$$M(2) = 7$$

$$M(3) = 10$$

$$M(4) = 13$$

and $M(5) = 16$

Suggest a rule that M is applying.

It must turn all five input numbers into the five output numbers.

If your rule only works on one or two numbers, it's no good !

[3 marks]

Question 12

Here is another “guess the rule” question.

$$N(0) = -1$$

$$N(1) = 0$$

$$N(2) = 3$$

$$N(3) = 8$$

$$N(4) = 15$$

and $N(5) = 24$

Suggest a function that N could be.

[3 marks]

1.5 Homework : Harder Questions

GCSE Mathematics
Functions I

Marks Available : 56

Question 1

If $f(x) = 13x + 5$, $x \in \mathbb{R}$, find;

(i) $f(1)$ (ii) $f(-1)$ (iii) $f(2)$

(iv) $f(10)$ (v) $f(0)$ (vi) $f(-3)$

[3 marks]

Question 2

If $g(x) = 17 - 2x$, $x \in \mathbb{R}$, find;

(i) $g(7)$ (ii) $g(10)$ (iii) $g(17)$

(iv) $g(50)$ (v) $g(-1)$ (vi) $g(-8)$

[3 marks]

Question 3

If $h(x) = \sqrt{101 - x}$, $x \in \mathbb{R}$, $x \leq 101$. find;

(i) $h(1)$ (ii) $h(100)$ (iii) $h(65)$

(iv) $h(52)$ (v) $h(20)$ (vi) $h(-20)$

[3 marks]

(vii) Try to find $h(105)$

Hence explain the restriction on the input (domain) that $x \leq 101$

[1 mark]

Question 4

If $k(x) = 10x^2 + 11$, $x \in \mathbb{R}$, find,

(i) $k(2)$ (ii) $k(-2)$ (iii) $k(3)$

(iv) $k(-3)$ (v) $k(5)$ (iv) $k(-5)$

[3 marks]

Question 5

If $L(x) = 4x^2 + 5x + 7$, $x \in \mathbb{R}$, find,

(i) $L(1)$ (ii) $L(3)$ (iii) $L(10)$

(iv) $L(-1)$ (v) $L(7)$ (iv) $L(-3)$

[3 marks]

Question 6

If $m(x) = x^3 + x^2 + x$, $x \in \mathbb{R}$, find,

(i) $m(2)$ (ii) $m(3)$ (iii) $m(-1)$

(iv) $m(10)$ (v) $m(5)$ (iv) $m(-3)$

[3 marks]

Question 7

If $n(x) = \frac{24}{x}$, $x \in \mathbb{R}$, $x \neq 0$, find,

(i) $n(6)$ (ii) $n(8)$ (iii) $n(24)$

(iv) $n(-4)$ (v) $n(48)$ (vi) $n(0.5)$

[3 marks]

(vii) Try to find $n(0)$
Hence explain the restriction on the domain that $x \neq 0$

[1 mark]

Question 8

If $p(x) = \sqrt{3x}$, $x \in \mathbb{R}$, $x \geq 0$ find,

(i) $p(3)$ (ii) $p(12)$ (iii) $p(27)$

(iv) $p(48)$ (v) $p\left(\frac{1}{3}\right)$ (vi) $p\left(\frac{25}{3}\right)$

[3 marks]

(vii) Try to find $p(-3)$
Hence explain the restriction on the input that $x \geq 0$

[1 mark]

Question 9

If $q(x) = x^3 + x$, $x \in \mathbb{R}$, find,

(i) $q(4)$ (ii) $q\left(\frac{1}{2}\right)$ (iii) $q\left(\frac{1}{10}\right)$

[3 marks]

Question 10

If $r(x) = x^{\frac{1}{2}}$, $x \in \mathbb{R}$, $x \geq 0$, find,

(i) $r(25)$ (ii) $r(81)$ (iii) $r\left(\frac{1}{4}\right)$

[3 marks]

(iv) Explain the restriction on the domain that $x \geq 0$

[1 mark]

Question 11

If $s(x) = x^{-2}$, $x \in \mathbb{R}$, $x \neq 0$, find,

(i) $s(2)$ (ii) $s(5)$ (iii) $s(4)$

[3 marks]

(iv) Explain the restriction on the input that $x \neq 0$

[1 mark]

Question 12

If $t(x) = x^{\frac{1}{2}} + x$, $x \in \mathbb{R}$, $x \geq 0$, find,

(i) $t(9)$ (ii) $t(36)$ (iii) $t(121)$

[3 marks]

(iv) Explain the restriction on the domain that $x \geq 0$

[1 mark]

Question 13

If $u(x) = x^{-1} + 1$, $x \in \mathbb{R}$, $x \neq 0$, find

(i) $u(3)$ (ii) $u(10)$ (iii) $u(-10)$

(iv) $u(5)$ (v) $u(-5)$ (iv) $u(-100)$

[3 marks]

Question 14

If $f(x) = x^2$, $x \in \mathbb{R}$, find

(i) $f(z+1)$

HINT: Use FOIL to work out $(z+1)(z+1)$

[1 mark]

(ii) $f(z+5)$

[1 mark]

(iii) $f(2z+3)$

[1 mark]

Question 15

If $f(x) = x + 1$, $x \in \mathbb{R}$, find,

(i) $f(3)$

(ii) $f(-3)$

(iii) $f(w^2)$

(iv) $f(w^2 + w)$

[4 marks]

Question 16

If $f(x) = x^2 + x$, $x \in \mathbb{R}$, find,

(i) $f(4)$

(ii) $f(-4)$

(iii) $f(z + 1)$

(iv) $f(z^2 + z)$

[4 marks]

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1.6 Answers

GCSE Mathematics Functions I

1.6.1 Solutions to 1.4 Exercise

Answer 1

(i)	17	(ii)	21	(iii)	25
(iv)	13	(v)	53	(vi)	5

[3 marks]

Only integers may be used as inputs

[1 mark]

Answer 2

(i)	0.5	(ii)	0.5	(iii)	8
(iv)	50	(v)	0	(vi)	4.5

[3 marks]

An integer squared can not be negative

[1 mark]

Answer 3

(i)	17	(ii)	9	(iii)	16
(iv)	7	(v)	27	(vi)	18

[3 marks]

Domain restricted to avoid trying to take the square root of a negative number.

[1 mark]

Answer 4

(i)	17	(ii)	11	(iii)	32
(iv)	41	(v)	5	(vi)	- 1

[3 marks]

Answer 5

(i)	2	(ii)	10	(iii)	14
(iv)	- 12	(v)	26	(vi)	9

[3 marks]

Answer 6

(i)	2	(ii)	1	(iii)	0.5
(iv)	- 3	(v)	60	(vi)	- 60

[3 marks]

Domain is restricted to avoid division by zero, which has no meaning in mathematics.

[1 mark]

Answer 7**(i)** 1**(iv)** 7**(ii)** 3**(v)** 2**(iii)** 5**(vi)** 0**[3 marks]****Answer 8****(i)** 7**(iv)** 19**(ii)** 7**(v)** 103**(iii)** 19**(vi)** 103**[3 marks]****Answer 9****(i)** 4**(iv)** 229**(ii)** − 1**(v)** 20 299**(iii)** 64**(vi)** − 2**[3 marks]****Answer 10****(i)** 49**(iv)** 3**(ii)** 20**(v)** 90**(iii)** − 2**(vi)** 5**[3 marks]****Answer 11**

$$M(x) = 3x + 1$$

[3 marks]**Answer 12**

$$N(x) = x^2 - 1$$

[3 marks]**NOTE:**

$$f (input\ number) = output\ number$$

The set of numbers allowed into f is termed the **domain**The set of numbers that can come out of f is termed the **range**

1.6.2 Solutions to 1.5 Homework : Harder Questions

GCSE Mathematics Functions I

Answer 1

(i)	18	(ii)	− 8	(iii)	31
(iv)	135	(v)	5	(vi)	− 34

[3 marks]

Answer 2

(i)	3	(ii)	− 3	(iii)	− 17
(iv)	− 83	(v)	19	(vi)	33

[3 marks]

Answer 3

(i)	10	(ii)	1	(iii)	6
(iv)	7	(v)	9	(vi)	11

[3 marks]

Domain is all real numbers such that $x \leq 101$ because otherwise you could try to take the square root of a negative number which is not real.

It could crash your computer program, for example.

[1 mark]

Answer 4

(i)	51	(ii)	51	(iii)	101
(iv)	101	(v)	261	(vi)	261

[3 marks]

Answer 5

(i)	16	(ii)	58	(iii)	457
(iv)	6	(v)	238	(vi)	28

[3 marks]

Answer 6

(i)	14	(ii)	39	(iii)	− 1
(iv)	1110	(v)	155	(vi)	− 21

[3 marks]

Answer 7

(i)	4	(ii)	3	(iii)	1
(iv)	− 6	(v)	0.5	(vi)	48

[3 marks]

Domain is all real numbers except zero (i.e. $x \neq 0$) because otherwise you could try to divide by zero which has no meaning in mathematics. It's undefined.

A division by zero error caused a car factory robot to kill a worker in 1994.

[1 mark]

Answer 8

(i)	3	(ii)	6	(iii)	9
(iv)	12	(v)	1	(vi)	5

[3 marks]

Restriction on the domain, $x \geq 0$, is to avoid trying to take $\sqrt{\quad}$ of a negative number.

[1 mark]

(i) 68 (ii) $\frac{5}{8}$ or 0.625 (iii) $\frac{1}{1000} + \frac{1}{10} = 0.101$

[3 marks]

(i) 5 (ii) 9 (iii) 0.5

[3 marks]

[1 mark]

(i) $\frac{1}{4}$ (ii) $\frac{1}{25}$ (iii) $\frac{1}{16}$ [3 marks]

[1 mark]

(i) 12 (ii) 42 (iii) 132

[3 marks]

[1 mark]

(i) $1\frac{1}{3}$	(ii) $1\frac{1}{10}$	(iii) $\frac{9}{10}$
(iv) $1\frac{1}{5}$	(v) $\frac{4}{5}$	(vi) $\frac{99}{100}$

[3 marks]

(i) $z^2 + 2z + 1$ (ii) $z^2 + 10z + 25$ (iii) $4z^2 + 12z + 9$
[3 marks]

(i) 4 **(ii)** -2
(iii) $w^2 + 1$ **(iv)** $w^2 + w + 1$

(i) 20

(iii) $z^2 + 3z + 2$

(ii) 12

(iv) $z^4 + 2 z^3 + 2 z^2 + z$

[4 marks]

Lesson 2

GCSE Mathematics Functions I

2.1 Algebraic Inputs

Previously we looked at what the following function did to various numbers;

$$f(x) = x^2 + 1 \quad x \in \mathbb{R}$$

In words, “whatever the input, square it and then add 1”.

For example,

$$\begin{aligned} f(5) &= 5^2 + 1 \\ &= 26 \end{aligned}$$

We can also drop algebraic expressions into function f .

The algebra dropped in will be squared and then have 1 added on.

For example, let's drop $4z$ into function f .

$$\begin{aligned} f(4z) &= (4z)^2 + 1 \\ &= (4z)(4z) + 1 \\ &= 16z^2 + 1 \end{aligned}$$

What will happen if $3z + 5$ is dropped into function f ?

Write the answer without any brackets for a BONUS MARK !



Dear Sir,

Here is my most excellent answer and also my thinking.

What I thought was that I have to square and add 1

So $(3z + 5)$ is to be squared and not to have brackets.

$$(3z + 5)(3z + 5) = 9z^2 + 30z + 25$$

Am I doing good, Sir ?

I bet you think I'm going to forget to add 1

Sir, guess what ? I'm NOT !

$$\begin{aligned} f(3z + 5) &= (3z + 5)^2 + 1 \\ &= 9z^2 + 30z + 25 + 1 \\ &= 9z^2 + 30z + 26 \end{aligned}$$

And so, please may I have the BONUS MARK !

Yours mathematically,

Bert

2.2 You Try

Here are five questions for you to try.

The answers are on the following page - Don't look yet !

Try each one yourself first, then check over the page to see if you got it correct.

If

$$h(x) = 4x + 13, \quad x \in \mathbb{R}$$

find expressions that do not involve brackets for;

Try 1 $h(5)$

[1 mark]

Try 2 $h(3z)$

[1 mark]

Try 3 $h(3z + 1)$

[1 mark]

Try 4 $h(z^2 + 7)$

[1 mark]

Try 5 $h(x^5 + 7)$

[1 mark]

Reminder :

$$h(x) = 4x + 13, \quad x \in \mathbb{R}$$

In words: “Multiply the input by four and then add on 13”

So the answers are :

Try 1 $h(5)$

$$\begin{aligned} h(5) &= 4 \times 5 + 13 \\ &= 33 \end{aligned}$$

[1 mark]

Try 2 $h(3z)$

$$\begin{aligned} h(3z) &= 4 \times 3z + 13 \\ &= 12z + 13 \end{aligned}$$

[1 mark]

Try 3 $h(3z + 1)$

$$\begin{aligned} h(3z + 1) &= 4(3z + 1) + 13 \\ &= 12z + 4 + 13 \\ &= 12z + 17 \end{aligned}$$

[1 mark]

Try 4 $h(z^2 + 7)$

$$\begin{aligned} h(z^2 + 7) &= 4(z^2 + 7) + 13 \\ &= 4z^2 + 28 + 13 \\ &= 4z^2 + 41 \end{aligned}$$

[1 mark]

Try 5 $h(x^5 + 7)$

$$\begin{aligned} h(x^5 + 7) &= 4(x^5 + 7) + 13 \\ &= 4x^5 + 28 + 13 \\ &= 4x^5 + 41 \end{aligned}$$

[1 mark]

How did you do ?

Dear Sir,
I did do done good !
Yours mathematically,
Bert

2.3 Exercise

Marks Available : 40

Question 1

If

$$f(x) = 5x + 11 \quad x \in \mathbb{R}$$

find expressions that do not involve brackets for;

(i) $f(3)$ (ii) $f(5)$ (iii) $f(12)$

(iv) $f(2z)$

(v) $f(6z + 5)$

(vi) $f(3z + 7)$

(vii) $f(-2)$ (viii) $f(-1)$ (ix) $f(0.1)$

(x) $f(4z - 2)$

(xi) $f(8z - 3)$

[6 marks]

Question 2

If

$$g(x) = 8x - 10 \quad x \in \mathbb{R}$$

find expressions that do not involve brackets for;

(i) $g(2)$ (ii) $g(6)$ (iii) $g(0)$

(iv) $g(3z)$

(v) $g(5z + 2)$

(vi) $g(7 + 2z^2)$

(vii) $g(-2)$ (viii) $g(-5)$ (ix) $g(0.1)$

(x) $g(-4z)$

(xi) $g(8\sqrt{z} - 1)$

[6 marks]

Question 3

If

$$h(x) = x^2 \quad x \in \mathbb{R}$$

find expressions that do not involve brackets for;

(i) $h(3)$ (ii) $h(-3)$ (iii) $h(8)$

(iv) $h(3z)$

(v) $h(5z + 1)$

HINT : FOIL

(vi) $h(2z + 7)$

(vii) $h\left(\frac{3}{2}\right)$ (viii) $h\left(-\frac{3}{10}\right)$ (ix) $h\left(\frac{1}{2}\right)$

(x) $h(-4z)$

(xi) $h(6z - 2)$

[6 marks]

Question 4

If

$$k(x) = 7x + 3 \quad x \in \mathbb{R}$$

find expressions that do not involve brackets for;

(i) $k(7)$

(ii) $k(6)$

(iii) $k(-11)$

(iv) $k(3x)$

(v) $k(2x + 10)$

(vi) $k(5x + 1)$

(vii) $k\left(\frac{1}{10}\right)$

(viii) $k\left(-\frac{1}{10}\right)$

(ix) $k\left(\frac{1}{2}\right)$

(x) $k(4x - 1)$

(xi) $k(7x - 2)$

[6 marks]

Question 5

If

$$m(x) = x^2 + 5 \quad x \in \mathbb{R}$$

find expressions that do not involve brackets for;

(i) $m(7)$ (ii) $m(-7)$ (iii) $m(20)$

(iv) $m(3x)$

(v) $m(-3x)$

(vi) $m(5x + 1)$

(vii) $m(-1)$ (viii) $m\left(\frac{1}{2}\right)$ (ix) $m\left(\frac{3}{2}\right)$

(x) $m(4x + 7)$

(xi) $m(7x - 2)$

[6 marks]

Question 6

If

$$n(x) = 4x^2 + 1 \quad x \in \mathbb{R}$$

find expressions that do not involve brackets for;

(i) $n(3)$ (ii) $n(-3)$ (iii) $n(100)$

(iv) $n(3x)$

(v) $n(-3x)$

(vi) $n(7x + 1)$

(vii) $n(0)$ (viii) $n\left(\frac{1}{2}\right)$ (ix) $n\left(\frac{3}{2}\right)$

(x) $n(2x + 3)$

(xi) $n(5x - 2)$

[10 marks]

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2.4 Homework

GCSE Mathematics Functions I

These questions are harder.

If you get stuck on one, miss it out and come back to it later.

Marks Available : 32

Question 1

If $f(x) = 8x + 5$, $x \in \mathbb{R}$, find;

- (i) $f(4)$ (ii) $f(-11)$ (iii) $f(0.25)$

[3 marks]

Question 2

If $g(x) = 13 - x$, $x \in \mathbb{R}$, find;

- (i) $g(7)$ (ii) $g(25)$ (iii) $g(-5)$

[3 marks]

Question 3

If $h(x) = \sqrt{25 - x^2}$, $x \in \mathbb{R}$, $-5 \leq x \leq 5$, find,

- (i) $h(4)$ (ii) $h(0)$ (iii) $h(-5)$

[3 marks]

- (iv) Try to find $h(7)$

Hence explain the restriction on the domain that $-5 \leq x \leq 5$

[1 mark]

Question 4

If $k(x) = 8x^2 + 6x + 13$, $x \in \mathbb{R}$, find,

- (i) $k(5)$ (ii) $k(-1)$ (iii) $k(0.5)$

[3 marks]

Question 5

If $m(x) = x^3 + x^2$, $x \in \mathbb{R}$, find,

- (i) $m(3)$ (ii) $m(-1)$ (iii) $m(0.5)$

[3 marks]

Question 6

If $n(x) = \frac{12}{x}$, $x \in \mathbb{R}$, $x \neq 0$, find,

- (i) $n(6)$ (ii) $n(0.25)$ (iii) $n(36)$

[3 marks]

- (iv) Try to find $n(0)$ and hence explain the restriction on the domain that $x \neq 0$

[1 mark]

Question 7

If $f(x) = 3x^2$, $x \in \mathbb{R}$, find,

- (i) $f(11)$ (ii) $f(w + 1)$ (iii) $f(2x)$

Write your answers without any brackets.

[3 marks]

Question 8

If $f(x) = x^3 + x$, $x \in \mathbb{R}$, find,

(i) $f(4)$ (ii) $f(3w)$ (iii) $f(x^2)$

Write your answers without any brackets.

[3 marks]

Question 9

If $f(x) = x^2 + 1$, $x \in \mathbb{R}$, find,

(i) $f(14)$ (ii) $f(w + 4)$ (iii) $f(3\sqrt{x})$

Write your answers without any brackets.

[3 marks]

Question 10

If $f(x) = x^3$, $x \in \mathbb{R}$, find $f(2x + 1)$

Express your answer without using any brackets.

[3 marks]

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2.5 Answers

GCSE Mathematics Functions I

2.5.1 Solutions (Introduction)

$$\begin{aligned}f(x) &= x^2 + 1 \\f(3z + 1) &= (3z + 1)^2 + 1 \\&= (3z + 1)(3z + 1) + 1 \\&= 9z^2 + 6z + 1 + 1 \\&= 9z^2 + 6z + 2\end{aligned}$$

2.5.2 Solutions (Exercise)

Answer 1

- | | | | | | |
|-------|------------|--------|----|-------|------|
| (i) | 26 | (ii) | 36 | (iii) | 71 |
| (iv) | $10z + 11$ | | | | |
| (v) | $30z + 36$ | | | | |
| (vi) | $15z + 46$ | | | | |
| (vii) | 1 | (viii) | 6 | (ix) | 11.5 |
| (x) | $20z + 1$ | | | | |
| (xi) | $40z - 4$ | | | | |

[6 marks]

Answer 2

- | | | | | | |
|-------|-------------------|--------|-----|-------|------|
| (i) | 6 | (ii) | 38 | (iii) | -10 |
| (iv) | $24z - 10$ | | | | |
| (v) | $40z + 6$ | | | | |
| (vi) | $46 + 16z^2$ | | | | |
| (vii) | -26 | (viii) | -50 | (ix) | -9.2 |
| (x) | $-32z - 10$ | | | | |
| (xi) | $64\sqrt{z} - 18$ | | | | |

[6 marks]

Answer 3

- | | | | | | |
|-------|----------------------|--------|------------------------|-------|----------------------|
| (i) | 9 | (ii) | 9 | (iii) | 64 |
| (iv) | $9z^2$ | | | | |
| (v) | $25z^2 + 10z + 1$ | | | | |
| (vi) | $4z^2 + 28z + 49$ | | | | |
| (vii) | $\frac{9}{4} = 2.35$ | (viii) | $\frac{9}{100} = 0.09$ | (ix) | $\frac{1}{4} = 0.25$ |
| (x) | $16z^2$ | | | | |
| (xi) | $36z^2 - 24z + 4$ | | | | |

[6 marks]

Answer 4

- | | | | | | |
|---------|------------|----------|-----|---------|------|
| (i) | 52 | (ii) | 45 | (iii) | - 74 |
| (iv) | $21x + 3$ | | | | |
| (v) | $14x + 73$ | | | | |
| (vi) | $35x + 10$ | | | | |
| (vii) | 3.7 | (viii) | 2.3 | (ix) | 6.5 |
| (x) | $28x - 4$ | | | | |
| (xi) | $49x - 11$ | | | | |

[6 marks]**Answer 5**

- | | | | | | |
|---------|--------------------|----------|------|---------|------|
| (i) | 54 | (ii) | 54 | (iii) | 405 |
| (iv) | $9x^2 + 5$ | | | | |
| (v) | $9x^2 + 5$ | | | | |
| (vi) | $25x^2 + 10x + 6$ | | | | |
| (vii) | 6 | (viii) | 5.25 | (ix) | 7.25 |
| (x) | $16x^2 + 56x + 54$ | | | | |
| (xi) | $49x^2 - 28x + 9$ | | | | |

[6 marks]**Answer 6**

- | | | | | | |
|---------|---------------------|----------|----|---------|--------|
| (i) | 37 | (ii) | 37 | (iii) | 40 001 |
| (iv) | $36x^2 + 1$ | | | | |
| (v) | $36x^2 + 1$ | | | | |
| (vi) | $196x^2 + 56x + 5$ | | | | |
| (vii) | 1 | (viii) | 2 | (ix) | 10 |
| (x) | $16x^2 + 48x + 37$ | | | | |
| (xi) | $100x^2 - 80x + 17$ | | | | |

[10 marks]

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2.4.3 Solutions to 2.4 Homework

GCSE Mathematics Functions I

Answer 1

- (i) 37 (ii) -83 (iii) 7

[3 marks]

Answer 2

- (i) 6 (ii) -12 (iii) 18

[3 marks]

Answer 3

- (i) 3 (ii) 5 (iii) 0
(iv) The square root of a negative number is not a real number

[4 marks]

Answer 4

- (i) 243 (ii) 15 (iii) 18

[3 marks]

Answer 5

- (i) 36 (ii) 0 (iii) $\frac{3}{8}$

[3 marks]

Answer 6

- (i) 2 (ii) 48 (iii) $\frac{6}{13}$
(iv) Division by zero is not defined

[4 marks]

Answer 7

- (i) 363 (ii) $3w^2 + 6w + 3$ (iii) $12x^2$

[3 marks]

Answer 8

- (i) 68 (ii) $27w^3 + 3w$ (iii) $x^6 + x^2$

[3 marks]

Answer 9

- (i) 197 (ii) $w^2 + 8w + 17$ (iii) $9x + 1$

[3 marks]

Answer 10

$$\begin{aligned}(2x + 1)^3 &= 1 \times (2x)^0 \times (1)^3 \\&\quad + 3 \times (2x)^1 \times (1)^2 \\&\quad + 3 \times (2x)^2 \times (1)^1 \\&\quad + 1 \times (2x)^3 \times (1)^0 \\&= 1 + 6x + 12x^2 + 8x^3\end{aligned}$$

[3 marks]

3.1 Composite Functions (Numerical Questions)

We have got used to the idea of substituting a number into a function, and the function 'doing stuff' to the number. Typically, a different number then comes out.

We're now going to take that output number and feed it back around to the input again. Alternatively we could use it as the input to another, completely different, function.

3.2 A numerical example

Let p and q be the functions;

$$p(x) = 8x - 20 \quad x \in \mathbb{R}$$

$$q(x) = 3x - 7 \quad x \in \mathbb{R}$$

Watch the teaching Video : [http://www.NumberWonder.co.uk/Video/v9002\(3\).mp4](http://www.NumberWonder.co.uk/Video/v9002(3).mp4)



Evaluate each of the following;

(i) $p(6)$

(ii) $q(6)$

[1, 1 marks]

(iii) $p q(3)$

[2 marks]

(iv) $q p(3)$

[2 marks]

3.3 Exercise

Marks Available : 54

Question 1

Let f and g be the functions;

$$f(x) = 5x - 12 \quad x \in \mathbb{R}$$

$$g(x) = 4x + 3 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $f(3)$ (ii) $g(6)$ (iii) $f(10)$

(iv) $g(16)$ (v) $ff(4)$ (vi) $gg(5)$

(vii) $fg(1)$ (viii) $fg(0)$ (ix) $gf(0)$

[9 marks]

Question 2

Let m and n be the functions;

$$m(x) = x^2 - 75 \quad x \in \mathbb{R}$$

$$n(x) = \frac{4x}{5} \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $m(1)$

(ii) $n(15)$

(iii) $m(-1)$

(iv) $n(-15)$

(v) $m(m(9))$

(vi) $n(n(50))$

(vii) $m(n(0))$

(viii) $m(n(10))$

(ix) $m(n(m(10)))$

[9 marks]

Question 3

Let s and t be the functions;

$$s(x) = 6x + 1 \quad x \in \mathbb{R}$$

$$t(x) = (x + 1)^2 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $s(8)$ (ii) $t(6)$ (iii) $s(9)$

(iv) $t(-4)$ (v) $s \circ s(1)$ (vi) $t \circ t(2)$

(vii) $s \circ t(9)$ (viii) $s \circ t(2)$ (ix) $t \circ s(-1)$

[9 marks]

Question 4

Let two functions, m and n , be;

$$m(x) = 10x - 8 \quad x \in \mathbb{R}$$

$$n(x) = 100 - x^2 \quad x \in \mathbb{R}$$

Find each of the following;

(i) $m(3)$

(ii) $n(7)$

(iii) $n(-6)$

(iv) $m \circ n(8)$

(v) $m \circ m(4)$

(vi) $n \circ m(2)$

[9 marks]

Question 5

Let two functions, s and t , be;

$$s(x) = x^2 + x \quad x \in \mathbb{R}$$

$$t(x) = 3x - 2 \quad x \in \mathbb{R}$$

Find each of the following;

(i) $s(5)$ (ii) $s \circ s(1)$ (iii) $t \circ t \circ t(1)$

(iv) $t \circ s(10)$ (v) $s \circ t(3)$ (vi) $t \circ s(-1)$

[9 marks]

Question 6

If $f(x) = \sqrt{2x + 11}$, $x \in \mathbb{R}$, $x \geq -\frac{11}{2}$

and $g(x) = 5x$, $x \in \mathbb{R}$

what is;

(i) $f(7)$ (ii) $gf(19)$ (iii) $fg(11)$

(iv) $fg(-0.2)$ (v) $gf\left(-\frac{11}{2}\right)$ (vi) $fg(x)$

[9 marks]

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GCSE Mathematics

Functions I

Question 1

Find; **(i)** $f(2)$ **(ii)** $f(-1)$

Question 2

Find; **(i)** $g(4)$ **(ii)** $g(-9)$

Question 3

$$f(x) = \frac{16}{x}, \quad x \in \mathbb{R}, \quad x \neq 0$$

Remember : $gf(16)$ means put 16 into function f first, then into function g .

(i) $gf(16)$ (ii) $fg(2)$ (iii) $ggf(2)$

$$\text{(iv)} \quad ffg(14) \qquad \text{(v)} \quad gfg(0) \qquad \text{(vi)} \quad fgf(8)$$

[6 marks]

Question 4

Let two functions, e and z , be;

$$e(x) = (x + 3)^2, \quad x \in \mathbb{R}$$

$$\text{and } z(x) = 7x - 4, \quad x \in \mathbb{R}$$

Find each of the following;

(i) $e(-5)$ (ii) $e z(2)$ (iii) $z z(-1)$

(iv) $z e e(-2)$ (v) $z z(x)$ (vi) $e z(x)$

[6 marks]

Question 5

Let two functions, f and g be,

$$f(x) = x^2, \quad x \in \mathbb{R}$$

$$\text{and } g(x) = 50 - x, \quad x \in \mathbb{R}$$

Work out;

(i) $f(-5)$ (ii) $g(-5)$ (iii) $f g(10)$

(iv) $g f(10)$ (v) $f f f(2)$ (vi) $g g g g(7)$

(vii) Find $f g(x)$ and write your answer without brackets.

(viii) By trial and improvement find a value of x such that $f(x) = g(x)$

[8 marks]

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3.5 Answers

GCSE Mathematics Functions I

3.5.1 Solutions to 3.2 A Numerical Example

(i)	28	(ii)	11	(iii)	− 4
(iv)	5				

[1, 1, 2, 2 marks]

3.5.2 Solutions to 3.3 Exercise

Answer 1

(i)	3	(ii)	27	(iii)	38
(iv)	67	(v)	28	(vi)	95
(vii)	23	(viii)	3	(ix)	− 45

[9 marks]

Answer 2

(i)	− 74	(ii)	12	(iii)	− 74
(iv)	− 12	(v)	− 39	(vi)	32
(vii)	− 75	(viii)	− 11	(ix)	325

[9 marks]

Answer 3

(i)	49	(ii)	49	(iii)	55
(iv)	9	(v)	43	(vi)	100
(vii)	601	(viii)	55	(ix)	16

[9 marks]

Answer 4

(i)	22	(ii)	51	(iii)	64
(iv)	352	(v)	312	(vi)	− 44

[9 marks]

Answer 5

(i)	30	(ii)	42	(iii)	1
(iv)	328	(v)	56	(vi)	− 2

[9 marks]

Answer 6

(i)	5	(ii)	35	(iii)	11
(iv)	3	(v)	0	(vi)	$\sqrt{10x + 11}$

[9 marks]

3.5.3 Solutions to 3.4 Homework

GCSE Mathematics Functions I

Answer 1

(i) $\frac{1}{2}$

(ii) $\frac{1}{5}$

[4 marks]

Answer 2

(i) 6

(ii) 6

[4 marks]

Answer 3

(i) 3

(ii) 4

(iii) 12

(iv) 16

(v) 10

(vi) 4

[6 marks]

Answer 4

(i) 4

(ii) 169

(iii) -81

(iv) 108

(v) $49x - 32$

(vi) $49x^2 - 14x + 1$

[6 marks]

Answer 5

(i) 25

(ii) 55

(iii) 1600

(iv) -50

(v) 256

(vi) 7

[3 marks]

(vii) $2500 - 100x + x^2$

[2 marks]

(viii)

The exact answer will be given by,

$$x^2 = 50 - x$$

$$x^2 + x - 50 = 0$$

$$x = \frac{-1 \pm \sqrt{201}}{2}$$

$$x = 6.5887, 7.5887$$

[3 marks]

4.1 What was the Input ?

In the work on functions so far, the questions have all revolved around the inputs of either a number or an algebraic expression into the function and deducing what would, in consequence, be the output.

With some functions, but not all, it's possible to reverse this process.

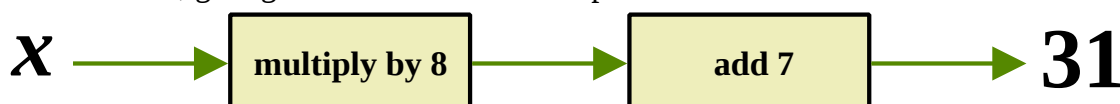
In other words, given what has come out, which could be a number or some algebra, what was it that went in ?

4.2 The Game of Opposites

Here is how such a question might be asked;

“If $f(x) = 8x + 7$, find x such that $f(x) = 31$. ($x \in \mathbb{R}$)”

A flow diagram is an excellent way of understanding exactly what this question is asking. As so often in maths, what is not known, in this case the input, is called x . The flow from left to right shows that the x is first multiplied by 8 and then 7 is added on, giving the number 31 as an output.



To work out x , instead of flowing left to right, the flow is reversed and goes from right to left. It's now a “Game of Opposites” !

The opposite of adding 7 is subtracting 7.

The opposite of multiplying by 8 is dividing by 8.

Here is the opposites flow diagram.



Decide, in your mind what the value of x must be.

A more mathematical way to do this question is as follows,

$$8x + 7 = 31$$

$$8x = 31 - 7 \quad \text{Subtract 7 from both sides}$$

$$8x = 24$$

$$x = \frac{24}{8} \quad \text{Divide both sides by 8}$$

$$x = 3$$

Here is a second example. Try it yourself, then check with my answer over the page.

If $f(x) = 6x + 11$, find x such that $f(x) = 53$. ($x \in \mathbb{R}$)

$$\text{Solve, } 6x + 11 = 53$$

$$6x = 53 - 11$$

$$6x = 42$$

$$x = \frac{42}{6}$$

$$x = 7$$

4.3 Exercise

Try doing some questions using a flow diagram, and some using just algebra. The algebraic method always works but often the flow diagram provides is a much quicker and easier short cut to the answer.

Marks Available: 78

Question 1

If $f(x) = 4x + 13$, find x such that $f(x) = 41$ ($x \in \mathbb{R}$)

[3 marks]

Question 2

If $g(x) = 18 + 5x$, find x such that $g(x) = 103$ ($x \in \mathbb{R}$)

[3 marks]

Question 3

If $h(x) = 3(8x - 3)$, find x such that $h(x) = 135$ ($x \in \mathbb{R}$)

[3 marks]

Question 4

If $k(x) = \frac{5x}{4} + 7$, find x such that $k(x) = 37$ ($x \in \mathbb{R}$)

[3 marks]

Question 5

If $m(x) = \frac{3x}{5} - 1$, find x such that $m(x) = 32$ ($x \in \mathbb{R}$)

[3 marks]

Question 6

If $n(x) = 68 - 7x$, find x such that $n(x) = 26$ ($x \in \mathbb{R}$)

[3 marks]

Question 7

If $p(x) = 6(2x + 3)$, find x such that $p(x) = 48$ ($x \in \mathbb{R}$)

[3 marks]

Question 8

If $q(x) = 6(13 - x)$, find x such that $q(x) = 72$ ($x \in \mathbb{R}$)

[3 marks]

Question 9

If $r(x) = \frac{x + 17}{3}$, find x such that $r(x) = 40$ ($x \in \mathbb{R}$)

[3 marks]

Question 10

If $s(x) = \frac{3x + 2}{5}$, find x such that $s(x) = 10$ ($x \in \mathbb{R}$)

[3 marks]

Question 11

If $t(x) = 5x^3$, find x such that $t(x) = 320$ ($x \in \mathbb{R}$)

[3 marks]

Question 12

If $u(x) = \frac{21 - x}{4}$, find x such that $u(x) = 6$ ($x \in \mathbb{R}$)

[3 marks]

Question 13

If $v(x) = 14\sqrt{x} + 111$, $x \in \mathbb{R}$, $x \geq 0$, find x such that $v(x) = 181$

[3 marks]

Question 14

If $w(x) = \frac{3x - 5}{x - 4}$, $x \in \mathbb{R}$, $x \neq 4$, find x such that $w(x) = 4$

[3 marks]

Question 15

If $a(x) = \frac{\sqrt{x} + 7}{2}$, $x \in \mathbb{R}$, $x \geq 0$, find x such that $a(x) = 16$

[3 marks]

Question 16

If $b(x) = \frac{x+2}{3} + 9$, find x such that $b(x) = 14$ ($x \in \mathbb{R}$)

[3 marks]

Question 17

If $c(x) = \frac{105}{x}$, $x \in \mathbb{R}$, $x \neq 0$, find x such that $c(x) = 7$

[3 marks]

Question 18

If $d(x) = \frac{5x+1}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$, find x such that $d(x) = 4$

[3 marks]

Question 19

If $e(x) = \sqrt[3]{x} + 7$, find x such that $e(x) = 11$ ($x \in \mathbb{R}$)

[3 marks]

Question 20

If $f(x) = \frac{5x - 4}{x + 1}$, $x \in \mathbb{R}$, $x \neq -1$, find x such that $f(x) = 8$

[3 marks]

Question 21

If $g(x) = \frac{2}{x} + \frac{3}{x}$, $x \in \mathbb{R}$, $x \neq 0$, find x such that $g(x) = 20$

[3 marks]

Question 22

If $h(x) = \sqrt{13 + 3x}$, $x \in \mathbb{R}$, $x \geq -\frac{13}{3}$, find x such that $h(x) = 7$

[3 marks]

Question 23

If $k(x) = 29 - 24\sqrt{x}$, $x \in \mathbb{R}$, $x \geq 0$, find x such that $k(x) = 17$

[3 marks]

Question 24

If $p(x) = x^2 + x$, find all x such that $p(x) = 1$ ($x \in \mathbb{R}$)

Note: The equation $ax^2 + bx + c = 0$, where a, b and c are constants

has the solutions given by, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

[3 marks]

Question 25

If $m(x) = \frac{3}{x} + \frac{5}{2x}$, $x \in \mathbb{R}$, $x \neq 0$, find x such that $m(x) = 33$

[3 marks]

Question 26

If $n(x) = \frac{1}{3x} + \frac{3}{5x}$, $x \in \mathbb{R}$, $x \neq 0$, find x such that $n(x) = 7$

[3 marks]

4.4 Answers

GCSE Mathematics Functions I

4.4.1 Solutions to 4.3 Exercise

Each question is worth 3 marks giving 78 marks in total.

Answer 1 7	Answer 2 17	Answer 3 6	Answer 4 24	Answer 5 55
Answer 6 6	Answer 7 2.5	Answer 8 1	Answer 9 103	Answer 10 16
Answer 11 4	Answer 12 − 3	Answer 13 25	Answer 14 11	Answer 15 625
Answer 16 13	Answer 17 15	Answer 18 3	Answer 19 64	Answer 20 − 4
Answer 21 0.25	Answer 22 12	Answer 23 0.25	Answer 24 $\frac{1}{6}$	Answer 25 $\frac{2}{15}$

Answer 26

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

approximately 0.62 or − 1.62

Lesson 5

GCSE Mathematics Functions I

5.1 Composite Functions (Algebra Questions)

Two functions, f and g given by,

$$f(x) = 5x - 2 \quad x \in \mathbb{R}$$

$$g(x) = 4x + 1 \quad x \in \mathbb{R}$$

What is $fg(2)$?



[2 marks]

In this Lesson the function inputs will be chunks of algebra rather than just numbers.

Work out $fg(2z)$



[2 marks]

Now try to work out $fg(3z + 1)$



[2 marks]

My answer is over the page.
Turn over to see if you concur.

$$\begin{aligned}
 f \circ g (3z + 1) &= f (4 (3z + 1) + 1) && \text{because } g(x) = 4x + 1 \\
 &= f (12z + 4 + 1) \\
 &= f (12z + 5) \\
 &= 5 (12z + 5) - 2 && \text{because } f(x) = 5x - 2 \\
 &= 60z + 25 - 2 \\
 &= 60z + 23
 \end{aligned}$$

[2 marks]

5.2 Exercise

Marks Available: 80

Question 1

Let p and q be the functions;

$$p(x) = 3x + 7 \quad x \in \mathbb{R}$$

$$q(x) = 10x + 9 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $p \circ q (0)$

(ii) $p \circ q (1)$

(iii) $p \circ q (5z)$

(iv) $p \circ q (2z + 3)$

(v) $q \circ q (-1)$

(vi) $p \circ p \circ p (-3)$

[6 marks]

Question 2

Let h and d be the functions;

$$h(x) = 3x + 7 \quad x \in \mathbb{R}$$

$$d(x) = 5x + 9 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $h \, d \, (4)$

(ii) $h \, d \, (-1)$

(iii) $h \, d \, (4z)$

(iv) $h \, d \, (3z + 3)$

[4 marks]

Question 3

Let f and g be the functions;

$$f(x) = 6x - 5 \quad x \in \mathbb{R}$$

$$g(x) = 2x + 11 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $g \, f \, (0)$

(ii) $g \, f \, (2)$

(iii) $g \, f \, (3z)$

(iv) $g \, f \, (5z + 1)$

[4 marks]

Question 4

$$f(x) = x + 3 \quad x \in \mathbb{R}$$

$$g(x) = 4x \quad x \in \mathbb{R}$$

Find $fg(x)$

[2 marks]

Question 5

$$f(x) = 3x + 2 \quad x \in \mathbb{R}$$

$$g(x) = 5x - 4 \quad x \in \mathbb{R}$$

Find an expression for $gf(x)$ that does not contain any brackets.

[2 marks]

Question 6

$$f(x) = 7x - 5 \quad x \in \mathbb{R}$$

$$g(x) = 10 - x \quad x \in \mathbb{R}$$

Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

Question 7

$$f(x) = x^2 + 5 \quad x \in \mathbb{R}$$

$$g(x) = 2x - 1 \quad x \in \mathbb{R}$$

Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

Question 8

$$f(x) = 7x - 6 \quad x \in \mathbb{R}$$

$$g(x) = \frac{3}{x+4} \quad x \in \mathbb{R}, x \neq -4$$

- (i) Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

- (ii) Find an expression for $gf(x)$ that does not contain any brackets.

[2 marks]

Question 9

$$f(x) = 5x + 2 \quad x \in \mathbb{R}$$

$$g(x) = 4x - 9 \quad x \in \mathbb{R}$$

- (i) Calculate $fg(3)$

[2 marks]

- (ii) Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

- (iii) Now use your part (ii) formula to calculate $fg(3)$ again.
Check it agrees with your answer to part (i) !

[1 mark]

Question 10

$$f(x) = x^2 - 6 \quad x \in \mathbb{R}$$

$$g(x) = 3x + 1 \quad x \in \mathbb{R}$$

- (i) Show carefully that $fg(x) = 9x^2 + 6x - 5$

[2 marks]

- (ii) Hence solve the equation $fg(x) = 43$

[2 marks]

Question 11

$$f(x) = 2x^2 + 3x - 4 \quad x \in \mathbb{R}$$

$$g(x) = x - 5 \quad x \in \mathbb{R}$$

- (i) Show carefully that $fg(x) = 2x^2 - 17x + 31$

[2 marks]

- (ii) Hence solve the equation $fg(x) = 10 - 4x$

[2 marks]

Question 12

Let u and v be the functions;

$$u(x) = x^2 - 5 \quad x \in \mathbb{R}$$

$$v(x) = 5x + 2 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $v \, u \, (3)$

(ii) $v \, u \, (2)$

(iii) $v \, u \, (5z)$

(iv) $v \, u \, (2z + 1)$ **HINT : FOIL**

[8 marks]

Question 13

Let m and n be the functions,

$$m(x) = 9x - 5 \quad x \in \mathbb{R}$$

$$n(x) = \sqrt{x - 7} \quad x \in \mathbb{R}, \, x \geq 7$$

Evaluate each of the following,

(i) $m \, n \, (8)$

(ii) $m \, n \, (56)$

(iii) $m \, n \, (z^2 + 7)$

(iv) $m \, n \, (4z^2 + 7)$

[8 marks]

Question 14

$$f(x) = 3x + 7 \quad x \in \mathbb{R}$$

$$g(x) = \frac{5}{x - 2} \quad x \in \mathbb{R}, x \neq 2$$

- (i) Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

- (ii) Find an expression for $gf(x)$ that does not contain any brackets.

[2 marks]

Question 15

$$f(x) = 4x + 5 \quad x \in \mathbb{R}$$

$$g(x) = 3x - 7 \quad x \in \mathbb{R}$$

- (i) Calculate $fg(2)$

[2 marks]

- (ii) Find an expression for $fg(x)$ that does not contain any brackets.

[2 marks]

- (iii) Now use your part (ii) formula to calculate $fg(2)$ again.
Check it agrees with your answer to part (i) !

[1 mark]

Question 16

$$f(x) = x^2 + 8 \quad x \in \mathbb{R}$$

$$g(x) = 2x - 5 \quad x \in \mathbb{R}$$

- (i) Show carefully that $fg(x) = 4x^2 - 20x + 33$

[2 marks]

- (ii) Hence solve the equation $fg(x) = 17$

[2 marks]

Question 17

$$f(x) = 3x^2 - 2x + 5 \quad x \in \mathbb{R}$$

$$g(x) = x + 4 \quad x \in \mathbb{R}$$

- (i) Show carefully that $fg(x) = 3x^2 + 22x + 45$

[2 marks]

- (ii) Hence solve the equation $fg(x) = 27x + 43$

[2 marks]

Question 18

Let f and g be the functions;

$$f(x) = 7 - 6x \quad x \in \mathbb{R}$$

$$g(x) = 5x + 7 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $fg(-1)$

(ii) $fg(-0.2)$

(iii) $fg(4z + 1)$

(iv) $fg(1 - z^2)$

[8 marks]

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(Questions 4 to 11, and 14 to 17 JCA)

5.3 Answers

GCSE Mathematics Functions I

5.3.1 Solutions to 5.1 Introductory Example)

 $fg(2) = 43$

[2 marks]

 $fg(2z) = f(8z + 1)$
 $= 40z + 3$

[2 marks]

 $fg(3z + 1) = f(12z + 5)$
 $= 60z + 23$

[2 marks]

5.3.2 Solutions to 5.2 Exercise

Answer 1

(i) 34

(ii) 64

(iii) $150z + 34$

(iv) $60z + 124$

(v) -1

(vi) 10

[6 marks]

Answer 2

(i) 94

(ii) 19

(iii) $60z + 34$

(iv) $45z + 79$

[4 marks]

Answer 3

(i) 1

(ii) 25

(iii) $36z + 1$

(iv) $60z + 13$

[4 marks]

Answer 4

$4x + 3$

Answer 5

$15x + 6$

Answer 6

$65 - 7x$

[2, 2, 2 marks]

Answer 7

$4x^2 - 4x + 6$

[2 marks]

Answer 8

(i) $\frac{21}{x+4} - 6$

(ii) $\frac{3}{7x-2}$

[2, 2 marks]

Answer 9

(i) 17

(ii) $20x - 43$

(iii) 17 again

[2, 2, 1 marks]

Answer 10

(i) $9x^2 + 6x - 5$

(ii) $2, \frac{8}{3}$

[2, 2 marks]

Answer 11

- (i) show (ii) 3.5, 3

[2, 2 marks]

Answer 12

- (i) 22 (ii) -3
 (iii) $125z^2 - 23$ (iv) $20z^2 + 20z - 18$

[8 marks]

Answer 13

- (i) 4 (ii) 58
 (iii) $9z - 5$ (iv) $18z - 5$

[8 marks]

Answer 14

- (i) $\frac{15}{x-2} + 7$ (ii) $\frac{5}{3x+5}$

[2, 2 marks]

Answer 15

- (i) 1
 (ii) $12x - 23$
 (iii) 1 again

[2, 2, 1 marks]

Answer 16

- (i) $4x^2 - 20x + 33$
 (ii) 1, 4

[2, 2 marks]

Answer 17

- (i) show (ii) $1, \frac{2}{3}$

[2, 2 marks]

Answer 18

- (i) -5 (ii) -29
 (iii) $-120z - 65$ (iv) $30z^2 - 65$

[8 marks]

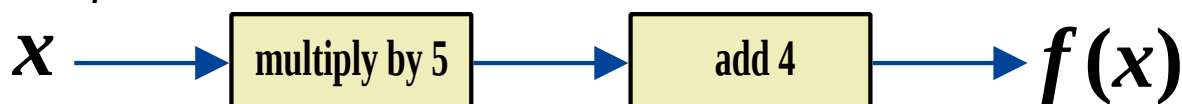
6.1 Inverse Functions from Flow Diagrams

Study the flow diagram below.

We're going to write down two algebraic expressions suggested by this flow diagram.

Firstly, what function does the flow diagram represent ?

Secondly, if you were to go backwards through the flow diagram, what *inverse function* would be obtained ?



Teaching Video : [http://www.NumberWonder.co.uk/Video/v9002\(6\).mp4](http://www.NumberWonder.co.uk/Video/v9002(6).mp4)



$$f(x) =$$

[1 mark]



$$f^{-1}(x) =$$

[1 mark]

Having extracted the algebra we need from the flow diagram we can now answer some easy questions. Determine the value of,

(i) $f(8)$

(ii) $f^{-1}(29)$



(iii) $f(11)$

(iv) $f^{-1}(44)$



Explain the connection between (i) and (iv)

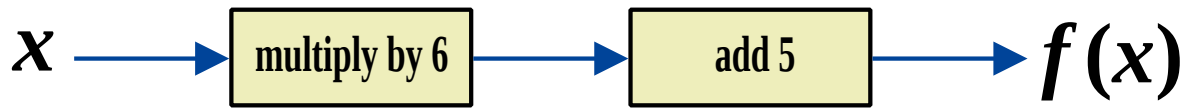


[5 marks]

6.2 Exercise

Marks Available: 54

Question 1



(a) Write down (i) $f(x)$

(ii) $f^{-1}(x)$

[2 marks]

(b) Determine the value of,

(i) $f(4)$

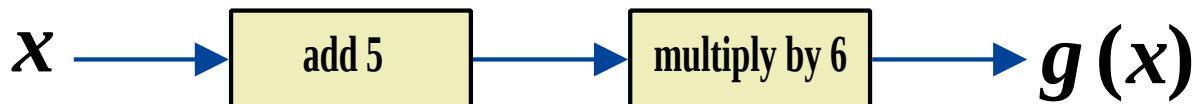
(ii) $f^{-1}(41)$

(iii) $f(9)$

(iv) $f^{-1}(71)$

[4 marks]

Question 2



(a) Write down (i) $g(x)$

(ii) $g^{-1}(x)$

[2 marks]

(b) Determine the value of,

(i) $g(4)$

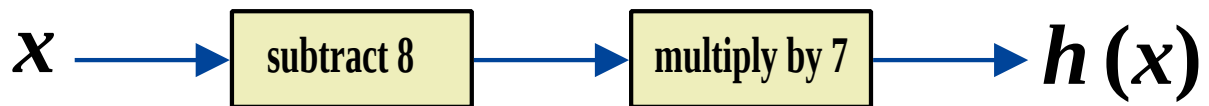
(ii) $g^{-1}(48)$

(iii) $g(15)$

(iv) $g^{-1}(72)$

[4 marks]

Question 3



- (a) Write down
- (i) $h(x)$
- (ii) $h^{-1}(x)$

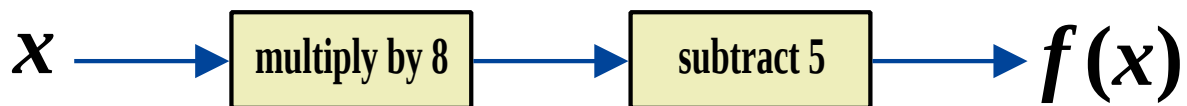
[2 marks]

- (b) Determine the value of,

- (i) $h(13)$ (ii) $h^{-1}(77)$
- (iii) $h(28)$ (iv) $h^{-1}(490)$

[4 marks]

Question 4



- (a) Write down
- (i) $f(x)$
- (ii) $f^{-1}(x)$

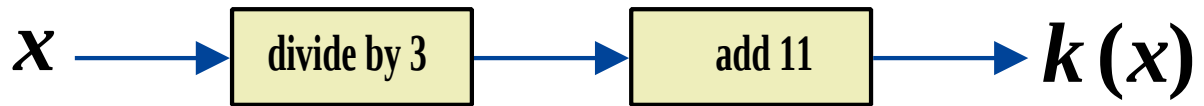
[2 marks]

- (b) Determine the value of,

- (i) $f(6)$ (ii) $f^{-1}(19)$
- (iii) $f(1.5)$ (iv) $f^{-1}(83)$

[4 marks]

Question 5



- (a) Write down
- (i) $k(x)$
- (ii) $k^{-1}(x)$

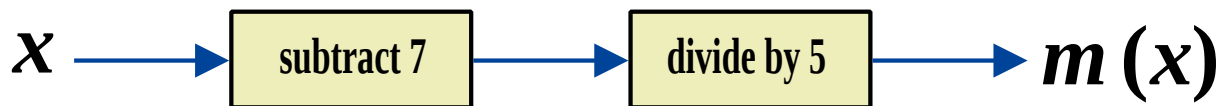
[2 marks]

- (b) Determine the value of,

- (i) $k(66)$ (ii) $k^{-1}(15)$
- (iii) $k(21)$ (iv) $k^{-1}(19)$

[4 marks]

Question 6



- (a) Write down
- (i) $m(x)$
- (ii) $m^{-1}(x)$

[2 marks]

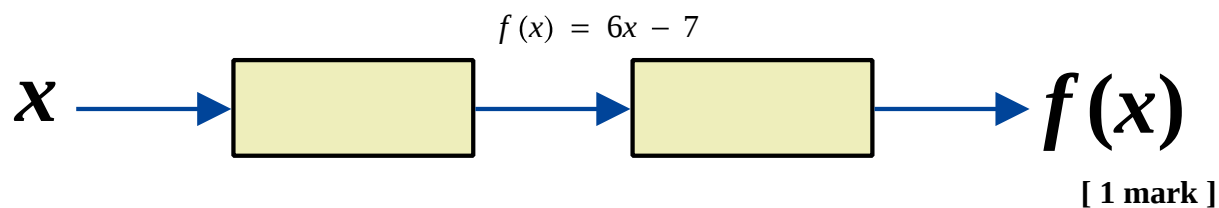
- (b) Determine the value of,

- (i) $m(22)$ (ii) $m^{-1}(1)$
- (iii) $m(37)$ (iv) $m^{-1}(8)$

[4 marks]

Question 7

- (a) Fill in the flow diagram for the function



- (b) Write down $f^{-1}(x)$

[1 mark]

- (c) Determine the value of,

(i) $f(10)$

(ii) $f^{-1}(11)$

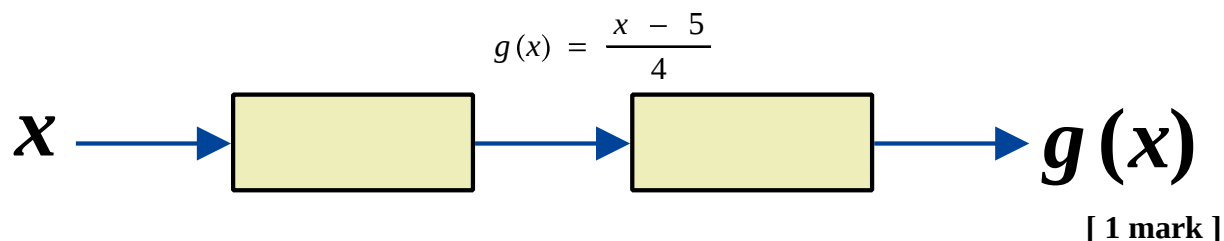
(iii) $f(9)$

(iv) $f^{-1}(29)$

[4 marks]

Question 8

- (a) Fill in the flow diagram for the function



- (b) Write down $g^{-1}(x)$

[1 mark]

- (c) Determine the value of,

(i) $g(13)$

(ii) $g^{-1}(11)$

(iii) $g(37)$

(iv) $g^{-1}(7)$

[4 marks]

Question 9

Consider the function, $f(x) = 8x + 3$

Find an expression for the inverse function $f^{-1}(x)$

HINT : Draw a flow diagram.

[3 marks]

Question 10

Consider the function, $g(x) = \frac{x}{7} + 3$

Find an expression for the inverse function $g^{-1}(x)$

HINT : Draw a flow diagram.

[3 marks]

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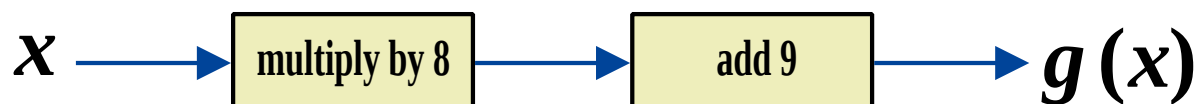
Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk

6.3 Homework

GCSE Mathematics Functions I

Marks Available: 21

Question 1



- (a) Write down
- (i) $g(x)$
- (ii) $g^{-1}(x)$

[2 marks]

- (b) Determine the value of,

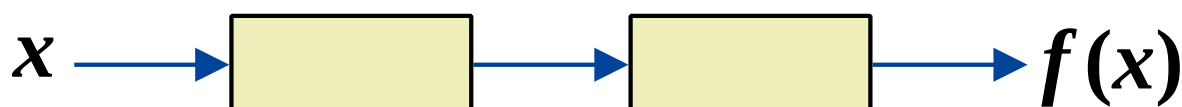
- (i) $g(5)$
- (ii) $g^{-1}(33)$
- (iii) $g(0.5)$
- (iv) $g^{-1}(1)$

[4 marks]

Question 2

- (a) Fill in the flow diagram for the function

$$f(x) = 3x - 7$$



[1 mark]

- (b) Write down $f^{-1}(x)$

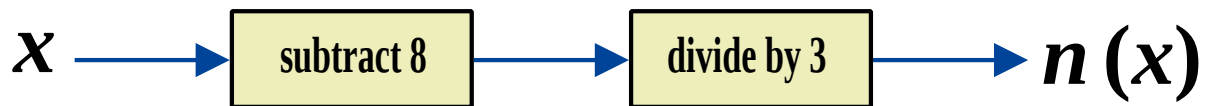
[1 mark]

- (c) Determine the value of,

- (i) $f(12)$
- (ii) $f^{-1}(11)$
- (iii) $f(1)$
- (iv) $f^{-1}(29)$

[4 marks]

Question 3



(a) Write down (i) $n(x)$

(ii) $n^{-1}(x)$

[2 marks]

(b) Determine the value of,

(i) $n(23)$

(ii) $n^{-1}(9)$

(iii) $n(41)$

(iv) $n^{-1}(-2)$

[4 marks]

Question 4

Consider the function, $k(x) = 9x + 7$ $x \in \mathbb{R}$

Find an expression for the inverse function $k^{-1}(x)$

HINT : Draw a flow diagram.

[3 marks]

6.4 Answers

GCSE Mathematics Functions I

6.4.1 Solutions to the Introductory Example

$$f(x) = 5x + 4 \qquad f^{-1}(x) = \frac{x - 4}{5}$$

$$(i) \quad 44 \qquad (ii) \quad 5$$

$$(iii) \quad 59 \qquad (iv) \quad 8$$

8 in gives 44 out so in the inverse 44 in gets back to the 8.

[7 marks]

6.4.2 Solutions to 6.2 Exercise

Answer 1

$$(a) \quad (i) \quad 6x + 5 \qquad (ii) \quad \frac{x - 5}{6}$$

$$(b) \quad (i) \quad 29 \qquad (ii) \quad 6$$
$$(iii) \quad 59 \qquad (iv) \quad 11$$

[6 marks]

Answer 2

$$(a) \quad (i) \quad 6(x + 5) = 6x + 30 \qquad (ii) \quad \frac{x}{6} - 5$$

$$(b) \quad (i) \quad 54 \qquad (ii) \quad 3$$
$$(iii) \quad 120 \qquad (iv) \quad 7$$

[6 marks]

Answer 3

$$(a) \quad (i) \quad 7(x - 8) = 7x - 56 \qquad (ii) \quad \frac{x}{7} + 8$$

$$(b) \quad (i) \quad 35 \qquad (ii) \quad 19$$
$$(iii) \quad 140 \qquad (iv) \quad 78$$

[6 marks]

Answer 4

$$(a) \quad (i) \quad 8x - 5 \qquad (ii) \quad \frac{x + 5}{8}$$

$$(b) \quad (i) \quad 43 \qquad (ii) \quad 3$$
$$(iii) \quad 7 \qquad (iv) \quad 11$$

[6 marks]

Answer 5

$$(a) \quad (i) \quad \frac{x}{3} + 11 \qquad (ii) \quad 3(x - 11) = 3x - 33$$

$$(b) \quad (i) \quad 33 \qquad (ii) \quad 12$$
$$(iii) \quad 18 \qquad (iv) \quad 24$$

[6 marks]

Answer 6

(a) (i) $\frac{x - 7}{5}$

(ii) $5x + 7$

(b) (i) 3

(ii) 12

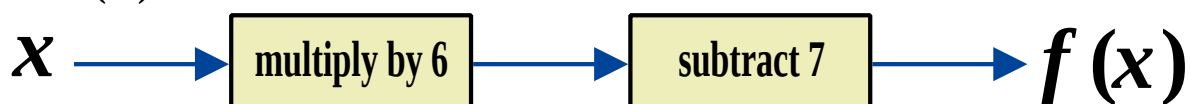
(iii) 6

(iv) 47

[6 marks]

Answer 7

(a)



(b) $\frac{x + 7}{6}$

(c) (i) 53

(ii) 3

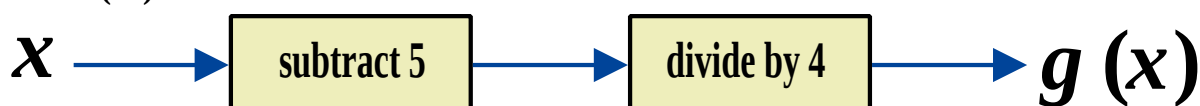
(iii) 47

(iv) 6

[6 marks]

Answer 8

(a)



(b) $4x + 5$

(c) (i) 2

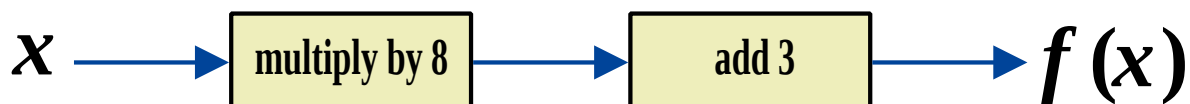
(ii) 49

(iii) 8

(iv) 33

[6 marks]

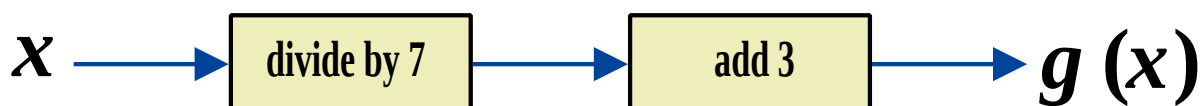
Answer 9



$\frac{x - 3}{8}$

[3 marks]

Answer 10



$7(x - 3)$ or $7x - 21$

[3 marks]

6.4.3 Solutions to 6.3 Homework

GCSE Mathematics Functions I

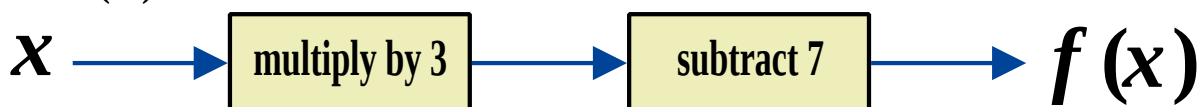
Answer 1

- | | | | | |
|-----|-------|----------|------|-------------------|
| (a) | (i) | $8x + 9$ | (ii) | $\frac{x - 9}{8}$ |
| (b) | (i) | 49 | (ii) | 3 |
| | (iii) | 13 | (iv) | -1 |

[6 marks]

Answer 2

(a)



(b) $\frac{x + 7}{3}$

- (c) (i) 29
(iii) -4

- (ii) 6
(iv) 12

[6 marks]

Answer 3

(a) (i) $\frac{x - 8}{3}$

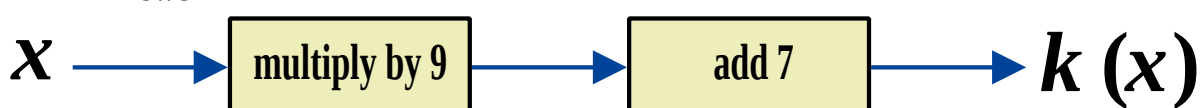
(ii) $3x + 8$

- (b) (i) 5
(iii) 11

- (ii) 35
(iv) 2

[6 marks]

Answer 4



$$k^{-1}(x) = \frac{x - 7}{9}$$

[3 marks]

7.1 More involved Inverse Functions from Flow Diagrams

Previously we've considered flow diagrams involving two action boxes.

In this Lesson, more than two action boxes may be involved.

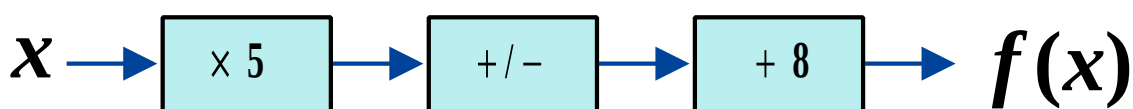
Previously the action boxes contained the arithmetical operations $+$, $-$, $\times$ and $\div$.

In this Lesson, other arithmetical operations may be involved.

Example

The following flow diagram contains the action boxes,

- ◇ Multiply by 5
- ◇ Change sign
- ◇ Add 8



Write down the function described by the flow diagram, and also the inverse function.

Teaching Video : [http://www.NumberWonder.co.uk/Video/v9002\(7\).mp4](http://www.NumberWonder.co.uk/Video/v9002(7).mp4)



$$f(x) =$$

[1 mark]



$$f^{-1}(x) =$$

[1 mark]

Having extracted the algebra we need from the flow diagram we can now answer a couple of easy questions.

Determine the value of,

(i) $f(1)$

(ii) $f^{-1}(3)$



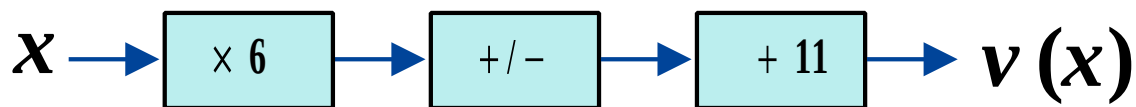
[1, 1 marks]

7.2 Exercise

Marks Available: 44

Question 1

The flow diagram action boxes are *Multiply by 6*, *Change sign* and *Add 11*.



(a) Write down (i) $v(x)$

(ii) $v^{-1}(x)$

[2 marks]

(b) Determine the value of,

(i) $v(1)$

(ii) $v^{-1}(23)$

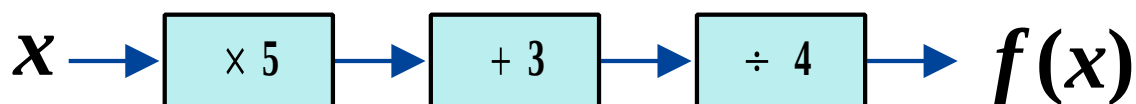
(iii) $v(-2)$

(iv) $v^{-1}(-1)$

[4 marks]

Question 2

The flow diagram action boxes are *Multiply by 5*, *Add 3* and *Divide by 4*.



(a) Write down (i) $f(x)$

(ii) $f^{-1}(x)$

[2 marks]

(b) Determine the value of,

(i) $f(5)$

(ii) $f^{-1}(12)$

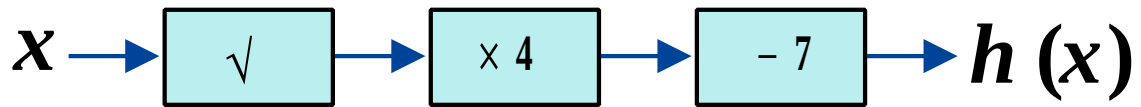
(iii) $f(-3)$

(iv) $f^{-1}(-8)$

[4 marks]

Question 3

The flow diagram action boxes are *Square root*, *Multiply by 4* and *Subtract 7*.



(a) Write down (i) $h(x)$

(ii) $h^{-1}(x)$

[2 marks]

(b) Determine the value of,

(i) $h(25)$ (ii) $h^{-1}(5)$

(iii) $h(81)$ (iv) $h^{-1}(57)$

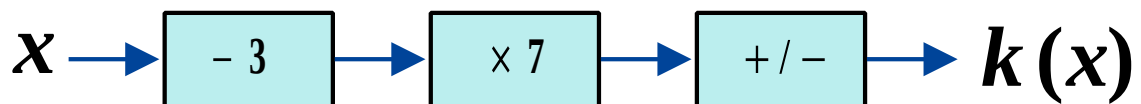
[4 marks]

(c) The domain of h is restricted such that $h \geq 0$.
Why?

[1 mark]

Question 4

The flow diagram action boxes are *Subtract 3*, *Multiply by 7* and *Change sign*.



(a) Write down (i) $k(x)$

(ii) $k^{-1}(x)$

[2 marks]

(b) Determine the value of,

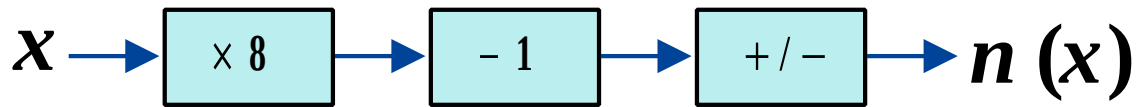
(i) $k(2)$ (ii) $k^{-1}(7)$

(iii) $k(5)$ (iv) $k^{-1}(-21)$

[4 marks]

Question 5

The flow diagram action boxes are *Multiply by 8*, *Subtract 1* and *Change sign*.



- (a) Write down
- (i) $n(x)$
- (ii) $n^{-1}(x)$

[2 marks]

- (b) Determine the value of,

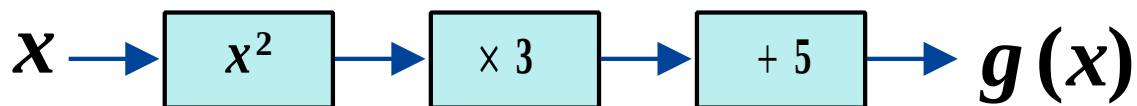
- (i) $n(3)$
- (ii) $n^{-1}(1)$
- (iii) $n(-1)$
- (iv) $n^{-1}(-7)$

[4 marks]

Question 6

The flow diagram action boxes are *Square*, *Multiply by 3* and *Add 5*.

The domain is restricted so that only positive real numbers are to be considered.



- (a) Write down
- (i) $g(x)$
- (ii) $g^{-1}(x)$

[2 marks]

- (b) Determine the value of,

- (i) $g(10)$
- (ii) $g^{-1}(8)$
- (iii) $g(5)$
- (iv) $g^{-1}(53)$

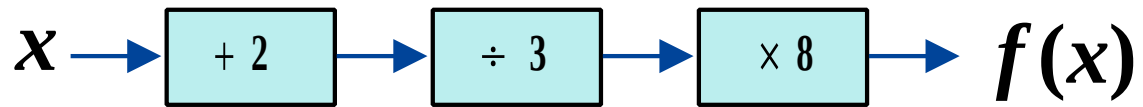
[4 marks]

- (c) Why was the domain in this question restricted ?

[1 mark]

Question 7

The flow diagram action boxes are *Add 2*, *Divide by 3* and *Multiply by 8*.



(a) Write down (i) $s(x)$

(ii) $s^{-1}(x)$

[2 marks]

(b) Determine the value of,

(i) $s(1)$

(ii) $s^{-1}(16)$

(iii) $s(7)$

(iv) $s^{-1}(72)$

[4 marks]

7.3 Answers

GCSE Mathematics Functions I

7.3.1 Solutions (Introductory example)

$$f(x) = -5x + 8$$

$$f^{-1}(x) = \frac{-x + 8}{5}$$

(i) $f(1) = 3$

(ii) $f^{-1}(3) = 1$

[4 marks]

7.3.2 Solutions (7.2 Exercise)

Answer 1

(a) (i) $v(x) = -6x + 11$

(ii) $v^{-1}(x) = \frac{-x + 11}{6}$

(b) (i) 5
(iii) 23

(ii) -2
(iv) 2

[6 marks]

Answer 2

(a) (i) $f(x) = \frac{5x + 3}{4}$

(ii) $f^{-1}(x) = \frac{4x - 3}{5}$

(b) (i) 7
(iii) -3

(ii) 9
(iv) -7

[6 marks]

Answer 3

(a) (i) $h(x) = 4\sqrt{x} - 7$

(ii) $h^{-1}(x) = \left(\frac{x + 7}{4}\right)^2$

(b) (i) 13
(iii) 29

(ii) 9
(iv) 256

[6 marks]

Answer 4

(a) (i) $k(x) = -7x + 21$

(ii) $k^{-1}(x) = -\frac{x}{7} + 3$

(b) (i) 7
(iii) -14

(ii) 2
(iv) 6

[6 marks]

Answer 5

(a)	(i)	$n(x) = -8x + 1$	(ii)	$n^{-1}(x) = \frac{-x + 1}{8}$
(b)	(i)	-23	(ii)	0
	(iii)	9	(iv)	1

[6 marks]**Answer 6**

(a)	(i)	$g(x) = 3x^2 + 5$	(ii)	$g^{-1}(x) = \sqrt{\frac{x-5}{3}}$
(b)	(i)	305	(ii)	1
	(iii)	80	(iv)	4
(c)	Otherwise not know if the reverse of x^2 is $+\sqrt{x}$ or $-\sqrt{x}$ Furthermore, the domain of the inverse is $x \geq 5$ to avoid the taking of the square root of a negative number.			

[7 marks]**Answer 7**

(a)	(i)	$s(x) = \frac{8(x+2)}{3}$	(ii)	$s^{-1}(x) = \frac{3x}{8} - 2$
(b)	(i)	8	(ii)	4
	(iii)	24	(iv)	25

[6 marks]

Lesson 8

GCSE Mathematics Functions I

8.1 Inverse Functions without Flow Diagrams

There is a more mathematical method that can be used to find the inverse of a function. Using it means solutions are less cluttered.

There are also functions which cannot easily be represented by flow diagrams. The mathematical method allows their inverses to be found also.

8.2 Example

$$f(x) = \frac{5}{2x} + 6, \quad x \in \mathbb{R}, \quad x \neq 0$$

- (i) Determine $f(5)$ (ii) Find $f^{-1}(x)$

Teaching Video : [http://www.NumberWonder.co.uk/Video/v9002\(8\).mp4](http://www.NumberWonder.co.uk/Video/v9002(8).mp4)



[4 marks]

- (iii) Use your part (i) answer to check your part (ii) answer.

[1 mark]

8.3 Exercise

Marks Available: 48

Question 1

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(i) $f(x) = 7 - 3x$

(ii) $g(x) = 8x + 3$

(iii) $h(x) = \frac{1}{2}x + 5$

(iv) $k(x) = \frac{x}{5} - 4$

[12 marks]

Question 2

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(i) $m(x) = 2(3 - 5x)$ (ii) $n(x) = \frac{x - 8}{3}$

(iii) $p(x) = \frac{7x}{4}$ (iv) $q(x) = \frac{3x}{5} + 4$

[12 marks]

Question 3

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(i) $r(x) = \frac{x}{11}$

(ii) $s(x) = \frac{1}{4x}, \quad x \neq 0$

(iii) $t(x) = \frac{3}{2x}, \quad x \neq 0$

(iv) $u(x) = \frac{1}{x} + 4, \quad x \neq 0$

[12 marks]

Question 4

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(i) $v(x) = 9 - \frac{1}{x}, \quad x \neq 0$ (ii) $w(x) = 5 - \frac{3}{x}, \quad x \neq 0$

(iii) $z(x) = \frac{2}{x} + 5, \quad x \neq 0$ (iv) $a(x) = \frac{x+4}{x-3}, \quad x \neq 3$

[12 marks]

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8.4 Answers

GCSE Mathematics Functions I

8.4.1 Solution to 8.2 Example

(i)

$$f(x) = \frac{5}{2x} + 6$$
$$f(5) = \frac{5}{2 \times 5} + 6$$
$$= 6.5$$

[1 mark]

(ii)

$$f(x) = \frac{5}{2x} + 6$$

rename $f(x)$ as y

$$y = \frac{5}{2x} + 6$$

swap $x \Leftrightarrow y$

$$x = \frac{5}{2y} + 6$$

make y the subject

$$x - 6 = \frac{5}{2y}$$
$$\frac{1}{x - 6} = \frac{2y}{5}$$
$$2y = \frac{5}{x - 6}$$
$$y = \frac{5}{2x - 12}$$

rename y as $f^{-1}(x)$

$$f^{-1}(x) = \frac{5}{2x - 12}$$

[3 marks]

(iii)

$$f^{-1}(x) = \frac{5}{2x - 12}$$
$$f^{-1}(6.5) = \frac{5}{2 \times 6.5 - 12}$$
$$= 5 \text{ as it should}$$

[1 mark]

8.4.2 Solution to 8.3 Exercise

Answer 1

(i) $f(x) = 7 - 3x$

$$y = 7 - 3x$$

$$\text{swap } x \Leftrightarrow y$$

$$x = 7 - 3y$$

$$3y = 7 - x$$

$$y = \frac{7 - x}{3}$$

$$f^{-1}(x) = \frac{7 - x}{3}$$

(ii) $g(x) = 8x + 3$

$$y = 8x + 3$$

$$\text{swap } x \Leftrightarrow y$$

$$x = 8y + 3$$

$$8y = x - 3$$

$$y = \frac{x - 3}{8}$$

$$g^{-1}(x) = \frac{x - 3}{8}$$

(iii)

$$h(x) = \frac{1}{2}x + 5$$

$$y = \frac{1}{2}x + 5$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{1}{2}y + 5$$

$$\frac{1}{2}y = x - 5$$

$$y = 2x - 10$$

$$h^{-1}(x) = 2x - 10$$

(iv)

$$k(x) = \frac{x}{5} - 4$$

$$y = \frac{x}{5} - 4$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{y}{5} - 4$$

$$\frac{y}{5} = x + 4$$

$$y = 5x + 20$$

$$k^{-1}(x) = 5x + 20$$

[12 marks]

Answer 2

(i)

$$m(x) = 2(3 - 5x)$$

$$y = 2(3 - 5x)$$

$$\text{swap } x \Leftrightarrow y$$

$$x = 2(3 - 5y)$$

$$x = 6 - 10y$$

$$10y = 6 - x$$

$$y = \frac{6 - x}{10}$$

$$m^{-1}(x) = \frac{6 - x}{10}$$

(iii)

$$p(x) = \frac{7x}{4}$$

$$y = \frac{7x}{4}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{7y}{4}$$

$$7y = 4x$$

$$y = \frac{4x}{7}$$

$$p^{-1}(x) = \frac{4x}{7}$$

(ii)

$$n(x) = \frac{x - 8}{3}$$

$$y = \frac{x - 8}{3}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{y - 8}{3}$$

$$y - 8 = 3x$$

$$y = 3x + 8$$

$$n^{-1}(x) = 3x + 8$$

(iv)

$$q(x) = \frac{3x}{5} + 4$$

$$y = \frac{3x}{5} + 4$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{3y}{5} + 4$$

$$5x = 3y + 20$$

$$3y = 5x - 20$$

$$y = \frac{5x - 20}{3}$$

$$q^{-1}(x) = \frac{5x - 20}{3}$$

[12 marks]

Answer 3**(i)**

$$r(x) = \frac{x}{11}$$

$$y = \frac{x}{11}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{y}{11}$$

$$y = 11x$$

$$r^{-1}(x) = 11x$$

(iii)

$$t(x) = \frac{3}{2x}$$

$$y = \frac{3}{2x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{3}{2y}$$

$$y = \frac{3}{2x}$$

$$t^{-1}(x) = \frac{3}{2x} \quad x \neq 0$$

(ii)

$$s(x) = \frac{1}{4x}$$

$$y = \frac{1}{4x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{1}{4y}$$

$$y = \frac{1}{4x}$$

$$s^{-1}(x) = \frac{1}{4x} \quad x \neq 0$$

(iv)

$$u(x) = \frac{1}{x} + 4$$

$$y = \frac{1}{x} + 4$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{1}{y} + 4$$

$$\frac{1}{y} = x - 4$$

$$y = \frac{1}{x - 4}$$

$$u^{-1}(x) = \frac{1}{x - 4} \quad x \neq 4$$

[12 marks]

Answer 4**(i)**

$$v(x) = 9 - \frac{1}{x}$$

$$y = 9 - \frac{1}{x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = 9 - \frac{1}{y}$$

$$\frac{1}{y} = 9 - x$$

$$y = \frac{1}{9 - x}$$

$$v^{-1}(x) = \frac{1}{9 - x} \quad x \neq 9$$

(ii)

$$w(x) = 5 - \frac{3}{x}$$

$$y = 5 - \frac{3}{x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = 5 - \frac{3}{y}$$

$$\frac{3}{y} = 5 - x$$

$$\frac{y}{3} = \frac{1}{5 - x}$$

$$y = \frac{3}{5 - x}$$

$$w^{-1}(x) = \frac{3}{5 - x} \quad x \neq 5$$

(iii)

$$z(x) = \frac{2}{x} + 5$$

$$y = \frac{2}{x} + 5$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{2}{y} + 5$$

$$\frac{2}{y} = x - 5$$

$$\frac{y}{2} = \frac{1}{x - 5}$$

$$y = \frac{2}{x - 5}$$

$$z^{-1}(x) = \frac{2}{x - 5} \quad x \neq 5$$

(iv)

$$a(x) = \frac{x + 4}{x - 3}$$

$$y = \frac{x + 4}{x - 3}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{y + 4}{y - 3}$$

$$xy - 3x = y + 4$$

$$xy - y = 3x + 4$$

$$y(x - 1) = 3x + 4$$

$$y = \frac{3x + 4}{x - 1}$$

$$a^{-1}(x) = \frac{3x + 4}{x - 1} \quad x \neq 1$$

[12 marks]

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Lesson 9

GCSE Mathematics Functions I

9.1 “Worst Case” Inverse Functions

In the GCSE examination there is a particularly awkward class of function to which the inverse may be requested.

The function referred to is known as a linear rational function and is of the form,

$$f(x) = \frac{ax + b}{cx + d} \quad a, b, c, d \in \mathbb{Z}, \quad x \in \mathbb{R}, \quad x \neq -\frac{d}{c}$$

9.2 Example

Find the inverse function of the following function,

$$f(x) = \frac{5x - 1}{2x + 3} \quad x \in \mathbb{R}, \quad x \neq -\frac{3}{2}$$

Teaching Video : [http://www.NumberWonder.co.uk/Video/v9002\(9\).mp4](http://www.NumberWonder.co.uk/Video/v9002(9).mp4)



[4 marks]

Check if your answer could be correct by working out $f(7)$ and then inserting the answer into your proposed inverse function.

[1 mark]

9.3 Exercise

Marks Available: 32

Question 1

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(i) $b(x) = \frac{7 - x}{x}, \quad x \neq 0$

[4 marks]

(ii) $c(x) = \frac{5 - 3x}{x}, \quad x \neq 0$

[4 marks]

Question 2

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(i) $d(x) = \frac{x+3}{x+2}, \quad x \neq -2$

[4 marks]

(ii) $e(x) = \frac{x+1}{x-2}, \quad x \neq 2$

[4 marks]

Question 3

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(i) $f(x) = \frac{1}{2x} + \frac{1}{3x}, \quad x \neq 0$

[4 marks]

(ii) $g(x) = \frac{3}{5x} + \frac{1}{4x} + 2, \quad x \neq 0$

[4 marks]

Question 4

Find the inverse of each of the following functions.

In each case the domain is the set of real numbers, $x \in \mathbb{R}$

(i) $h(x) = \frac{x^2 + 5x + 6}{x^2 + 6x + 8}, \quad x \neq -4, -2$

HINT: separately factorise the numerator and the denominator

[4 marks]

(ii) $k(x) = \frac{x^2 - 4}{x^2 - 4x + 4}, \quad x \neq 2$

[4 marks]

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9.4 Answers

GCSE Mathematics Functions I

9.4.1 Solution to 9.2 Example (Teaching Video)

$$f(x) = \frac{5x - 1}{2x + 3}, \quad x \in \mathbb{R}, \quad x \neq -\frac{3}{2}$$

$$y = \frac{5x - 1}{2x + 3}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{5y - 1}{2y + 3}$$

$$x(2y + 3) = 5y - 1$$

$$2xy + 3x = 5y - 1$$

$$3x + 1 = 5y - 2xy$$

$$3x + 1 = y(5 - 2x)$$

$$y = \frac{3x + 1}{5 - 2x}$$

$$f^{-1}(x) = \frac{3x + 1}{5 - 2x}, \quad x \neq 1$$

$$\text{Note: } f(7) = 2 : f^{-1}(2) = 7$$

[4, 1 marks]

9.4.2 Solutions to 9.3 Exercise

Answer 1

$$(i) \quad b(x) = \frac{7 - x}{x}$$

$$y = \frac{7 - x}{x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{7 - y}{y}$$

$$xy = 7 - y$$

$$xy + y = 7$$

$$y(x + 1) = 7$$

$$y = \frac{7}{x + 1}$$

$$b^{-1}(x) = \frac{7}{x + 1}, \quad x \neq -1$$

$$(ii) \quad c(x) = \frac{5 - 3x}{x}$$

$$y = \frac{5 - 3x}{x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{5 - 3y}{y}$$

$$xy = 5 - 3y$$

$$xy + 3y = 5$$

$$y(x + 3) = 5$$

$$y = \frac{5}{x + 3}$$

$$c^{-1}(x) = \frac{5}{x + 3}, \quad x \neq -3$$

[4, 4 marks]

Answer 2**(i)**

$$d(x) = \frac{x+3}{x+2}$$

$$y = \frac{x+3}{x+2}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{y+3}{y+2}$$

$$xy + 2x = y + 3$$

$$xy - y = 3 - 2x$$

$$y(x-1) = 3-2x$$

$$y = \frac{3-2x}{x-1}$$

$$d^{-1}(x) = \frac{3-2x}{x-1}, \quad x \neq 1$$

(ii)

$$e(x) = \frac{x+1}{x-2}$$

$$y = \frac{x+1}{x-2}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{y+1}{y-2}$$

$$xy - 2x = y + 1$$

$$xy - y = 1 + 2x$$

$$y(x-1) = 2x+1$$

$$y = \frac{2x+1}{x-1}$$

$$e^{-1}(x) = \frac{2x+1}{x-1}, \quad x \neq 1$$

[4, 4 marks]**Answer 3****(i)**

$$f(x) = \frac{1}{2x} + \frac{1}{3x}$$

$$y = \frac{1}{2x} + \frac{1}{3x}$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{1}{2y} + \frac{1}{3y}$$

$$6xy = 3 + 2$$

$$6xy = 5$$

$$y = \frac{5}{6x}$$

$$f^{-1}(x) = \frac{5}{6x}, \quad x \neq 0$$

(ii)

$$g(x) = \frac{3}{5x} + \frac{1}{4x} + 2$$

$$y = \frac{3}{5x} + \frac{1}{4x} + 2$$

$$\text{swap } x \Leftrightarrow y$$

$$x = \frac{3}{5y} + \frac{1}{4y} + 2$$

$$20xy = 12 + 5 + 40y$$

$$20xy - 40y = 17$$

$$y(20x - 40) = 17$$

$$y = \frac{17}{20x - 40}$$

$$g^{-1}(x) = \frac{17}{20x - 40}, \quad x \neq 2$$

[4, 4 marks]

Answer 4**(i)**

$$h(x) = \frac{x^2 + 5x + 6}{x^2 + 6x + 8}$$

$$y = \frac{x^2 + 5x + 6}{x^2 + 6x + 8}$$

swap $x \Leftrightarrow y$

$$x = \frac{y^2 + 5y + 6}{y^2 + 6y + 8}$$

$$x = \frac{(y + 2)(y + 3)}{(y + 2)(y + 4)}$$

$$x = \frac{y + 3}{y + 4}$$

$$xy + 4x = y + 3$$

$$xy - y = 3 - 4x$$

$$y = \frac{3 - 4x}{x - 1}$$

$$h^{-1}(x) = \frac{3 - 4x}{x - 1}, \quad x \neq 1$$

(ii)

$$k(x) = \frac{x^2 - 4}{x^2 - 4x + 4}$$

$$y = \frac{x^2 - 4}{x^2 - 4x + 4}$$

swap $x \Leftrightarrow y$

$$x = \frac{y^2 - 4}{y^2 - 4y + 4}$$

$$x = \frac{(y - 2)(y + 2)}{(y - 2)(y - 2)}$$

$$x = \frac{y + 2}{y - 2}$$

$$xy - 2x = y + 2$$

$$xy - y = 2x + 2$$

$$y = \frac{2x + 2}{x - 1}$$

$$k^{-1}(x) = \frac{2x + 2}{x - 1}, \quad x \neq 1$$

[4, 4 marks]

Lesson 10

GCSE Mathematics Functions I

10.1 Revision

Marks Available: 60

Question 1

The function f is given by, $f(x) = x^2 - 1$, $x \in \mathbb{R}$

Calculate;

(i) $f(3)$

(ii) $f(8)$

(iii) $f(1)$

(iv) $f(0)$

(v) $f(5)$

(vi) $f(-5)$

(vii) $f(100)$

(viii) $f\left(\frac{1}{2}\right)$

(ix) $f\left(\frac{3}{2}\right)$

(x) $f(\sqrt{2})$

[6 marks]

Question 2

Sometimes the domain of a function is restricted.

Consider the function, $g(x) = \frac{10}{x+1}$, $x \in \mathbb{R}$, $x \neq -1$

(i) What real number is not allowed into this function ?

[1 mark]

(ii) Why is this function's domain restricted in this way ?

[2 marks]

Question 3

This question involves the functions;

$$f(x) = 3x - 2, \quad x \in \mathbb{R}$$

$$g(x) = \frac{10}{x + 1}, \quad x \in \mathbb{R}, \quad x \neq -1$$

Determine the value of,

(i) $f(13)$

(ii) $g(0)$

(iii) $ff(3)$

(iv) $fg(4)$

(v) $gf(1)$

[5 marks]

Question 4

If $v(x) = 3x^2 - 1$, $x \in \mathbb{R}$, find expressions that do not involve brackets for,

(i) $v(7)$

(ii) $v(10x)$

(iii) $v(x + 4)$

HINT : FOIL

[1, 2, 3 marks]

Question 5

If $m(x) = 6x + 7$, find x such that $m(x) = 25$ ($x \in \mathbb{R}$)

[3 marks]

Question 6

If $s(x) = \frac{3x + 2}{4}$, find x such that $s(x) = 11$ ($x \in \mathbb{R}$)

[3 marks]

Question 7

Let p and q be the functions,

$$p(x) = 4x + 7 \quad x \in \mathbb{R}$$

$$q(x) = 5x + 6 \quad x \in \mathbb{R}$$

Evaluate each of the following,

(i) $p \circ q(3)$

(ii) $p \circ q(-1)$

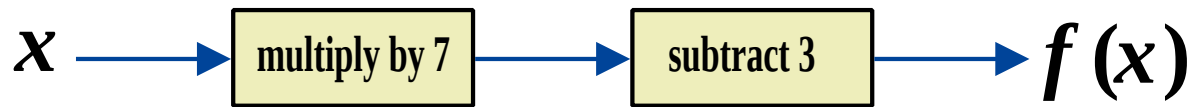
(iii) $p \circ q(4z)$

(iv) $p \circ q(3z + 1)$

[1, 1, 2, 2 marks]

Question 8

Consider the following flow diagram;



(a) Write down (i) $f(x)$

(ii) $f^{-1}(x)$

[1, 2 marks]

(b) Determine the value of,

(i) $f(5)$

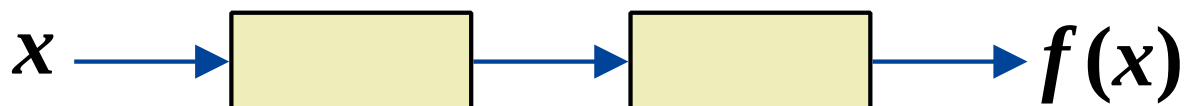
(ii) $f^{-1}(32)$

[1, 1 marks]

Question 9

(a) Fill in the flow diagram for the function

$$f(x) = 5(x - 9), \quad x \in \mathbb{R}$$



[1 mark]

(b) Write down $f^{-1}(x)$

[2 marks]

(c) Determine the value of,

(i) $f(12)$

(ii) $f^{-1}(25)$

[1, 1 marks]

Question 10

Consider the function, $k(x) = \frac{x}{5} + 7$, $x \in \mathbb{R}$

Find an expression for the inverse function, $k^{-1}(x)$

HINT : Draw a flow diagram.

[4 marks]

The GCSE examination often includes an awkward functions question.

Here is an example of a grade 8 question.

Question 11

For the function $f(x) = 3x + 2$, $x \in \mathbb{R}$, determine x such that $f(x) = f^{-1}(x)$

[5 marks]

Question 12

Consider the function, $f(x) = \frac{8}{3x} + 7$, $x \in \mathbb{R}$, $x \neq 0$

Find an expression for the inverse function $f^{-1}(x)$

[4 marks]

Question 13

Consider the function, $f(x) = \frac{x+5}{x+3}$, $x \in \mathbb{R}$, $x \neq -3$

Find an expression for the inverse function $f^{-1}(x)$

[5 marks]

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10.2 Solutions to the Revision

GCSE Mathematics Functions I

Answer 1

- (i) 8 (ii) 63 (iii) 0 (iv) -1 (v) 24
(vi) 24 (vii) 9999 (viii) $-\frac{3}{4}$ (ix) $\frac{5}{4}$ (x) 1

[6 marks]

Answer 2

- (i) -1 (ii) To avoid a division by zero (which is not defined).

[1, 2 marks]

Answer 3

- (i) 37 (ii) 10 (iii) 19 (iv) 4 (v) 5

[5 marks]

Answer 4

- (i) 146 (ii) $300x^2 - 1$ (iii) $3x^2 + 24x + 47$

[1, 2, 3 marks]

Answer 5

3

Answer 6

14

[3, 3 marks]

Answer 7

- (i) 91 (ii) 11 (iii) $80z + 31$ (iv) $60z + 51$

[1, 1, 2, 2 marks]

Answer 8

- (a) (i) $f(x) = 7x - 3$ (ii) $f^{-1}(x) = \frac{x + 3}{7}$
(b) (i) 32 (ii) 5

[1, 2, 1, 1 marks]

Answer 9

- (a) Diagram contains **Subtract 9**, then **Multiply by 5**.

- (b) $\frac{x}{5} + 9$ (c) (i) 15 (ii) 14

[1, 2, 1, 1 marks]

Answer 10

$5x - 35$

Answer 11

-1

[4, 5 marks]

Answer 12

$$f^{-1}(x) = \frac{8}{3(x - 7)}$$

Answer 13

$$f^{-1}(x) = \frac{5 - 3x}{x - 1}$$

[4, 5 marks]

Lesson 11

GCSE Mathematics Functions I

11.1 Homework

Marks Available: 60

Question 1

The function f is given by, $f(x) = x^2 + 3$ $x \in \mathbb{R}$

Calculate;

(i) $f(4)$

(ii) $f(9)$

(iii) $f(1)$

(iv) $f(0)$

(v) $f(6)$

(vi) $f(-6)$

(vii) $f(100)$

(viii) $f\left(\frac{1}{2}\right)$

(ix) $f\left(\frac{3}{2}\right)$

(x) $f(\sqrt{5})$

[6 marks]

Question 2

Sometimes the domain of a function is restricted.

Consider the function, $g(x) = \frac{12}{x+2}$, $x \in \mathbb{R}$, $x \neq -2$

(i) What number is not allowed into this function ?

[1 mark]

(ii) Why is this function's domain restricted in this way ?

[2 marks]

Question 3

This question involves the functions;

$$f(x) = 5x - 3 \quad x \in \mathbb{R}$$

$$g(x) = \frac{12}{x + 2} \quad x \in \mathbb{R}, \quad x \neq -2$$

Determine the value of,

(i) $f(8)$

(ii) $g(1)$

(iii) $ff(3)$

(iv) $fg(4)$

(v) $gf(1)$

[5 marks]

Question 4

If $H(x) = 4x^2 - 1$, $x \in \mathbb{R}$ find expressions that do not involve brackets for,

(i) $H(7)$

(ii) $H(10x)$

(iii) $H(x + 3)$

HINT : FOIL

[1, 2, 3 marks]

Question 5

If $w(x) = 8x + 7$ find x such that $w(x) = 31$ ($x \in \mathbb{R}$)

[3 marks]

Question 6

If $n(x) = \frac{2x + 5}{3}$ find x such that $n(x) = 13$ ($x \in \mathbb{R}$)

[3 marks]

Question 7

Let u and v be the functions;

$$u(x) = 7x + 4 \quad x \in \mathbb{R}$$

$$v(x) = 6x + 5 \quad x \in \mathbb{R}$$

Evaluate each of the following;

(i) $u \circ v(1)$

(ii) $u \circ v(-1)$

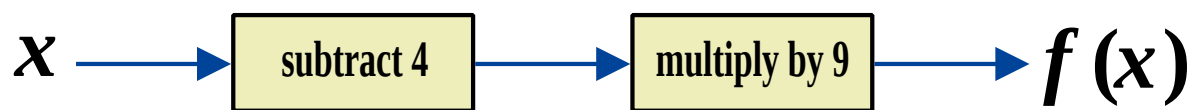
(iii) $u \circ v(3z)$

(iv) $u \circ v(4z + 1)$

[1, 1, 2, 2 marks]

Question 8

Consider the following flow diagram;



(a) Write down (i) $f(x)$

(ii) $f^{-1}(x)$

[1, 2 marks]

(b) Determine the value of,

(i) $f(8)$

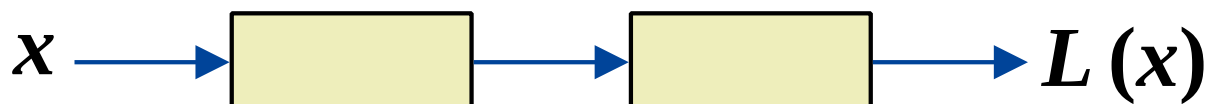
(ii) $f^{-1}(45)$

[1, 1 marks]

Question 9

(a) Fill in the flow diagram for the function

$$L(x) = 7(x - 8) \quad x \in \mathbb{R}$$



[1 mark]

(b) Write down $L^{-1}(x)$

[2 marks]

(c) Determine the value of,

(i) $L(17)$

(ii) $L^{-1}(35)$

[1, 1 marks]

Question 10

Consider the function $k(x) = \frac{x}{7} - 3$, $x \in \mathbb{R}$

Find an expression for the inverse function $k^{-1}(x)$

HINT : Draw a flow diagram.

[4 marks]

The GCSE examination often includes an awkward functions question.
Here is an example of a grade 8 question.

Question 11

For the function $f(x) = 4x - 1$, $x \in \mathbb{R}$ determine x such that $f(x) = f^{-1}(x)$

[5 marks]

Question 12

Consider the function, $f(x) = \frac{13}{5x} + 4$, $x \in \mathbb{R}$, $x \neq 0$

Find an expression for the inverse function $f^{-1}(x)$

[4 marks]

Question 13

Consider the function, $f(x) = \frac{x + 11}{x - 2}$, $x \in \mathbb{R}$, $x \neq 2$

Find an expression for the inverse function $f^{-1}(x)$

[5 marks]

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11.2 Solutions to the Homework

GCSE Mathematics Functions I

Answer 1

- (i) 19 (ii) 84 (iii) 4 (iv) 3 (v) 39
(vi) 39 (vii) 10003 (viii) 3.25 (ix) 5.25 (x) 8

[6 marks]

Answer 2

- (i) -2 (ii) To avoid a division by zero (which is not defined).

[1, 2 marks]

Answer 3

- (i) 37 (ii) 4 (iii) 57 (iv) 7 (v) 3

[5 marks]

Answer 4

- (i) 195 (ii) $400x^2 - 1$ (iii) $4x^2 + 24x + 35$

[1, 2, 3 marks]

Answer 5

3

Answer 6

17

[3, 3 marks]

Answer 7

- (i) 81 (ii) -3 (iii) $126z + 39$ (iv) $168z + 81$

[1, 1, 2, 2 marks]

Answer 8

- (a) (i) $f(x) = 9x - 36$ (ii) $f^{-1}(x) = \frac{x}{9} + 4$
(b) (i) 36 (ii) 9

[1, 2, 1, 1 marks]

Answer 9

- (a) Diagram contains **Subtract 8**, then **Multiply by 7**

- (b) $\frac{x}{7} + 8$ (c) (i) 63 (ii) 13

[1, 2, 1, 1 marks]

Answer 10

$7x + 21$

Answer 11

$\frac{1}{3}$

[4, 5 marks]

Answer 12

$$f^{-1}(x) = \frac{13}{5(x - 4)}$$

Answer 13

$$f^{-1}(x) = \frac{11 + 2x}{x - 1}$$

[4, 5 marks]

Lesson 12

GCSE Mathematics Functions I

12.1 Test

Marks Available: 86

Question 1

The function f is given by, $f(x) = x^2 - x$, $x \in \mathbb{R}$

Calculate;

(i) $f(7)$

(ii) $f(3)$

(iii) $f(10)$

(iv) $f(0)$

(v) $f(1)$

(vi) $f(4)$

(vii) $f(-4)$

(viii) $f(100)$

(ix) $f\left(\frac{1}{2}\right)$

(x) $f\left(-\frac{1}{2}\right)$

[6 marks]

Question 2

Sometimes the domain of a function is restricted.

Consider the function $g(x) = \frac{12}{x - 3}$, $x \in \mathbb{R}$, $x \neq 3$

(i) Which real number is not allowed into this function ?

[1 mark]

(ii) Why is this function's domain necessarily restricted in this way ?

[2 marks]

Question 3

This question involves the functions;

$$f(x) = 4x - 1 \quad x \in \mathbb{R}$$

$$g(x) = \frac{12}{x - 3} \quad x \in \mathbb{R}, \ x \neq 3$$

Determine the value of,

(i) $f(8)$

(ii) $g(5)$

(iii) $ff(5)$

(iv) $fg(9)$

(v) $gf(2)$

[5 marks]

Question 4

If $H(x) = 3x^2 + 1$, $x \in R$, find expressions that do not involve brackets for,

(i) $H(6)$

(ii) $H(10x)$

(iii) $H(x + 4)$ **HINT : FOIL**

[1, 2, 3 marks]

Question 5

If $w(x) = 7x + 5$, find x such that $w(x) = 33$ $(x \in \mathbb{R})$

[3 marks]**Question 6**

If $n(x) = \frac{3x + 2}{5}$, find x such that $n(x) = 7$ $(x \in \mathbb{R})$

[3 marks]**Question 7**

Let u and v be the functions,

$$u(x) = 4x + 7 \quad x \in \mathbb{R}$$

$$v(x) = 5x + 6 \quad x \in \mathbb{R}$$

Evaluate each of the following,

(i) $u \circ v(1)$

(ii) $u \circ v(-1)$

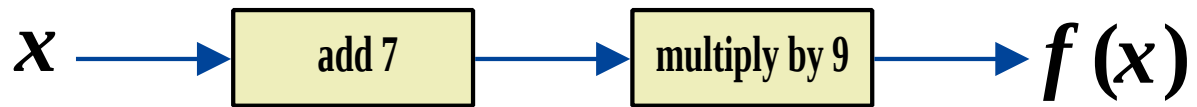
(iii) $u \circ v(5z)$

(iv) $u \circ v(3z + 2)$

[1, 1, 2, 2 marks]

Question 8

Consider the following flow diagram;



(a) Write down (i) $f(x)$

(ii) $f^{-1}(x)$

[1, 2 marks]

(b) Determine the value of,

(i) $f(-3)$

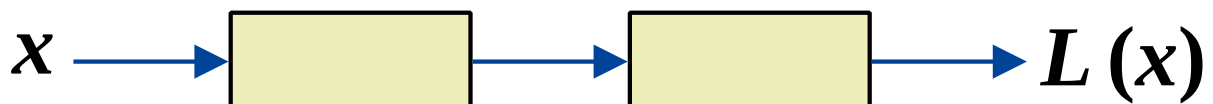
(ii) $f^{-1}(45)$

[1, 1 marks]

Question 9

(a) Fill in the flow diagram for the function,

$$L(x) = 11(x - 4), \quad x \in \mathbb{R}$$



[1 mark]

(b) Write down $L^{-1}(x)$

[2 marks]

(c) Determine the value of,

(i) $L(24)$

(ii) $L^{-1}(44)$

[1, 1 marks]

Question 10

Consider the function $k(x) = \frac{x}{13} - 7$, $x \in \mathbb{R}$

Find an expression for the inverse function $k^{-1}(x)$

HINT : Draw a flow diagram.

[4 marks]

The GCSE examination often includes an awkward functions question.
Here is an example of a grade 8 question.

Question 11

For the function $f(x) = 5x + 1$, $x \in \mathbb{R}$, determine x such that $f(x) = f^{-1}(x)$

[5 marks]

Question 12

If $p(x) = \frac{5x - 7}{3 - x}$, $x \in \mathbb{R}$, $x \neq 3$, find x such that $p(x) = 11$

[4 marks]

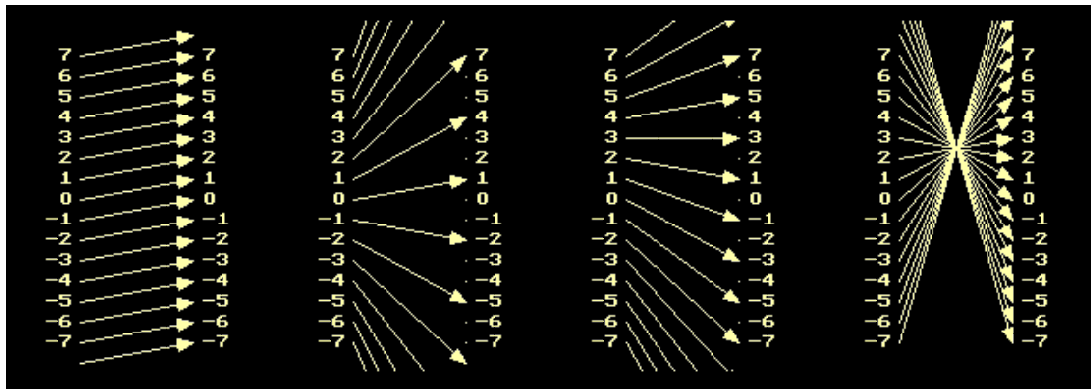
Question 13

You are not expected to have seen integer “arrow diagrams” like these before.

(i) Which one of the four diagrams is a mapping of the function,

$$w(x) = 5 - x, \quad x \in \mathbb{Z} ?$$

(ii) For each of the other three diagrams, suggest the functions being mapped.



[4 marks]

Question 14

Let f and g be the functions, $f(x) = 3x + 5, x \in \mathbb{R}$

$$g(x) = x^2 + 5, x \in \mathbb{R}$$

Find; (i) $fg(4)$

[2 marks]

(ii) $gff(-1)$

[2 marks]

(iii) $gf(x)$ and write the answer without using brackets.

[4 marks]

Question 15

Let v and w be the functions, $v(x) = 10 - x, x \in \mathbb{R}$

$$w(x) = \sqrt{x}, x \in \mathbb{R}, x \geq 0$$

(a) Find, (i) $vw(36)$

[2 marks]

(ii) $wv(-1)$

[2 marks]

(iii) $wv(x)$

[3 marks]

(b) If $vw(x) = 0$, what must be the value of x ?

[3 marks]

Question 16

Consider the function $f(x) = 8 - \frac{12}{7x}$, $x \in \mathbb{R}$, $x \neq 0$

Find an expression for the inverse function $f^{-1}(x)$

[4 marks]

Question 17

Consider the function $f(x) = \frac{13 - x}{5 + 2x}$, $x \in \mathbb{R}$, $x \neq -\frac{2}{5}$

Find an expression for the inverse function $f^{-1}(x)$

[5 marks]

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12.2 Solutions to 12.1 Test

GCSE Mathematics Functions I

Answer 1

(i) 42

(iii) 90

(v) 0

(vii) 20

(ix) $-\frac{1}{4}$

(ii) 6

(iv) 0

(vi) 12

(viii) 9900

(x) $\frac{3}{4}$

[6 marks]

Answer 2

(i) 3

(ii) To avoid a division by zero (which is not defined).

[1, 2 marks]

Answer 3

(i) 31

(iii) 75

(v) 3

(ii) 6

(iv) 7

[5 marks]

Answer 4

(i) 109

(ii) $300x^2 + 1$

(iii) $3x^2 + 24x + 49$

[1, 2, 3 marks]

Answer 5

4

Answer 6

11

[3, 3 marks]

Answer 7

(i) 51

(iii) $100z + 31$

(ii) 11

(iv) $60z + 71$

[1, 1, 2, 2 marks]

Answer 8

(a) (i) $f(x) = 9x + 63$

(ii) $f^{-1}(x) = \frac{x}{9} - 7$

(b) (i) 36

(ii) -2

[1, 2, 1, 1 marks]

Answer 9

(a) Diagram contains **Subtract 4**, then **Multiply by 11**

(b) $\frac{x}{11} + 4$ (c) (i) 220 (ii) 8

[1, 2, 1, 1 marks]

Answer 10

$13x + 91$

Answer 11

$-\frac{1}{4}$

[4, 4 marks]

Answer 12

$$x = 2.5$$

[4 marks]

Answer 13

(i) The fourth diagram

$$(\text{ii}) \quad x + 1$$

$$3x + 1$$

$$2x - 3$$

[4 marks]

Answer 14

$$(\text{i}) \quad 68$$

$$(\text{ii}) \quad 126$$

$$(\text{iii}) \quad 9x^2 + 30x + 30$$

[2, 2, 4 marks]

Answer 15

$$(\text{a}) \quad (\text{i}) \quad 4$$

$$(\text{ii}) \quad \sqrt{11} \text{ or } 3.317$$

$$(\text{iii}) \quad \sqrt{10 - x}$$

$$(\text{b}) \quad 100$$

[2, 2, 3, 3 marks]

Answer 16

$$f^{-1}(x) = \frac{12}{7(8 - x)}, \quad x \neq 8$$

[4 marks]

Answer 17

$$f^{-1}(x) = \frac{13 - 5x}{2x + 1}, \quad x \neq -\frac{1}{2}$$

[5 marks]