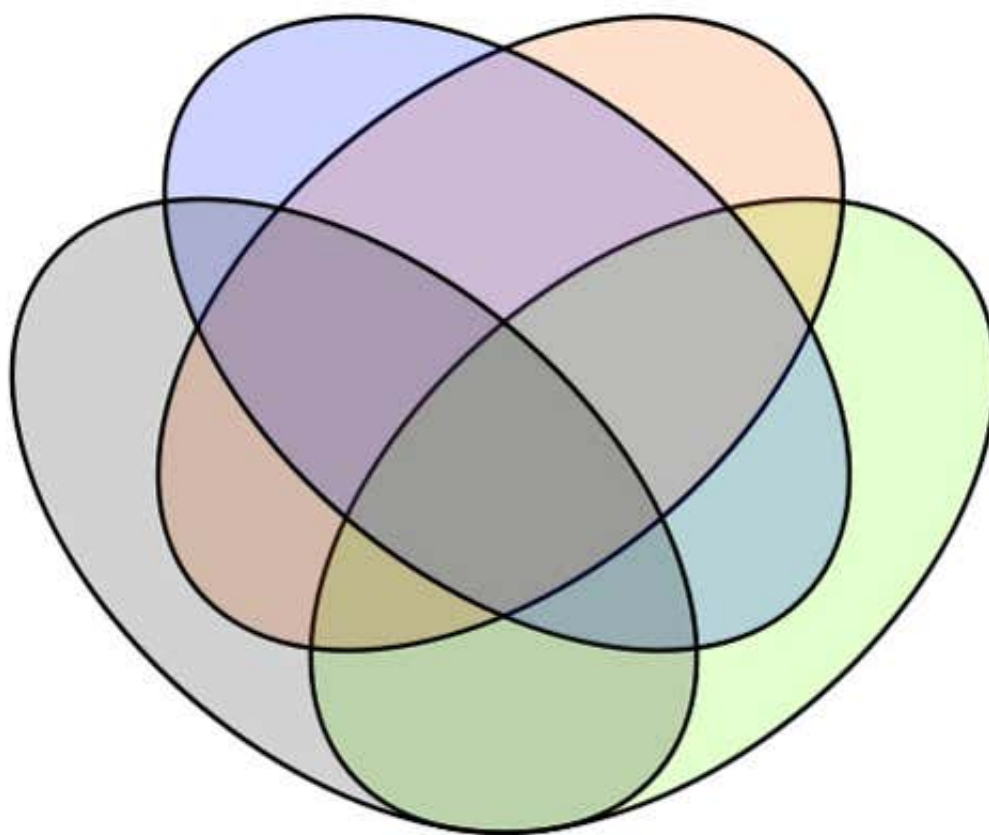


S E T T H E O R Y I



Venn Diagram for four sets

Lesson 1

GCSE Mathematics Set Theory I

1.1 Introduction : What is a Set ?

At a simple level, a set can be thought of as a collection of objects.

The objects are often, but not always, numbers.

If it is helpful, a set can be given a name.

Here are two descriptions of the set that I've called F .

$$F = \{ \text{The factors of 14} \}$$

$$F = \{ 1, 2, 7, 14 \}$$

Notice the use of curly set brackets, $\{ \}$, rather than curved, $()$.

1.2 Two Ways of Describing a Set

Think of another way of describing each of the following sets.

(i) $T = \{ \text{Traffic light colours} \}$

[1 mark]

(ii) $S = \{ \text{spring, summer, autumn, winter} \}$

[1 mark]

(iii) $P = \{ \text{Mars, Earth, Saturn, ...} \}$

[1 mark]

What does the dot, dot, dot indicate ?

[1 mark]

(iv) $M = \{ \text{Types of metal} \}$

[1 mark]

Teaching Video : <http://www.NumberWonder.co.uk/v9003/1.mp4>



Watch the Teaching Video and complete the above questions.

1.3 Exercise

Marks Available : 20 + 1 Bonus

Question 1

In the spirit of the introduction, give an alternative description of each of the following sets;

(i) $S = \{ \text{Days of the week with the letter "s" in their spelling} \}$

[1 mark]

(ii) $V = \{ a, e, i, o, u \}$

[1 mark]

(iii) $F = \{ \text{The factors of 24} \}$

HINT : There are eight

[1 mark]

(iv) $A = \{ +, -, \times, \div \}$

[1 mark]

(v) $P = \{ \text{The prime numbers less than 20} \}$

HINT : 1 is NOT prime

[1 mark]

(vi) $C = \{ \text{Values of copper coloured British coins in everyday use} \}$

[1 mark]

(vii) $G = \{ \alpha, \beta, \chi, \delta, \varepsilon, \phi, \gamma, \eta, \iota, \varphi, \kappa, \lambda, \mu, \nu, o, \pi, \theta, \rho, \sigma, \tau, v, \varpi, \omega, \xi, \psi, \zeta \}$

[1 mark]

(viii) $E = \{ \text{Even prime numbers} \}$

HINT : There's only one !

[1 mark]

(ix) $P = \{ 1, 2, 4, 8, 16, 32, 64, 128, \dots \}$

[1 mark]

Notice the dot, dot, dot.

BONUS MARK : What does the ... tell you about this set ?

[BONUS 1 mark]

(x) $O = \{ \text{Odd numbers} \}$

HINT : List a few then put ...

[1 mark]

(xi) $D = \{ \begin{array}{|c|} \hline \bullet \\ \hline \end{array} \begin{array}{|c|} \hline \bullet \quad \bullet \\ \hline \end{array} \begin{array}{|c|} \hline \bullet \quad \bullet \quad \bullet \\ \hline \end{array} \begin{array}{|c|} \hline \bullet \quad \bullet \quad \bullet \quad \bullet \\ \hline \end{array} \begin{array}{|c|} \hline \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \\ \hline \end{array} \begin{array}{|c|} \hline \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \\ \hline \end{array} \}$

[1 mark]

(xii) $F = \{ \text{Factors of 17} \}$

[1 mark]

(xiii) $M = \{ \text{Multiples of 5 that are *less than* 60} \}$

[1 mark]

(xiv) $R = \{ \text{I, V, X, L, C, D, M} \}$

[1 mark]

(xv) $R = \{ \text{red, orange, yellow, green, blue, indigo, violet} \}$

[1 mark]

(xvi) $S = \{ \text{hearts, clubs, spades, diamonds} \}$

[1 mark]

(xvii) $M = \{ \text{makes of mobile phone} \}$

HINT : List a few then put ...

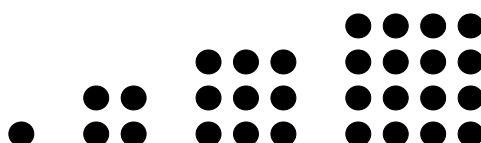
[1 mark]

(xviii) $C = \{ \text{ready salted, cheese & onion, prawn cocktail, ...} \}$

[1 mark]

(xix) $S = \{ \text{square numbers} \}$

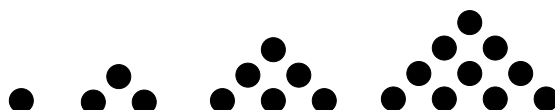
HINT :



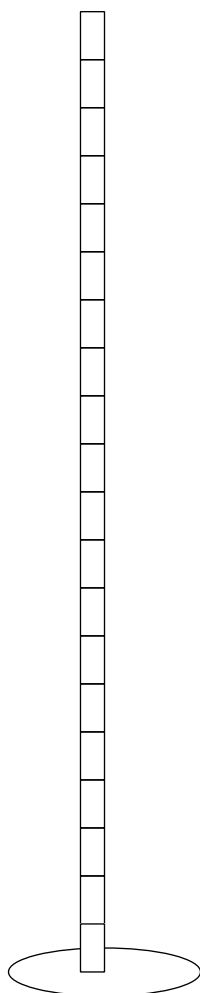
[1 mark]

(xx) $T = \{ \text{triangular numbers} \}$

HINT :



[1 mark]



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1.4 Homework

Marks Available : 20

These questions are harder than those of the previous exercise.
If you get stuck on one, miss it out and come back to it later.
You are NOT expected to get all these questions correct !

Question 1

In the spirit of the introduction, give an alternative description of each of the following sets;

(i) $P = \{ \text{Snow, rain, drizzle, hailstones, ...} \}$

[1 mark]

(ii) $I = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$

[1 mark]

(iii) $P = \{ 3, 6, 9, 12, 15, 18, 21, \dots \}$

[1 mark]

(iv) $F = \{ \text{Factors of 8} \}$

[1 mark]

(v) $B = \{ 0, 1, 10, 11, 100, 101, 110, 111, 1000, \dots \}$

HINT : Numbers used by computers.

[1 mark]

(vi) $C = \{ \text{Cube numbers} \}$

HINT : Give the first ten then put ...

[1 mark]

(vii) $P = \{ \text{Palindromic words} \}$

[1 mark]

(viii) $F = \left\{ A = \pi r^2, A = \frac{1}{2}bh, A = lb, A = \frac{1}{2}(a + b)h, \dots \right\}$

[1 mark]

(ix) $M = \{ \text{Famous mathematicians} \}$

[1 mark]

(x) $M = \{ \text{Multiples of 7} \}$

HINT : Give the first ten then put ...

[1 mark]

(xi) $S = \{ \text{leech, clew, tack, luff, telltale, head, foot, ...} \}$

[1 mark]

(xii) $D = \{ \text{breeds of gundog} \}$

HINT : Google if necessary.

[1 mark]

(xiii) $P = \{ \text{triangle, square, pentagon, hexagon, ...} \}$

[1 mark]

(xiv) $Q = \{ \text{four sided shapes} \}$

HINT : Name as many as you can.

[1 mark]

(xv) $A = \{ \text{acute, obtuse, reflex} \}$

[1 mark]

(xvi) $K = \{ \text{monkey fist, reef, granny, bowline, half hitch, sheet bend, ...} \}$

[1 mark]

(xvii) $M = \{ \cdot -, -.., -\cdot-, -.., \cdot, ..-, \cdot, \dots \}$

[1 mark]

(xviii) $F = \{ 1, 2, 3, 6, 9, 18 \}$

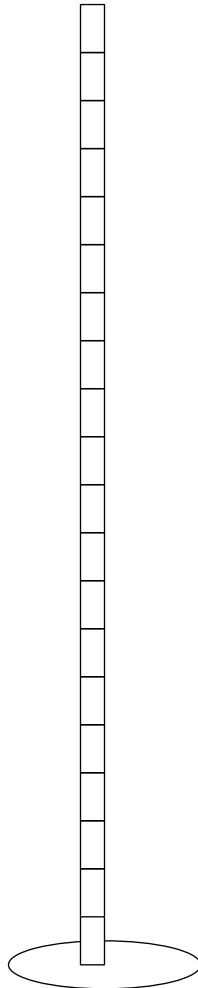
[1 mark]

(xix) $M = \{ 13, 26, 39, 52, 65, 78, \dots \}$

[1 mark]

(xx) $T = \{ \sin, \cos, \tan, \dots \}$

[1 mark]



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1.5 Answers

1.5.1 Solutions (Introductory Example)

- (i) $T = \{ \text{red, amber, green} \}$
- (ii) $S = \{ \text{seasons} \}$
- (iii) $P = \{ \text{planets in the solar system} \}$
The dot, dot, dot indicates that not all members of the set are listed
- (iv) $M = \{ \text{iron, steel, mercury, aluminium, ...} \}$

1.5.2 Solutions (1.3 Exercise)

1 mark for each correct answer + 1 Bonus

- (i) $S = \{ \text{Tuesday, Wednesday, Thursday, Saturday, Sunday} \}$
- (ii) $V = \{ \text{Vowels} \}$
- (iii) $F = \{ 1, 2, 3, 4, 6, 8, 12, 24 \}$
- (iv) $A = \{ \text{Simple arithmetic symbols} \}$
- (v) $P = \{ 2, 3, 5, 7, 11, 13, 17, 19 \}$
- (vi) $C = \{ 1p, 2p \}$
- (vii) $G = \{ \text{Greek letters} \}$
- (viii) $E = \{ 2 \}$
- (ix) $P = \{ \text{Powers of 2} \}$
The ... means that not all members of the set are listed.
- (x) $O = \{ 1, 3, 5, 7, 9, 11, 13, 15, ... \}$
- (xi) $D = \{ \text{Faces of a six sided die} \}$
- (xii) $F = \{ 1, 17 \}$
- (xiii) $M = \{ 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55 \}$
- (xiv) $R = \{ \text{Roman number symbols} \}$
- (xv) $R = \{ \text{colours of the rainbow} \}$
- (xvi) $S = \{ \text{playing card suits} \}$
- (xvii) $M = \{ \text{Apple iPhone, Nokia, BlackBerry, ...} \}$
- (xviii) $C = \{ \text{crisp flavours} \}$
- (xix) $S = \{ 1, 4, 9, 16, 25, 36, 49, ... \}$
- (xx) $T = \{ 1, 3, 6, 10, 15, 21, ... \}$

1.5.3 Solutions (1.4 Homework)

1 mark for each correct answer

- (i) $P = \{ \text{Forms of precipitation} \}$
- (ii) $I = \{ \text{Integers} \}$
- (iii) $P = \{ \text{Multiples of 3} \}$
- (iv) $F = \{ 1, 2, 4, 8 \}$
- (v) $B = \{ \text{Non negative integers in Binary} \}$
- (vi) $C = \{ 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, \dots \}$
- (vii) $P = \{ \text{mum, dad, oxo, abba, madam, nurun (girl's name), } \dots \}$
- (viii) $F = \{ \text{formulae for areas of 2D shapes} \}$
- (ix) $M = \{ \text{Pythagoras, Euclid, Euler, Fermat, Gauss, Riemann, Mandelbrot, } \dots \}$
- (x) $M = \{ 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, \dots \}$

- (xi) $S = \{ \text{parts of a sail} \}$
- (xii) $D = \{ \text{springer, pointer, retriever, } \dots \}$
- (xiii) $P = \{ \text{polygons} \}$
- (xiv) $Q = \{ \text{square, rectangle, kite, rhombus, parallelogram, trapezium, } \dots \}$
- (xv) $A = \{ \text{size of angle descriptions} \}$
- (xvi) $K = \{ \text{knots} \}$
- (xvii) $M = \{ \text{alphabet in Morse code} \}$
- (xviii) $F = \{ \text{Factors of 18} \}$
- (xix) $M = \{ \text{Multiples of 13} \}$
- (xx) $T = \{ \text{Mathematical trigonometric functions} \}$

2.1 Venn Diagrams

Previously, we considered one set at a time.

Today we'll look at two sets at a time.

The interest is in the connections between the two sets.

A Venn Diagram is an excellent way of making clear how the two sets are connected.

2.2 An Example Involving a Venn Diagram

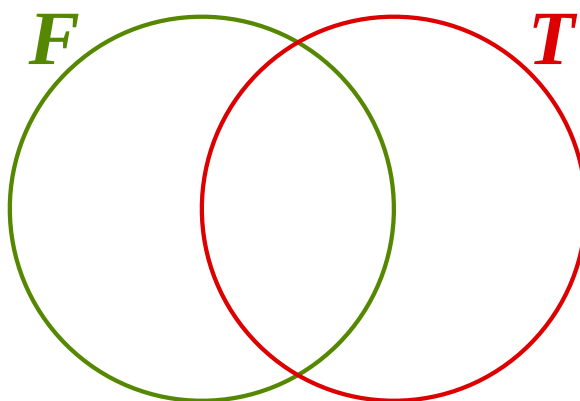
Draw a Venn Diagram to show the relationship between the sets F and T where

$$F = \{ \text{The factors of 15} \}$$

$$T = \{ \text{The factors of 25} \}$$

$$F = \{ \quad, \quad, \quad, \quad \}$$

$$T = \{ \quad, \quad, \quad \}$$



Teaching Video : <http://www.NumberWonder.co.uk/v9003/2.mp4>



2.3 Exercise

Marks Available : 60 + 20 Bonus

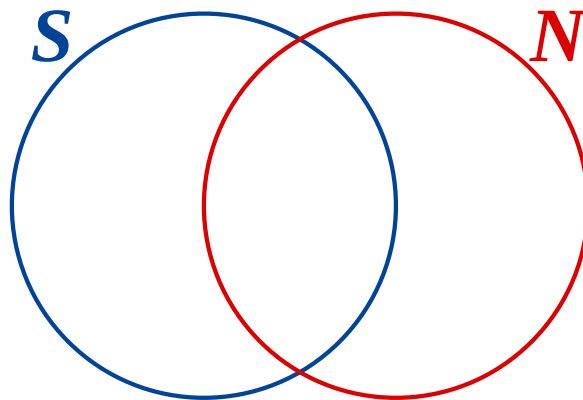
Question 1

Draw a Venn Diagram to show the relationship between the sets S and N where

$S = \{ \text{The factors of 6} \}$ and $N = \{ \text{The factors of 9} \}$

$S = \{ _, _, _, _ \}$

$N = \{ _, _, _ \}$



[7 marks]

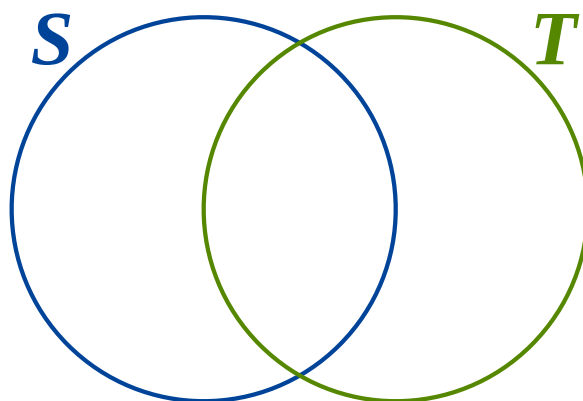
Question 2

Draw a Venn Diagram to show the relationship between the sets S and T where

$S = \{ \text{The factors of 16} \}$ and $T = \{ \text{The factors of 12} \}$

$S = \{ _, _, _, _, _ \}$

$T = \{ _, _, _, _, _, _ \}$



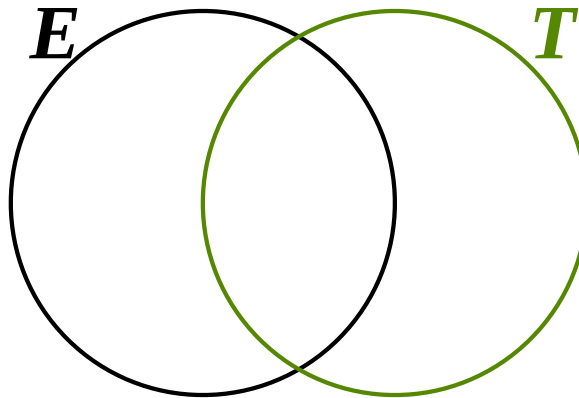
[7 marks]

Question 3

- (i) Draw a Venn Diagram to show the relationship between the sets E and T where $E = \{ \text{The factors of 8} \}$ and $T = \{ \text{The factors of 20} \}$

$$E = \{ \quad , \quad , \quad , \quad \}$$

$$T = \{ \quad , \quad , \quad , \quad , \quad , \quad \}$$



- [7 marks]
- (ii) What is the HCF (the highest common factor) of 8 and 20 ?
[1 mark]
- (iii) Find this number in your Venn Diagram and draw a cloud around it.
[1 mark]

Question 4

The factors of 210 are

$$T = \{ 1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 70, 105, 210 \}$$

The **prime factors** of 210 are the numbers in the above list which are **prime**.

What are the four **prime factors** of 210 ?

[4 marks]

Question 5

The number 12 has six factors, but only two **prime factors**.

What are the two **prime factors** of 12 ?

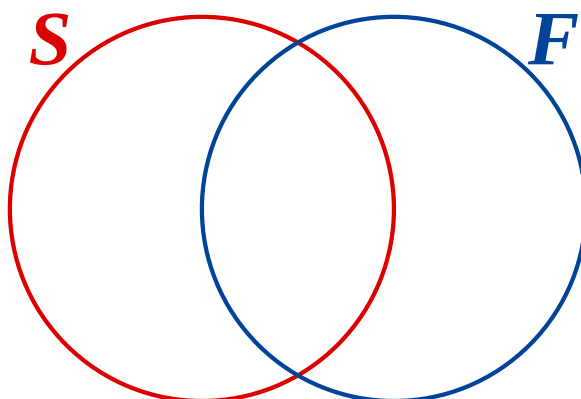
[2 marks]

Question 6

- (i) Draw a Venn Diagram show the relationship between the sets S and F where
 $S = \{ \text{The first five multiples of 6} \}$ & $F = \{ \text{The first seven multiples of 4} \}$

$$S = \{ \quad , \quad , \quad , \quad , \quad \}$$

$$F = \{ \quad , \quad , \quad , \quad , \quad , \quad , \quad \}$$



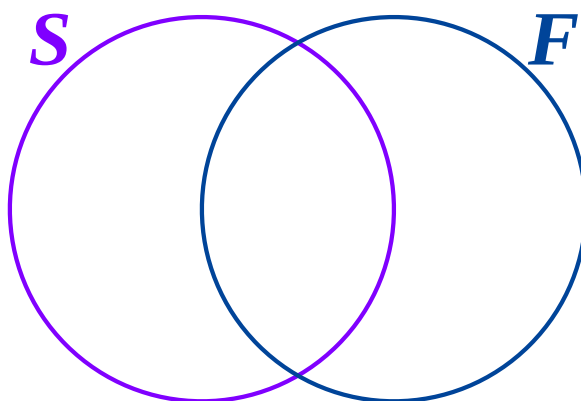
- (ii) What is the LCM (the lowest common multiple) of 6 and 4 ? [7 marks]
[1 mark]
- (iii) Find this number in your Venn Diagram and draw a cloud around it. [1 mark]

Question 7

On a Venn Diagram show the relationship between the sets S and F where

$$S = \{ \triangle , \times , O , \square \}$$

$$F = \{ \diamond , \square , \pi , \times \}$$



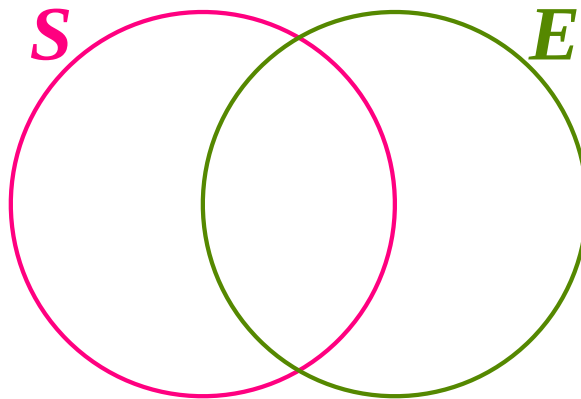
[4 marks]

Question 8

- (i) Draw a Venn Diagram show the relationship between the sets S and E where
 $S = \{ \text{The first nine multiples of 6} \}$ & $E = \{ \text{The first seven multiples of 8} \}$

$S = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

$E = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$



[7 marks]

- (ii) What is the LCM (the lowest common multiple) of 6 and 8 ?

[1 mark]

- (iii) Find this number in your Venn Diagram and draw a cloud around it.

[1 mark]

Question 9

Here are three sets,

$$A = \{ \text{factors of } 28 \}$$

$$B = \{ \text{prime factors of } 28 \}$$

$$C = \{ \text{prime factors of } 15 \}$$

List the members of sets A , B and C .

$$A = \{ \quad, \quad, \quad, \quad, \quad, \quad \}$$

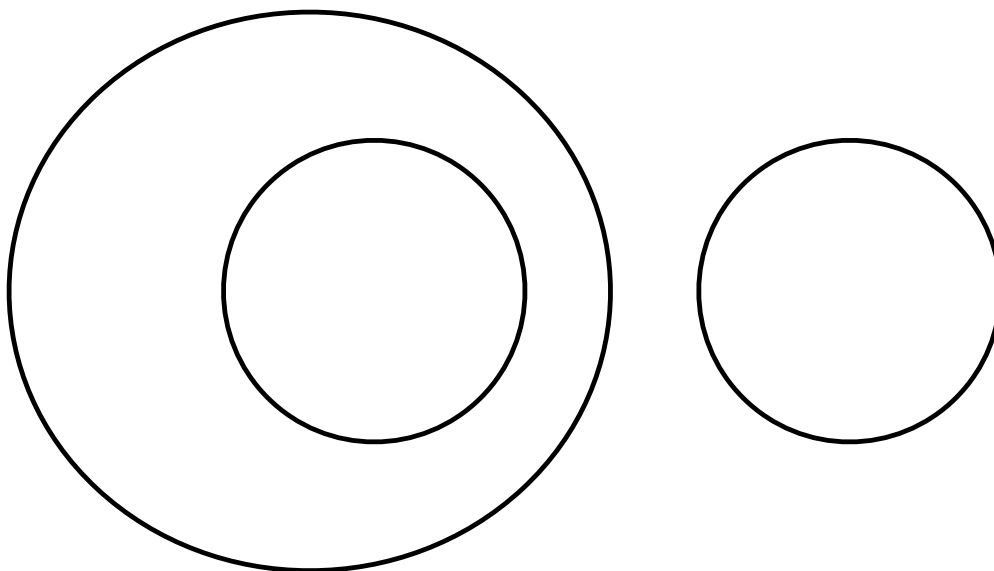
$$B = \{ \quad, \quad \}$$

$$C = \{ \quad, \quad \}$$

This time the Venn diagram has three hoops and can be drawn as shown below.

Decide which hoop is for set A , which for set B and which for set C .

Then complete the Venn diagram to show the relationship between sets A , B & C .



[8 marks]

Which set is a *subset* of another set ?

[1 mark]

Question 10 : BONUS QUESTION (Voluntary)

This question is crazy !

List the members of the following sets,

$A = \{ \text{The multiples of 35 less than 700} \}$

[3 marks]

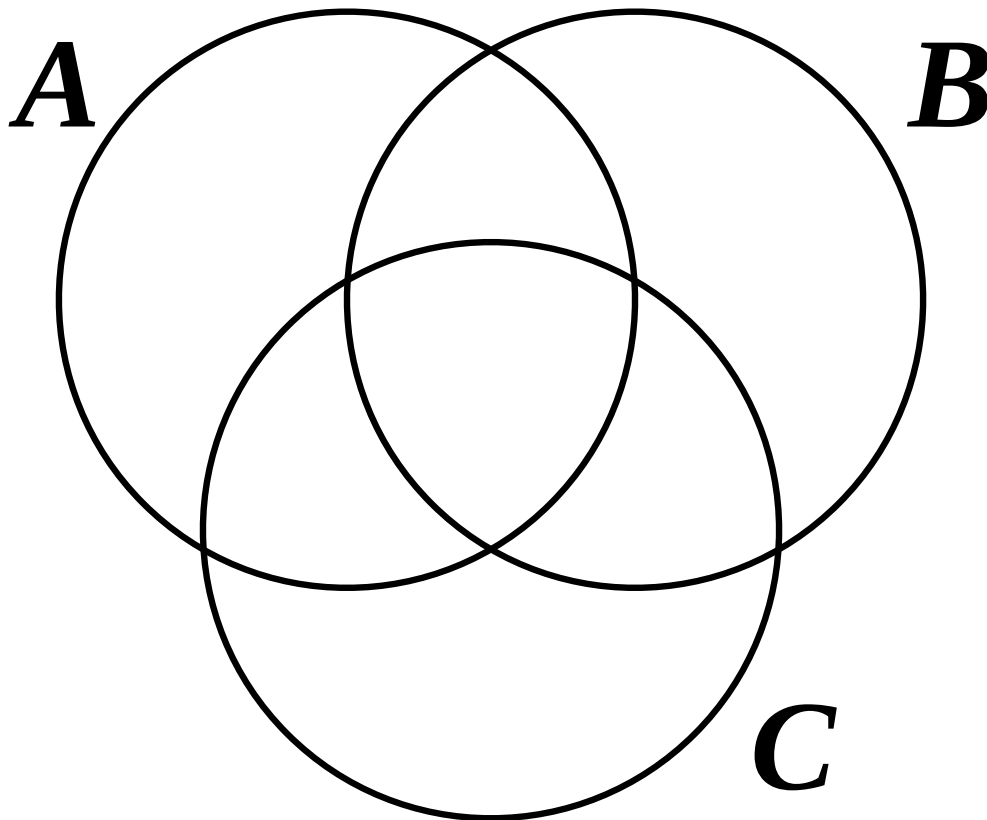
$B = \{ \text{The multiples of 42 less than 700} \}$

[3 marks]

$C = \{ \text{The multiples of 63 less than 700} \}$

[3 marks]

Use this Venn diagram to show the relationship between the sets A , B and C .



[4 marks]

Use the Venn Diagram to answer the following questions.

(i) What is the LCM of 35, 42 and 63 ?

[1 mark]

(ii) What is the LCM of 35 and 42 ?

[1 mark]

(iii) What is the LCM of 35 and 63 ?

[1 mark]

(iv) What is the LCM of 42 and 63 ?

[1 mark]



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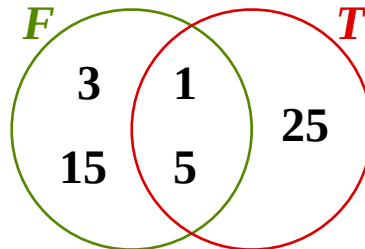
Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk

2.4 Answers

2.4.1 Solutions to 2.2 Example (Teaching Video)

$F = \{ \text{The factors of 15} \} = \{ 1, 3, 5, 15 \}$

$T = \{ \text{The factors of 25} \} = \{ 1, 5, 25 \}$



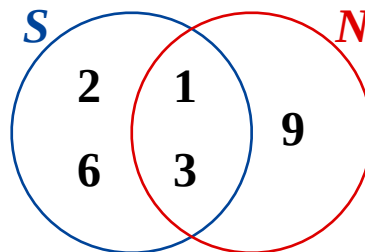
[7 marks]

2.4.2 Solutions to 2.3 Exercise

Answer 1

$S = \{ \text{The factors of 6} \} = \{ 1, 2, 3, 6 \}$

$N = \{ \text{The factors of 9} \} = \{ 1, 3, 9 \}$

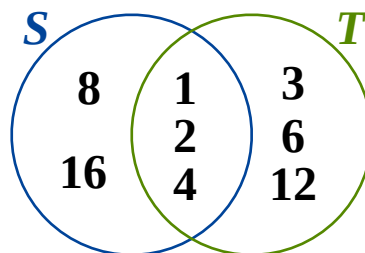


[7 marks]

Answer 2

$S = \{ \text{The factors of 16} \} = \{ 1, 2, 4, 8, 16 \}$

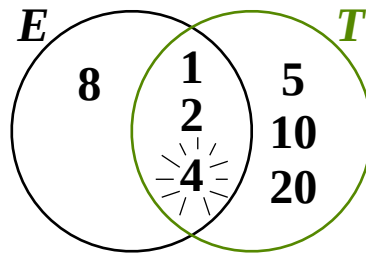
$T = \{ \text{The factors of 12} \} = \{ 1, 2, 3, 4, 6, 12 \}$



[7 marks]

Answer 3

- (i) $E = \{ \text{The factors of } 8 \} = \{ 1, 2, 4, 8 \}$
 $T = \{ \text{The factors of } 20 \} = \{ 1, 2, 4, 5, 10, 20 \}$



- (ii) $\text{HCF} \{ 8, 20 \} = 4$

[7 marks]

- (iii) Cloud around the “4”

[1 mark]

[1 mark]

Answer 4

prime factors (210) = { 2, 3, 5, 7 }

[4 marks]

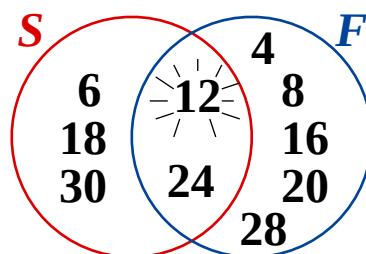
Answer 5

prime factors (12) = { 2, 3 }

[2 marks]

Answer 6

- (i) $S = \{ \text{The first five multiples of } 6 \} = \{ 6, 12, 18, 24, 30 \}$
 $F = \{ \text{The first seven multiples of } 4 \} = \{ 4, 8, 12, 16, 20, 24, 28 \}$



- (ii) $\text{LCM} \{ 6, 4 \} = 12$

[7 marks]

- (iii) Cloud around the “12”

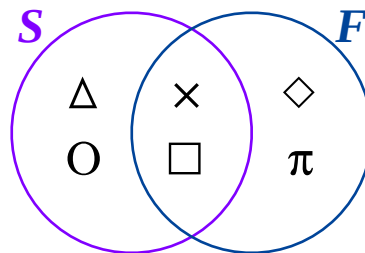
[1 mark]

[1 mark]

Answer 7

$$S = \{ \triangle, \times, O, \square \}$$

$$F = \{ \diamond, \square, \pi, \times \}$$

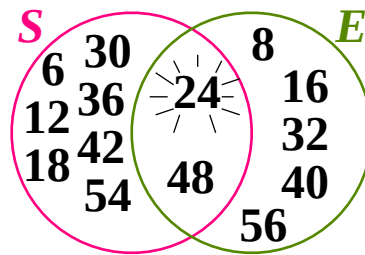


[4 marks]

Answer 8

(i) $S = \{ \text{The first nine multiples of 6} \} = \{ 6, 12, 18, 24, 30, 36, 42, 48, 54 \}$

$E = \{ \text{The first seven multiples of 8} \} = \{ 8, 16, 24, 32, 40, 48, 56 \}$



[7 marks]

(ii) $\text{LCM} \{ 6, 8 \} = 24$

[1 mark]

(iii) Cloud around the “24”

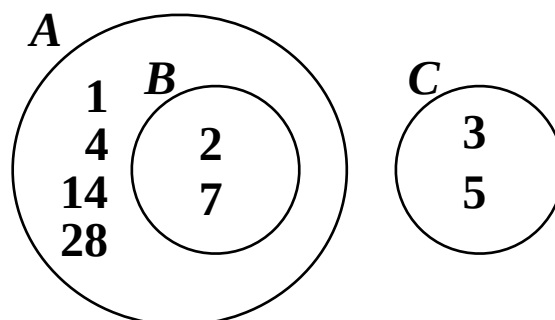
[1 mark]

Answer 9

$$A = \{ \text{factors of 28} \} = \{ 1, 2, 4, 7, 14, 28 \}$$

$$B = \{ \text{prime factors of 28} \} = \{ 2, 7 \}$$

$$C = \{ \text{prime factors of 15} \} = \{ 3, 5 \}$$



[8 marks]

B is a subset of A

[1 mark]

Answer 10

$A = \{ \text{The multiples of 35 less than 700} \}$

$B = \{ \text{The multiples of 42 less than 700} \}$

$C = \{ \text{The multiples of 63 less than 700} \}$

$A = \{ 35, 70, 105, 140, 175, 210, 245, 280, 315, 350, 385, 420, 455, 490, 525, 560, 595, 630, 665 \}$

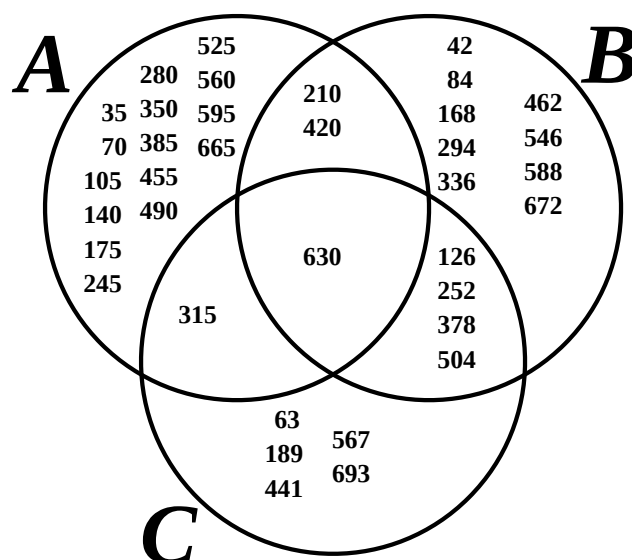
[3 marks]

$B = \{ 42, 84, 126, 168, 210, 252, 294, 336, 378, 420, 462, 504, 546, 588, 630, 672 \}$

[3 marks]

$C = \{ 63, 126, 189, 252, 315, 378, 441, 504, 567, 630, 693 \}$

[3 marks]



(i) $\text{LCM} \{ 35, 42, 63 \} = 630$

[1 mark]

(ii) $\text{LCM} \{ 35, 42 \} = 210$

[1 mark]

(iii) $\text{LCM} \{ 35, 63 \} = 315$

[1 mark]

(iv) $\text{LCM} \{ 42, 63 \} = 126$

[1 mark]

Lesson 3

GCSE Mathematics Set Theory I

3.1 The Elements of a Set

In Set Theory, great importance is attached to whether an object is in a set or not.

The objects that are in a set are called the elements of that set.

Another word for “element” is “member”

In Set Theory the symbol $\in$ means “is an element of”

Think of : $\in$ means “is an element of”

Consider the set of people's names beginning with the letter L

$$L = \{ \text{Libby, Louis, Lyla, Livi, ...} \}$$

I can write

$$Lyla \in L$$

This reads as

“Lyla is an element of set L ”

In other words,

“Lyla is a member of the club of people
who have a name begin with the letter L”

I can also write

$$\text{Archie} \notin L$$

This reads as

“Archie is NOT an element of set L ”

In other words,

“Archie is a NOT member of the club of people
who have a name begin with the letter L”

Here are four statements and you now have to say if they are TRUE or FALSE

(i) $Livi \in L$

(ii) $Livi \notin L$

(iii) $Alex \notin L$

(iv) $Alex \in L$

[4 marks]

As mathematicians we're interested in doing this sort of thing with numbers.

The answer is that the first and third are true and the other two false.

Did you get it correct ?

3.2 Exercise

Marks Available : 100

Question 1

Let $P = \{ \text{Prime Numbers} \}$

i.e. $P = \{ 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, \dots \}$

For each of the following, decide if the given statement is TRUE or FALSE

(i) $7 \in P$

(vi) $17 \in P$

(ii) $9 \notin P$

(vii) $2 \notin P$

(iii) $27 \in P$

(viii) $91 \in P$

(iv) $1 \notin P$

(ix) $369 \notin P$

(v) $49 \notin P$

(x) $\pi \in P$

[10 marks]

Question 2

Let $S = \{ \text{Square Numbers} \}$

i.e. $S = \{ 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, \dots \}$

For each of the following, decide if the given statement is TRUE or FALSE.

(i) $2 \in S$

(vi) $101 \in S$

(ii) $15 \notin S$

(vii) $1 \notin S$

(iii) $36 \in S$

(viii) $400 \in S$

(iv) $91 \notin S$

(ix) $\sqrt{16} \in S$

(v) $100 \notin S$

(x) $\pi \in S$

[10 marks]

The next question is going to ask you whether certain numbers are in the following sequence or not

3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67, 71, 79, 83, ...

Each term is four more than the term before.

Here is a clever “Matched Columns” technique that focuses on the right hand end of the numbers. By starting a new row each time a number in the sequence ends with a 3 a pattern emerges:

3	7	11	15	19
23	27	31	35	39
43	47	51	55	59
63	67	71	75	79
83 ...				

As well as what is there, notice what is not there: there are no numbers that end in a 2 for example.

In fact, can you now see that there are no even numbers in the sequence ?

Good luck with the question...

Question 3

Let $B = \{ \text{The sequence of numbers that begins } 3, 7, 11, 15, 19, 23, \dots \}$

For each of the following, decide if the given statement is TRUE or FALSE.

- | | |
|----------------------|----------------------|
| (i) $15 \in B$ | (vi) $38 \in B$ |
| (ii) $9 \notin B$ | (vii) $25 \in B$ |
| (iii) $24 \in B$ | (viii) $55 \in B$ |
| (iv) $31 \notin B$ | (ix) $104 \in B$ |
| (v) $21 \in B$ | (x) $526 \notin B$ |

[10 marks]

Question 4

Let,

$A = \{ \text{The first five triangular numbers} \}$

$B = \{ \text{The factors of 15} \}$

$C = \{ \text{The *prime* factors of 390} \}$

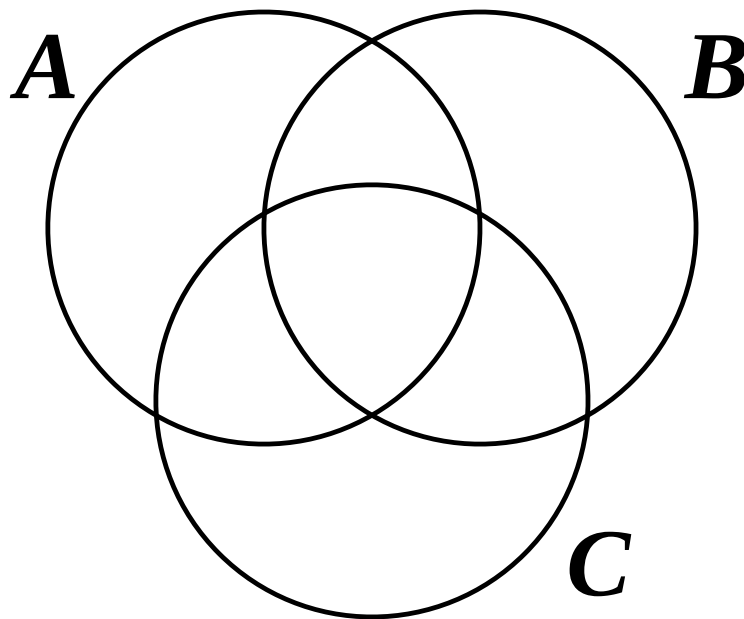
List the elements of sets A , B and C below,

$A = \{ \text{_____, _____, _____, _____, _____} \}$

$B = \{ \text{_____, _____, _____, _____} \}$

$C = \{ \text{_____, _____, _____, _____} \}$

Use the three hoop Venn Diagram to show the relationship between sets A , B and C .



[10 marks]

Question 5

Let $H = \{ \text{The sequence of numbers that begins } 1, 5, 9, 13, \dots \}$

Use the question 3 “Matched columns” technique to help answer the next question...
Begin by writing out the first 25 members of the set in the spaces below,

_____, _____, _____, _____, _____,

_____, _____, _____, _____, _____,

_____, _____, _____, _____, _____,

_____, _____, _____, _____, _____,

_____, _____, _____, _____, _____,

For each of the following, decide if the given statement is TRUE or FALSE.

(i) $13 \in H$

(vi) $27 \in H$

(ii) $19 \notin H$

(vii) $63 \in H$

(iii) $29 \in H$

(viii) $4568 \in H$

(iv) $18 \notin H$

(ix) $91 \in H$

(v) $85 \in H$

(x) $207 \notin H$

[10 marks]

Question 6

Let,

$D = \{ \text{The factors of 36} \}$

$E = \{ \text{The first eight triangular numbers} \}$

$F = \{ \text{The first six multiples of 6} \}$

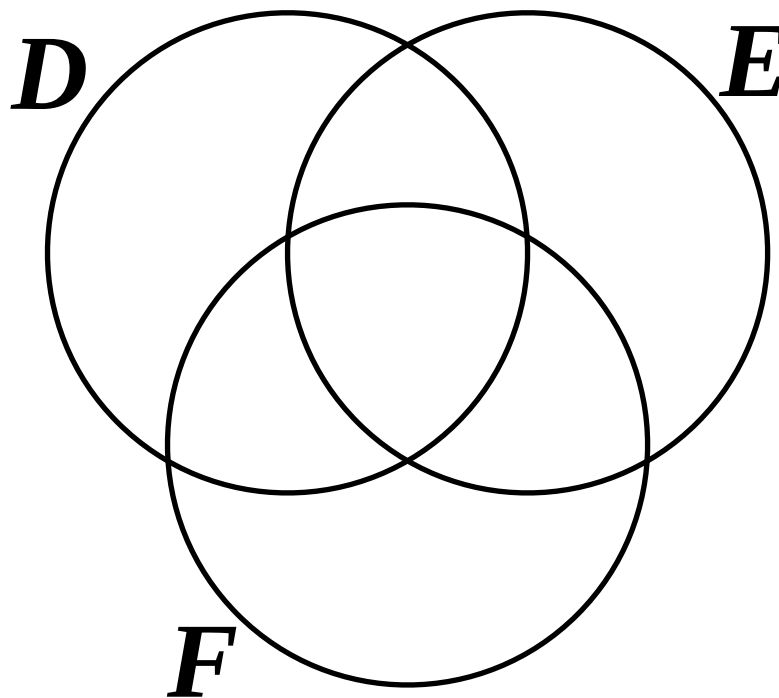
List the elements of sets D , E and F below,

$D = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

$E = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

$F = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

Use the Venn Diagram to show the relationship between sets D , E and F .



[10 marks]

Question 7

Let $D = \{ \text{The sequence of numbers that begins } 2, 4, 8, 16, \dots \}$

Use the “Matched Columns” technique to help answer the next question.
Begin by writing out the first 16 members of the set in the spaces below,

_____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ ,

For each of the following, decide if the given statement is TRUE or FALSE.

(i) $42 \in D$

(vi) $2^5 \in D$

(ii) $32 \notin D$

(vii) $2^{200} \in D$

(iii) $512 \in D$

(viii) $1\,200\,875 \in D$

(iv) $8008 \notin D$

(ix) $3^4 \in D$

(v) $2^3 \in D$

(x) $2^{43} - 1 \notin D$

[10 marks]

Question 8

Let $P = \{ \text{The sequence of numbers that begins } 3, 6, 12, 24, \dots \}$

Use the “Matched Columns” technique to help answer the next question...

Begin by writing out the first 13 members of the set in the spaces below,

_____, _____, _____, _____, _____,

_____, _____, _____, _____,

_____, _____, _____, _____,

For each of the following, decide if the given statement is TRUE or FALSE.

(i) $3 \in P$ (vi) $2^3 \times 3 \in P$

(ii) $33 \in P$ (vii) $2^7 \times 3 \in P$

(iii) $33\,333 \in P$ (viii) $12\,288 \in P$

(iv) $2 \times 3 \in P$ (ix) $10^8 \in P$

(v) $2^2 \times 3 \in P$ (x) $2^{40} \notin P$

[10 marks]

Question 9

Apply the “Matched Columns” technique to the square numbers.

i.e. $S = \{ 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, \dots \}$

You'll need to zig-zag the numbers.

_____ , _____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ , _____ ,

_____ , _____ , _____ , _____ , _____ ,

[4 marks]

What digits can a square number NOT end in ?

[4 marks]

Question 10

The “Matched Columns” trick does NOT work with triangular numbers. :-(

However, just as four endings are not possible with square numbers, four endings are not possible with triangular numbers.

Play around until you can see what those impossible endings are.

What digits can a triangular number NOT end in ?

[6 marks]

Question 11

2 is the least positive integer to have two factors, $\{ 1, 2 \}$

4 is the least positive integer to have three factors, $\{ 1, 2, 4 \}$

6 is the least positive integer to have four factors, $\{ 1, 2, 3, 6 \}$

You might think that this *least integer to have n factors* sequence goes, 2, 4, 6, 8.

But don't jump to conclusions !

Show that 8 is not the least integer to have five factors.

What is the least integer to have five factors ?

The sequence is becoming even more weird...

What is the least integer to have six factors ?

Continue the sequence a little further, if you have time.

[6 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk

3.3 Answers to 3.2 Exercise

Answer 1

- (i) T
- (ii) T
- (iii) F
- (iv) T
- (v) T

- (vi) T
- (vii) F
- (viii) F
- (ix) T
- (x) F

$$7 \times 13 = 91$$

[10 marks]

Answer 2

- (i) F
- (ii) T
- (iii) T
- (iv) T
- (v) F

- (vi) F
- (vii) F
- (viii) T
- (ix) T
- (x) F

[10 marks]

Answer 3

- (i) T
- (ii) T
- (iii) F
- (iv) F
- (v) F

- (vi) F
- (vii) F
- (viii) T
- (ix) F
- (x) T

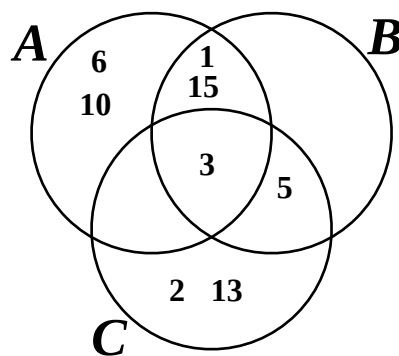
[10 marks]

Answer 4

$$A = \{ 1, 3, 6, 10, 15 \}$$

$$B = \{ 1, 3, 5, 15 \}$$

$$C = \{ 2, 3, 5, 13 \}$$



[10 marks]

Answer 5

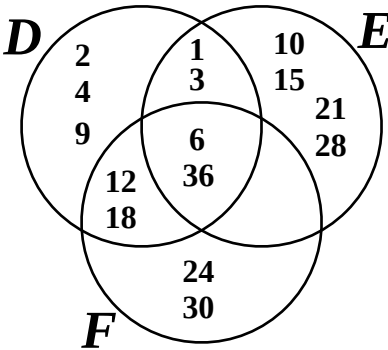
1	5	9	13	17
21	25	29	33	37
41	45	49	53	57
61	65	69	73	77
81	85			

- | | | | |
|---------|---|----------|---|
| (i) | T | (vi) | F |
| (ii) | T | (vii) | F |
| (iii) | T | (viii) | F |
| (iv) | T | (ix) | F |
| (v) | T | (x) | T |

[10 marks]

Answer 6

$D = \{ 1, 2, 3, 4, 6, 9, 12, 18, 36 \}$
 $E = \{ 1, 3, 6, 10, 15, 21, 28, 36 \}$
 $F = \{ 6, 12, 18, 24, 30, 36 \}$



[10 marks]

Answer 7

2	4	8	16
32	64	128	256
512	1024	2048	4096
8192	16384	32768	65536

- | | | | |
|---------|---|----------|---|
| (i) | F | (vi) | T |
| (ii) | F | (vii) | T |
| (iii) | T | (viii) | F |
| (iv) | T | (ix) | F |
| (v) | T | (x) | T |

[10 marks]

Answer 8

3	6	12	24	48
	96	192	384	768
	1536	3072	6144	12288

(i)	T	(vi)	T
(ii)	T	(vii)	T
(iii)	F	(viii)	T
(iv)	F	(ix)	F
(v)	T	(x)	T

[10 marks]**Answer 9**

	1	4	9	16	25
100	81	64	49	36	
	121	144	169	196	225
400	361	324	289	256	
	441	484	529	576	625
900	841	784	729	676	
	961	1024	1089	1156	1225
1600	1521	1444	1369	1296	
	1681	1764	1849	1936	2025
2500	2401	2304	2209	2116	
	2601	2704	2809	2916	3025
3600	3481	3364	3249	3136	
	3721	3844	3969	4096	4225

[4 marks]

Square numbers never end in 2, 3, 7 or 8

[4 marks]**Answer 10**

Triangular numbers never end in a 2, 4, 7 or 9

1	3	6	10	15	21	28	36	45
55	78	91	105	120	136	153	136	171
190	210	231	253	276	300	325	351	378

[6 marks]**Answer 11**

2, 4, 6, 16, 12, 64, 24, 36, 48, 128, 60, ...

[6 marks]

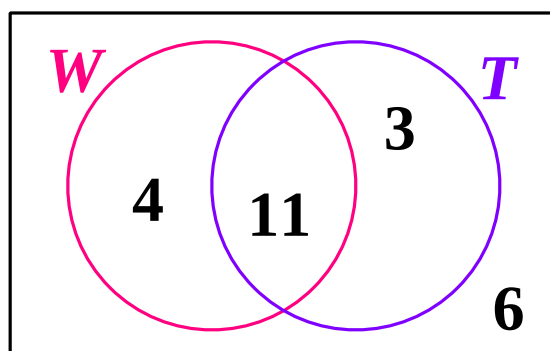
4.1 Notation : Intersection, NOT



Are you a Star Wars or a Star Trek fan ?

Here is a Venn Diagram that shows how a class of 24 pupils answered that question.
In the diagram

- W is the hoop containing fans of Star Wars
- T is the hoop containing fans of Star Trek



Here is a list of what a question may ask you for;

- $n(W)$ number of pupils who are fans of Star Wars
- $n(T)$ number of pupils who are fans of Star Trek
- $n(W')$ number of pupils who are NOT fans of Star Wars
- $n(T')$ number of pupils who are NOT fans of Star Trek
- $n(W \cap T)$ number of pupils who are fans of Star Wars and Star Trek
- $n(W \cap T')$ number of pupils who are fans of Star Wars but are NOT of Star Trek
- $n(W' \cap T)$ number of pupils who are NOT fans of Star Wars but are of Star Trek
- $n(W' \cap T')$ number of pupils who are NOT fans of Star Wars and NOT of Star Trek

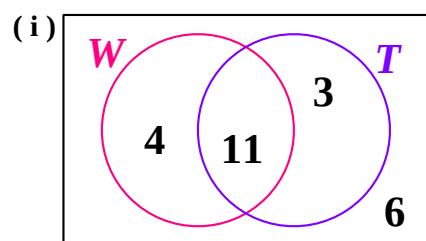
4.2 Example

Teaching Video : <http://www.NumberWonder.co.uk/v9003/5.mp4>

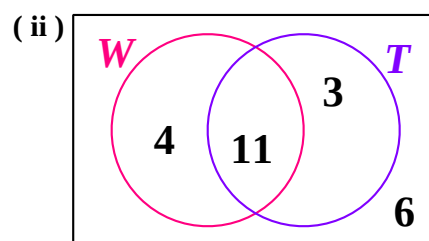


Complete the question as you watch the video

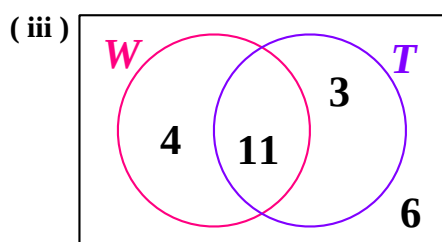
In each Venn Diagram, shade in the region specified and hence give the total number of pupils in that region,



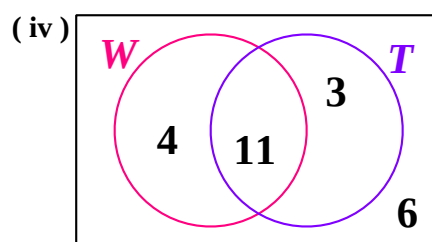
$$n(W) =$$



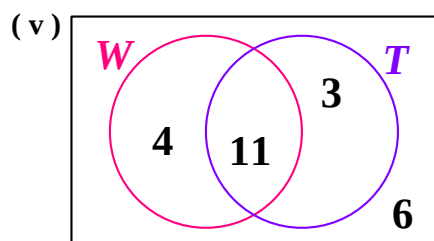
$$n(W') =$$



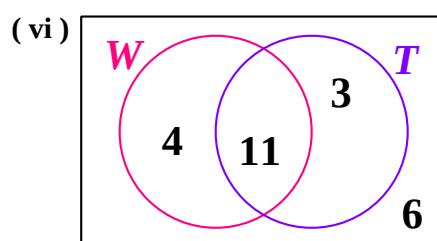
$$n(T) =$$



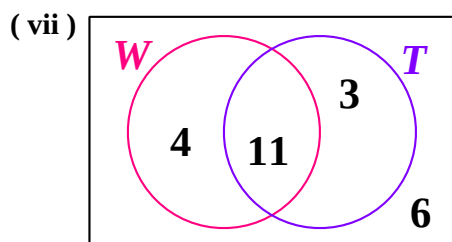
$$n(T') =$$



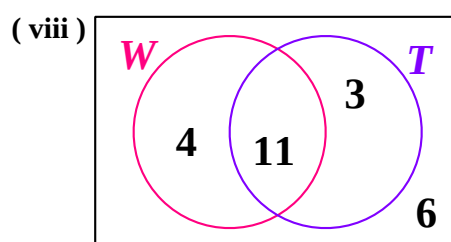
$$n(W \cap T) =$$



$$n(W \cap T') =$$



$$n(W' \cap T) =$$



$$n(W' \cap T') =$$

[16 marks]

4.3 You Try

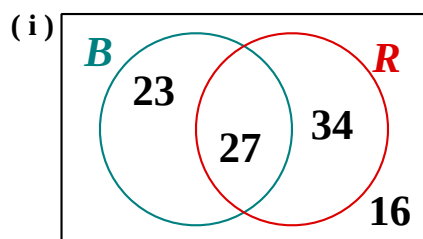


Are you a fan of The Beatles or The Rolling Stones ?

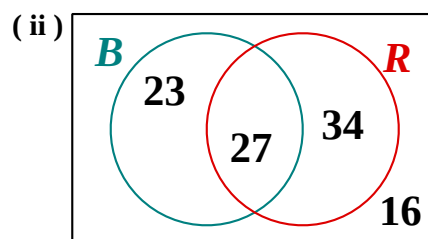
Here is a Venn Diagram that shows how 100 sixth form students responded.

- B is the hoop containing Beatles fans
- R is the hoop containing Rolling Stones fans

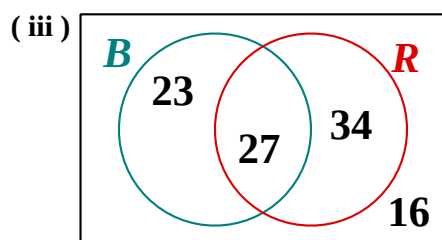
In each Venn Diagram, shade in the region specified and hence give the total number of sixth form students in that region.



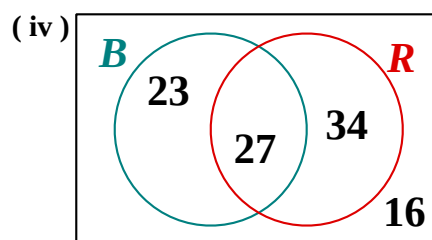
$$n(B \cap R) =$$



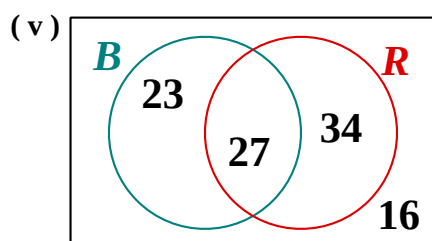
$$n(R) =$$



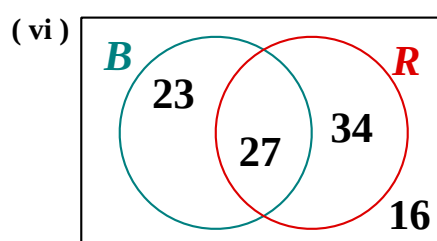
$$n(B' \cap R') =$$



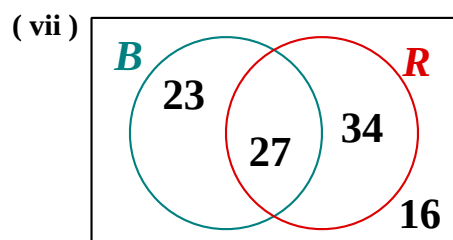
$$n(R') =$$



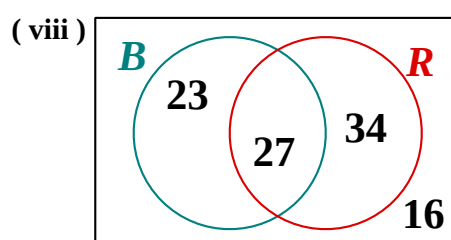
$$n(B' \cap R) =$$



$$n(B) =$$



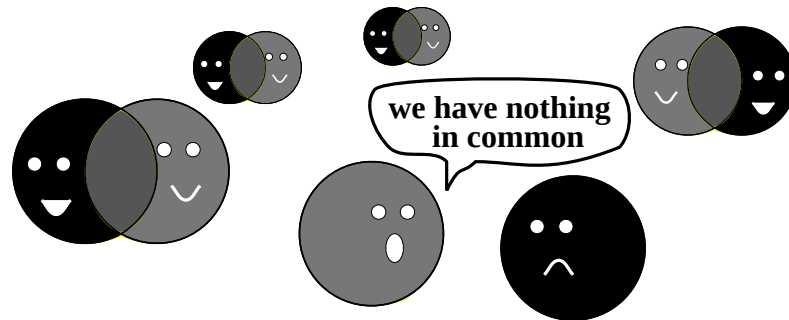
$$n(B \cap R') =$$



$$n(B') =$$

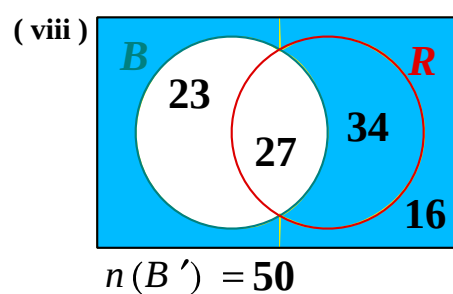
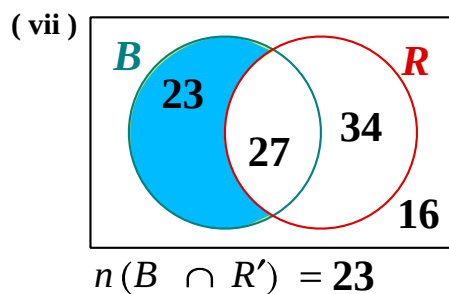
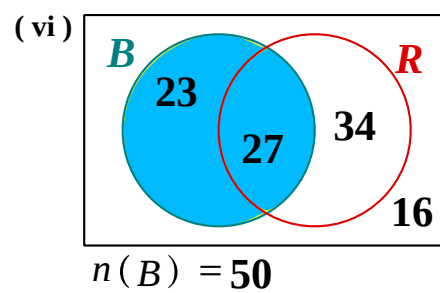
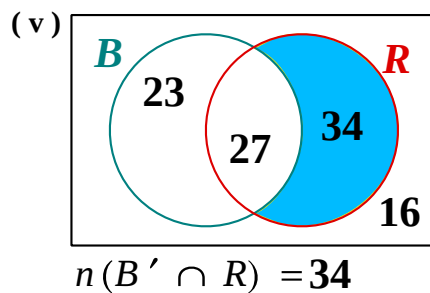
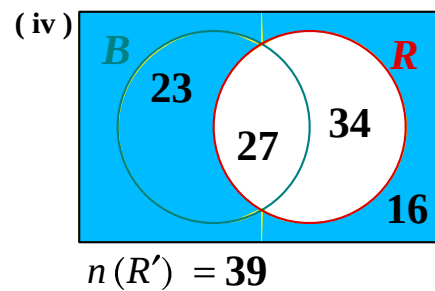
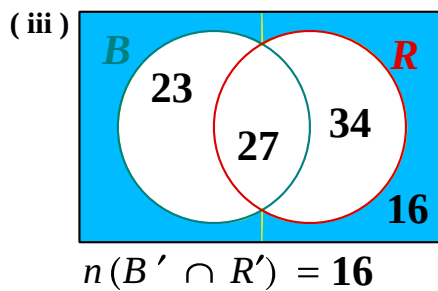
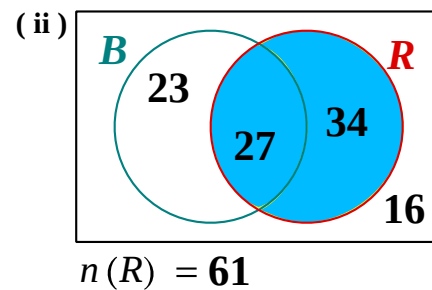
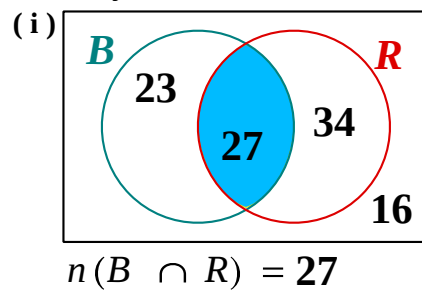
[16 marks]

4.4 Venn Diagram Joke !



Harold had to face the painful truth ; He and Daisy
were never going to be a Venn diagram.

4.5 You Try Answers



How did you do ?

[16 marks]

Tick the box that applies: ☐ Ace ☐ One Hiccup ☐ Com Si Com Sa ☐ Pants

4.6 Exercise

Marks Available : 24

Question 1

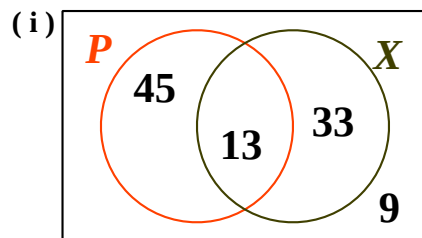


Are you an enthusiast of Play Station 5 or XBOX series X ?

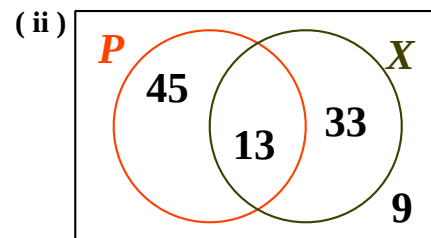
Here is a Venn Diagram that shows how 100 teenagers responded.

- P is the hoop containing Play Station 5 enthusiasts
- X is the hoop containing XBOX series X enthusiasts

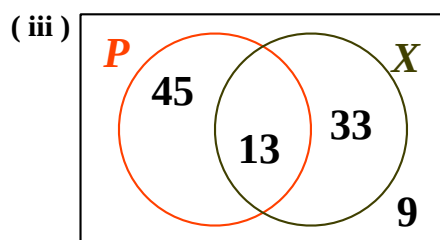
In each Venn Diagram, shade in the region specified and hence give the total number of teenagers in that region.



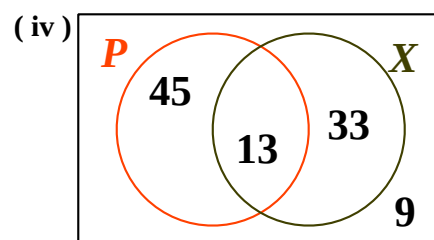
$$n(X) =$$



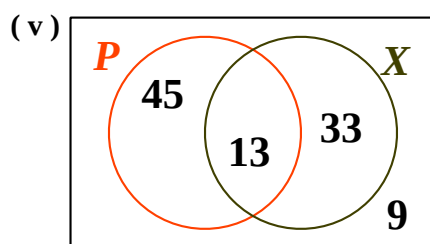
$$n(P' \cap X) =$$



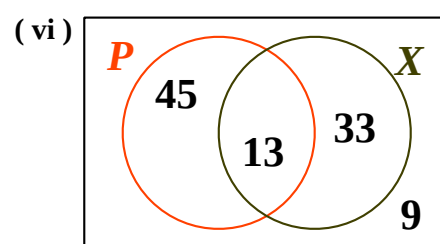
$$n(P \cap X') =$$



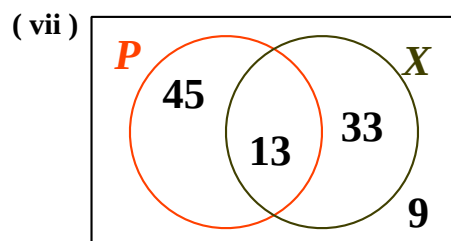
$$n(P) =$$



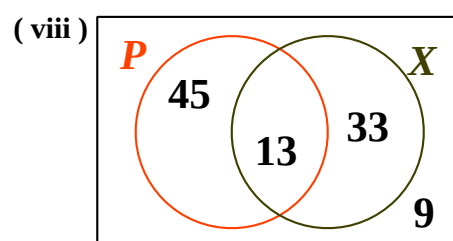
$$n(P') =$$



$$n(P' \cap X') =$$



$$n(X') =$$



$$n(P \cap X) =$$

[16 marks]

Question 2

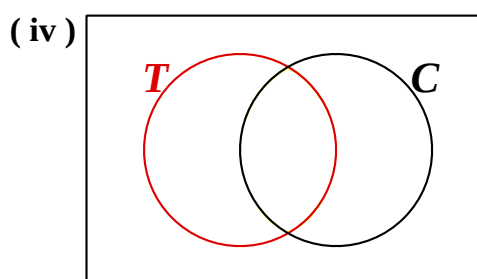
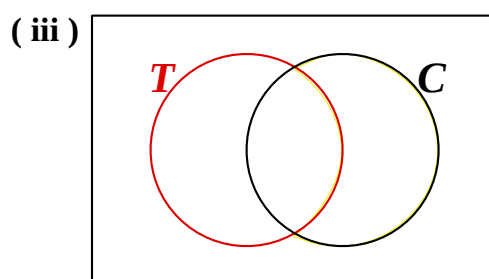
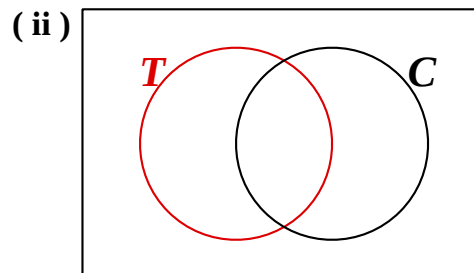
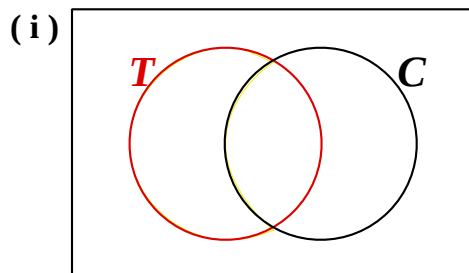
On the Venn Diagrams below, shade the part that represents;

(i) C

(ii) C'

(iii) $C \cap T$

(iv) $(C \cap T)'$



[4 marks]

Question 3

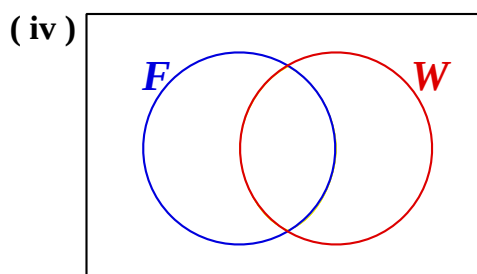
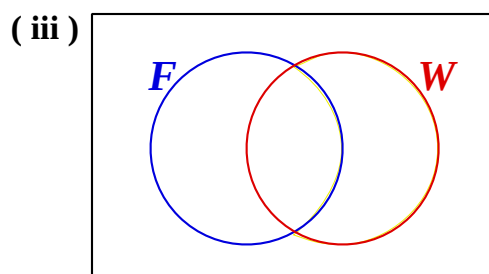
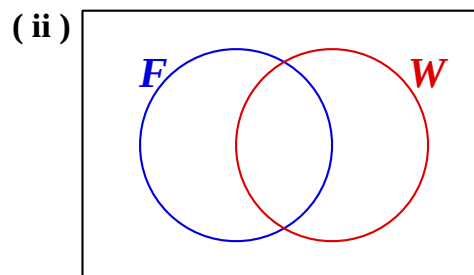
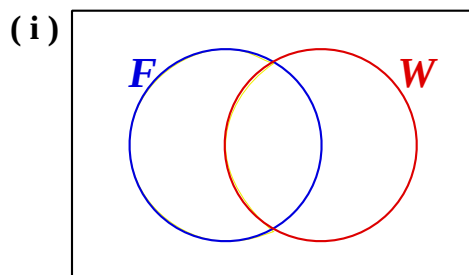
On the Venn Diagrams below, shade the part that represents;

(i) F'

(ii) $F' \cap W$

(iii) $F' \cap W'$

(iv) $(F' \cap W')'$



[4 marks]

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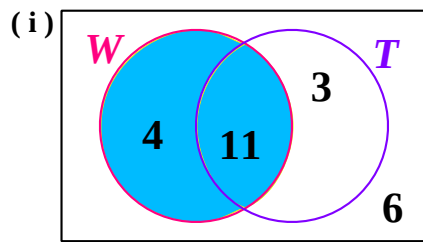
In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

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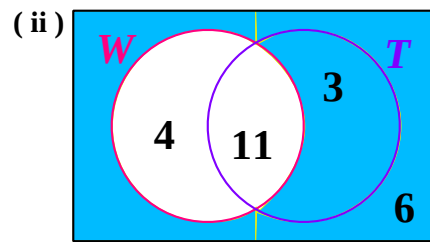
Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk

4.7 Answers

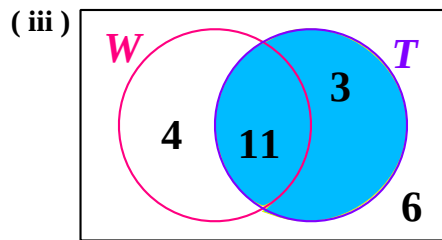
4.7.1 Solutions to 4.2 Example (Teaching Video)



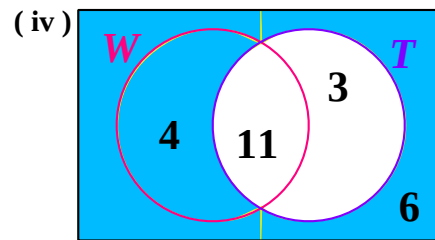
$$n(W) = 15$$



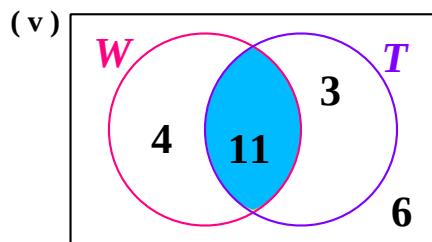
$$n(W') = 9$$



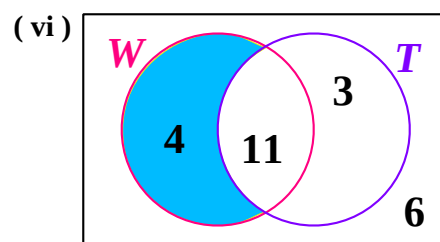
$$n(T) = 14$$



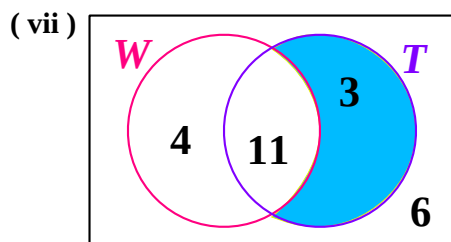
$$n(T') = 10$$



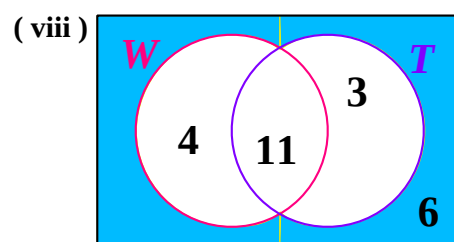
$$n(W \cap T) = 11$$



$$n(W \cap T') = 4$$



$$n(W' \cap T) = 3$$

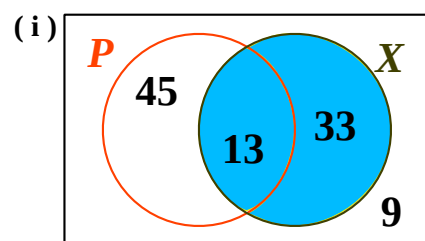


$$n(W' \cap T') = 6$$

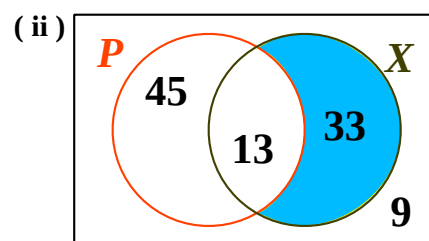
[16 marks]

4.7.2 Solutions to 5.6 Exercise

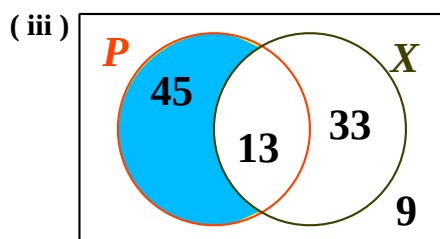
Answer 1



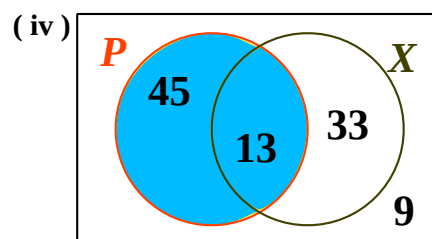
$$n(X) = 46$$



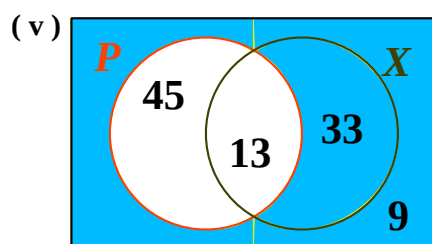
$$n(P' \cap X) = 33$$



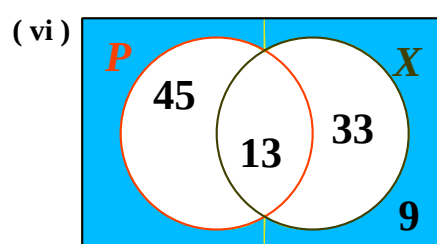
$$n(P \cap X') = 45$$



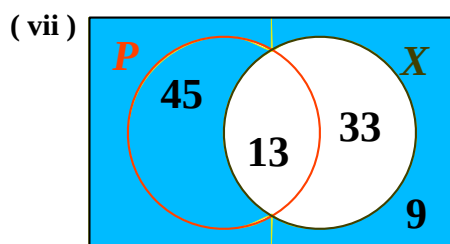
$$n(P) = 58$$



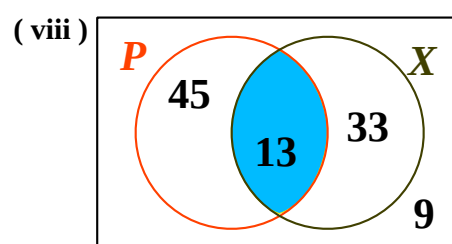
$$n(P') = 42$$



$$n(P' \cap X') = 9$$



$$n(X') = 54$$



$$n(P \cap X) = 13$$

[16 marks]

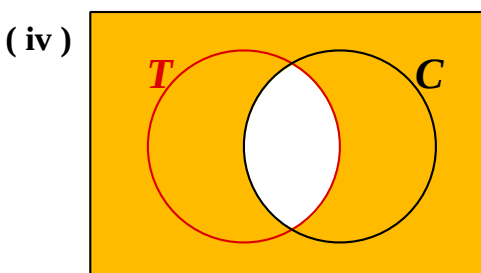
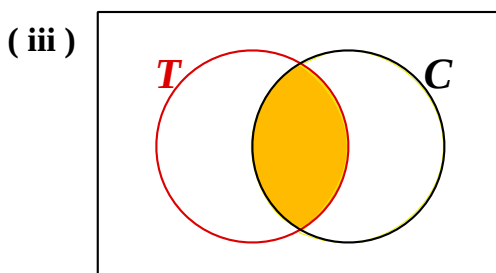
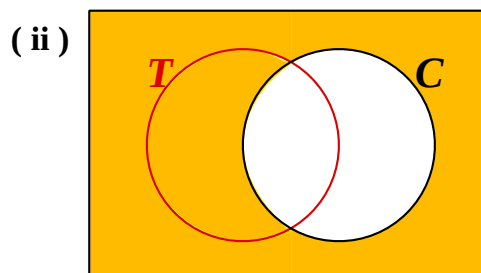
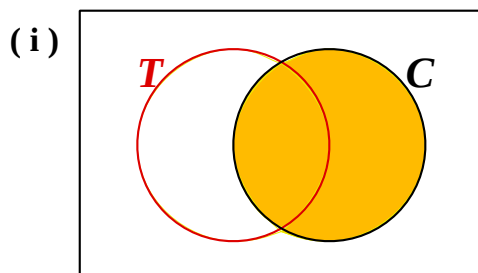
Answer 2

(i) C

(ii) C'

(iii) $C \cap T$

(iv) $(C \cap T)'$



[4 marks]

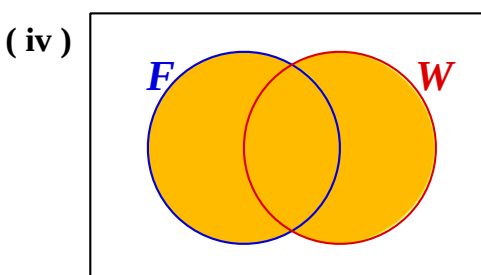
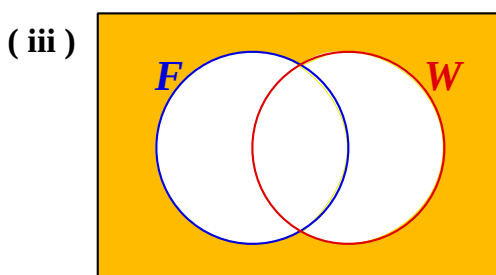
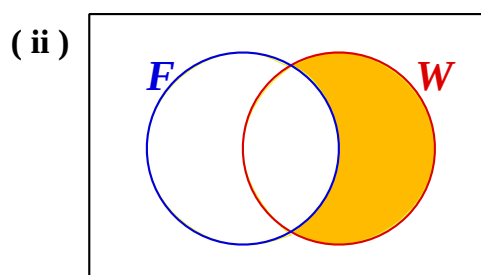
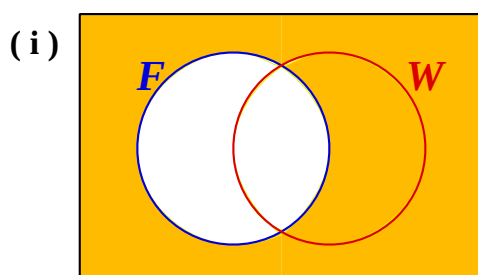
Answer 3

(i) F'

(ii) $F' \cap W$

(iii) $F' \cap W'$

(iv) $(F' \cap W')'$



[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk

5.1 Venn Diagram Problem Solving



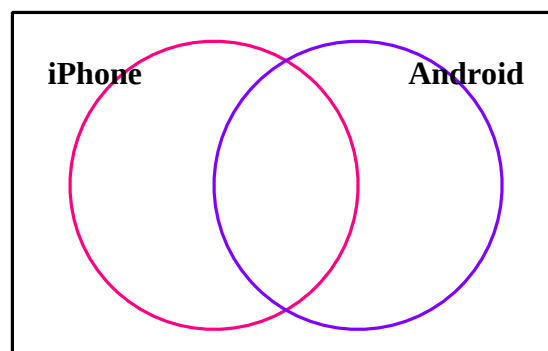
A group of 75 students were asked if they'd like to own an iPhone™
They were also asked if they'd like to own an Android™ phone

- ◇ 40 said they'd like to own an iPhone™
- ◇ 50 said they'd like to own an Android™ phone
- ◇ 35 said they'd like to own both an iPhone™ and an Android™ phone

How many students didn't wish to own either phone ?

[5 marks]

Teaching Video : <http://www.NumberWonder.co.uk/v9003/4.mp4>



5.2 Exercise

Marks Available : 56

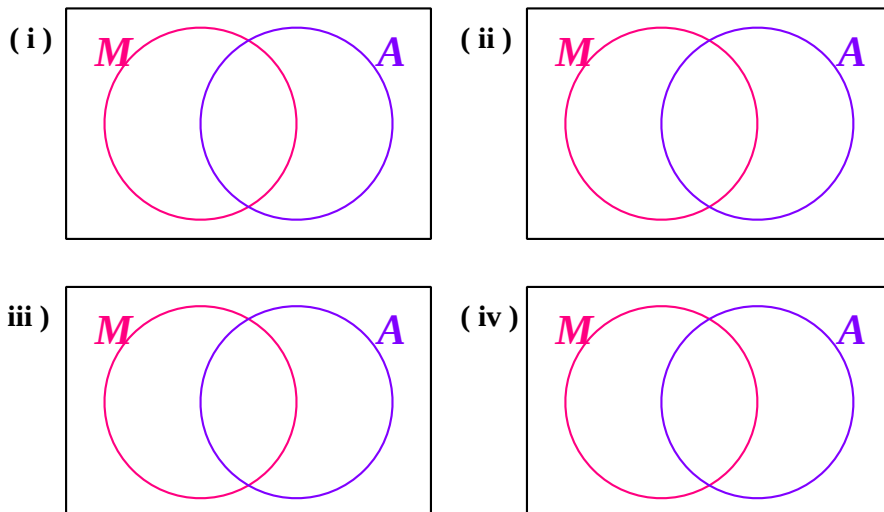
Question 1

At a school there is a maths club, the **MathMagicians**, *M*.

There is also an art club, the **Artisans**, *A*.

On a Venn Diagrams below, shade the part that represents;

- (i) The pupils who belong to the Maths club.
- (ii) The pupils who belong to both clubs.
- (iii) The pupils who belong to only one of the two clubs.
- (iv) The pupils who do not belong to either club.



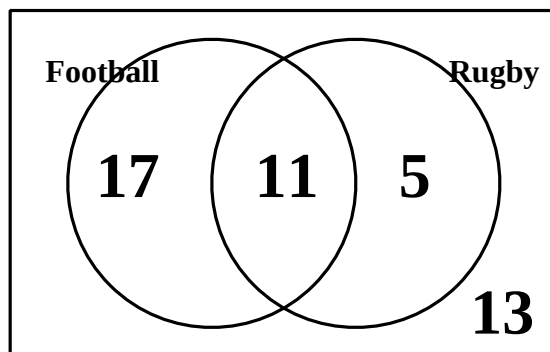
[4 marks]

Question 2

A group of students were asked if they played football.

They were also asked if they played rugby.

The Venn Diagram provides a summary of their replies.



- (i) How many students play football only ?
- (ii) How many students play football ?
- (iii) How many students don't play rugby ?
- (iv) How many students play exactly one of the two sports ?

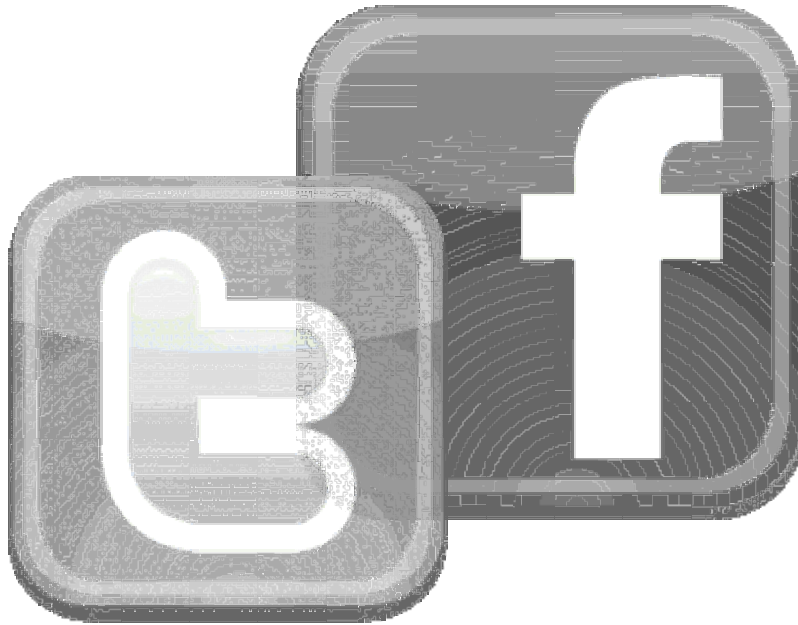
[1 mark]

[1 mark]

[1 mark]

[1 mark]

Question 3

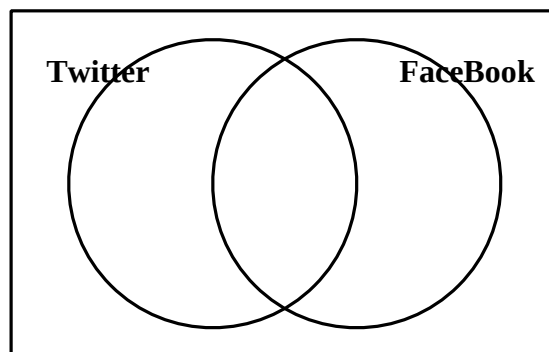


A group of 100 students were asked if they used Twitter™ regularly. They were also asked if they used FaceBook™ regularly.

Here is a summary of their answers:

- ◇ 36 said they used Twitter™ regularly.
- ◇ 44 said they used FaceBook™ regularly.
- ◇ 20 said they used both and Twitter™ and FaceBook™ regularly.

(i) Complete the Venn Diagram to help clarify the students' replies :



(ii) How many students use only Twitter™ regularly ?

[4 marks]

(iii) How many students use neither Twitter™ nor FaceBook™ ?

[1 mark]

(iv) How many students do not use FaceBook™ regularly ?

[1 mark]

[1 mark]

Question 4

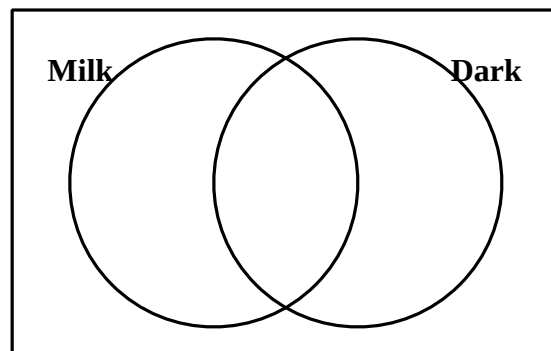


A class of 24 pupils were asked if they liked **milk** chocolate. They were also asked if they liked **dark** chocolate.

Here is a summary of their answers:

- ◇ 9 said they liked **milk** chocolate only
- ◇ 10 said they didn't like **milk** chocolate
- ◇ 3 said they did not like chocolate

(i) Complete the Venn Diagram to help clarify the pupils' replies.



[4 marks]

(ii) How many pupils like both **milk** and **dark** chocolate ?

[1 mark]

(iii) How many pupils like **dark** chocolate ?

[1 mark]

(iv) How many pupils don't like **dark** chocolate ?

[1 mark]

(v) In a similar group of 96 pupils, estimate how many would not like **milk** chocolate.

[1 mark]

Question 5

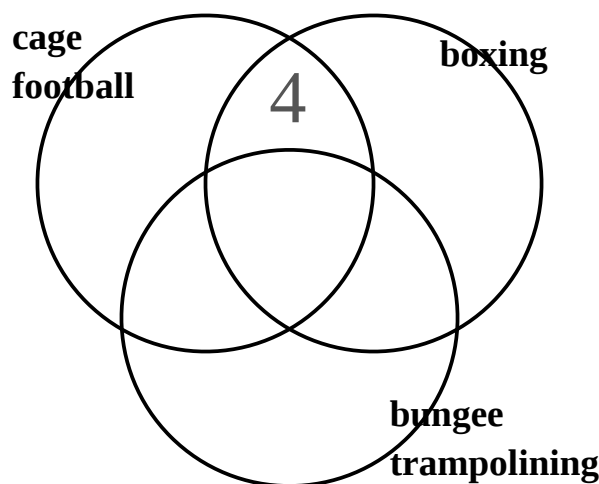
A sports club has 80 members.

They all play at least one of cage football, boxing, or bungee trampolining.

In talking to a newspaper about the sports club the manager states that:

- ◇ 16 members do all three activities.
- ◇ 20 members do cage football and boxing.
- ◇ 40 members do boxing and bungee trampolining.
- ◇ 24 members do bungee trampolining and cage football.
- ◇ 60 members do bungee trampolining.
- ◇ 46 members do boxing.

- (i) Complete the Venn Diagram to help clarify the manager's statements.
(The 4 is in there so you can check you're getting it right)



- (ii) How many members play only cage football ? [6 marks]
- (iii) How many members play cage football ? [1 mark]
- (iv) How many members play exactly one of the three activities ? [1 mark]
- (v) How many members play exactly two of the three activities ? [1 mark]

Question 6

In a class of 24 pupils there are 8 who are left handed.

There are 15 boys in the class

If 5 boys are left handed, how many girls are not left handed ?

[4 marks]

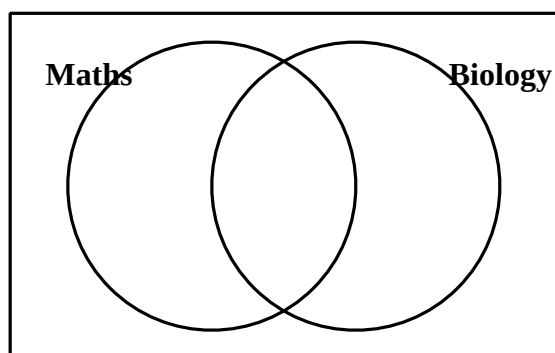
Question 7

A school has 200 students in its 6th form.

The director of studies reports that;

- ◇ 100 study maths.
- ◇ 58 study biology.
- ◇ 68 study neither maths nor biology.
- ◇ 142 don't study biology.

- (i) Complete the Venn Diagram to clarify the director of studies' report.



[4 marks]

- (ii) How many students study maths and biology ?

[1 mark]

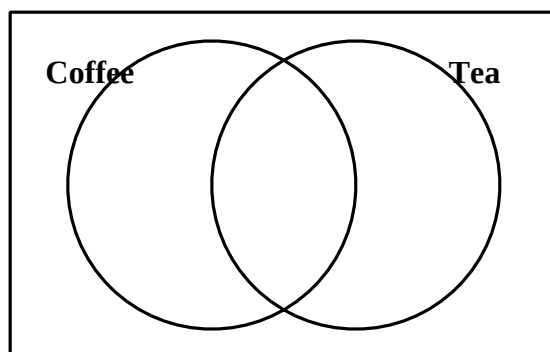
Question 8

In a survey, 100 members of the public are asked if they like coffee.

They are also asked if they like tea.

- ◇ 45 said they did not like coffee.
- ◇ 35 said they did not like tea.
- ◇ 28 said they liked neither coffee nor tea.

- (i) Complete the Venn diagram to clarify the survey's findings.



[4 marks]

- (ii) How many liked both coffee and tea ?

[1 mark]

Question 9



Sherlock Holmes is investigating a theft from a bank safe.
He knows that one of the bank's 40 workers is the thief.
(It was clearly an 'inside job')

At the crime scene, on top of the safe, he finds a pair of glasses.
These, deduces Holmes, were used by the thief to read the numbers on the combination lock of the safe, then placed on top, and left in error.

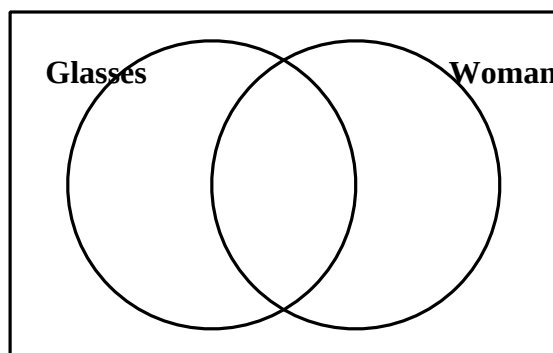
He also finds a used lipstick, which Holmes realises fell out of a woman's handbag when the money from the safe was stuffed into it.

Reasoning that men who work in a bank do not use lipstick or carry handbags, Holmes eliminates the 8 men with perfect eyesight from his list of suspects.

At the bank, 11 workers wear glasses and 23 are women.

Sherlock asks you, his trusty assistant, to draw a Venn Diagram in order to clarify the facts.

Do so, here :



[4 marks]

How many of the bank workers are women who wear glasses, and are thus suspected of committing the crime ?

[1 mark]

Question 10

Clare and Lucy are twin sisters.

They have 20 lipsticks in their collection.

Clare likes 16 of the lipsticks, and Lucy likes 12.

Complete this sentence:

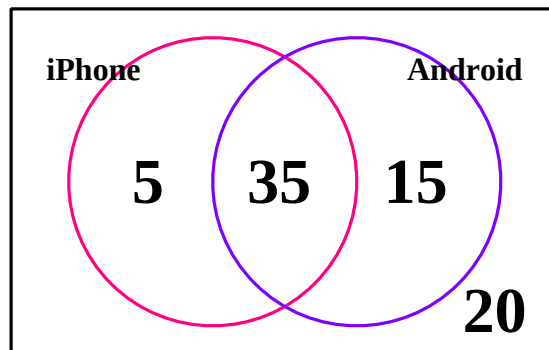
Clare and Lucy both like between ____ and ____ lipsticks.

[4 marks]

The answer to this question is not a single number.

5.3 Answers

5.3.1 Solution to 5.1 Example (Teaching Video)



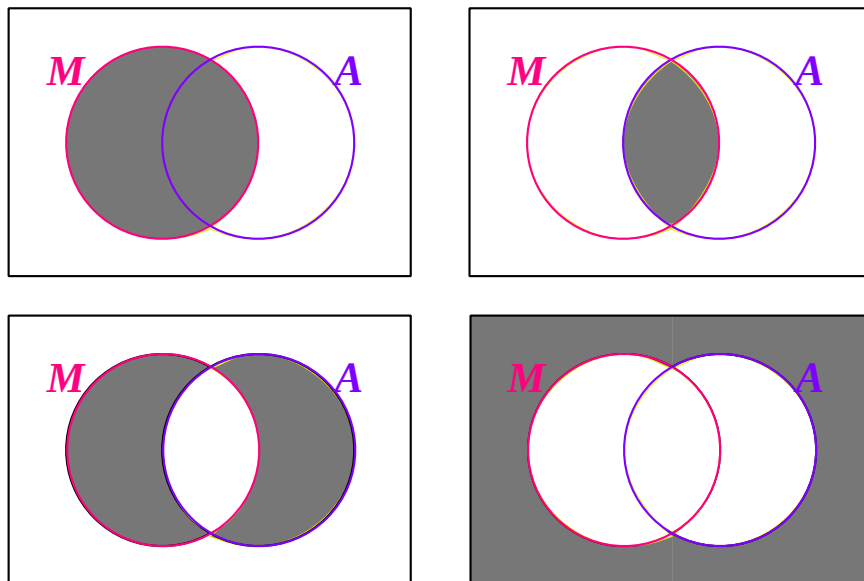
[4 marks]

20 did not wish to own either phone.

[1 mark]

5.3.2 Solutions to 4.2 Exercise

Answer 1



[4 marks]

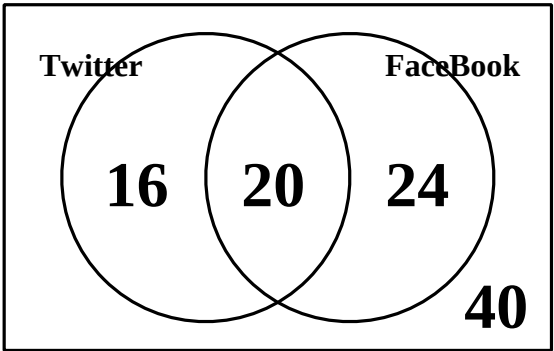
Answer 2

- (i) 17
- (ii) 28
- (iii) 30
- (iv) 22

[4 marks]

Answer 3

(i)



[4 marks]

(ii) 16

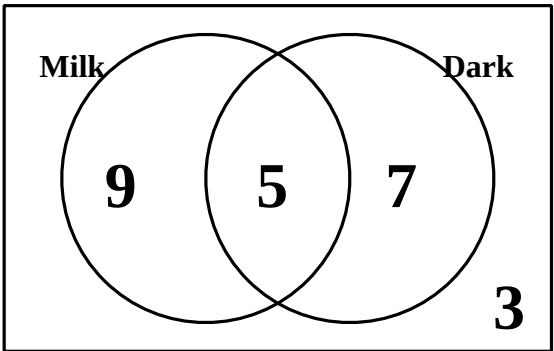
(iii) 40

(iv) 56

[1, 1, 1 marks]

Answer 4

(i)



[4 marks]

(ii) 5

(iii) 12

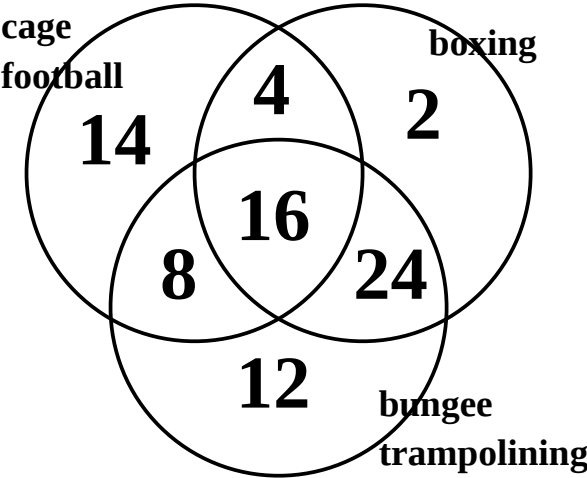
(iv) 12

(v) 40

[1, 1, 1, 1 marks]

Answer 5

(i)



[6 marks]

(ii) 14

(iii) 42

(iv) 28

(v) 36

[1, 1, 1, 1 marks]

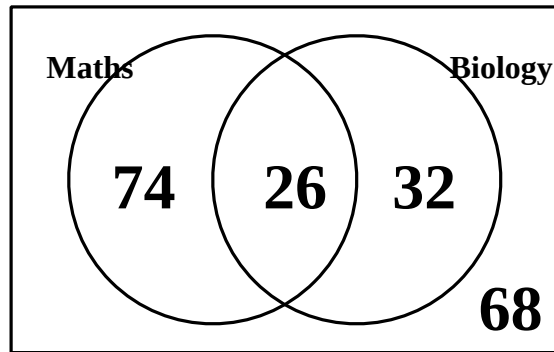
Answer 6

6

[4 marks]

Answer 7

(i)



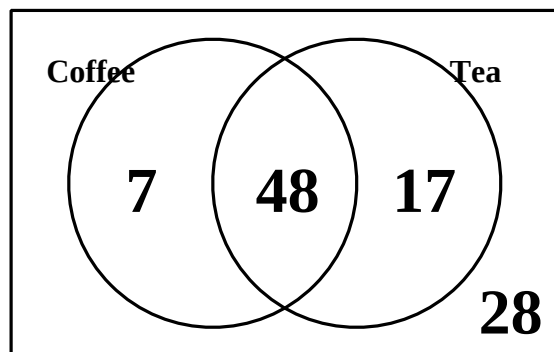
[4 marks]

(ii) 26

[1 mark]

Answer 8

(i)

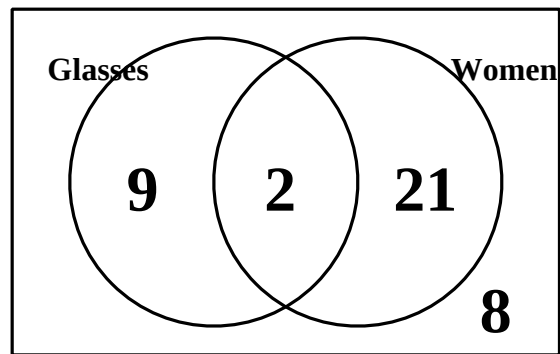


[4 marks]

(ii) 48

[1 mark]

Answer 9



[4 marks]

2 suspects

[1 mark]

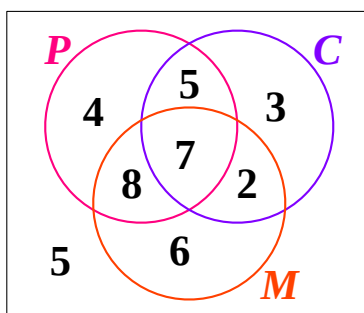
Answer 10

Clare and Lucy both like between 8 and 12 lipsticks

[4 marks]

6.1 The Three Hoop Venn Diagram

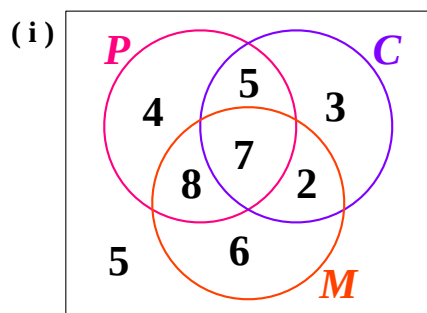
The number of sixth form students who study Physics or Chemistry or Maths at a small sixth form college is;



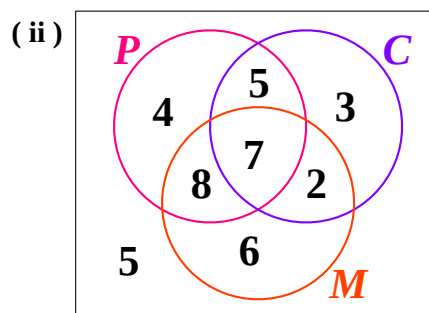
Where • P is the hoop containing Physicists
 • C is the hoop containing Chemists
 and • M is the hoop containing Mathematicians

Before watching the teaching video have a go at the following question;

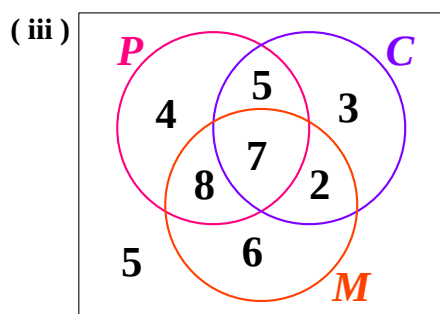
In each of the following Venn Diagram, shade in the region specified and hence give the total number of sixth form students in that region.



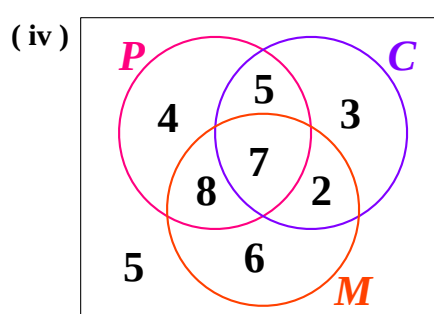
$$n(M) =$$



$$n(P \cap C) =$$



$$n(P \cap C \cap M') =$$



$$n(C \cap M)' =$$

[8 marks]

Teaching Video : <http://www.NumberWonder.co.uk/v9003/6.mp4>



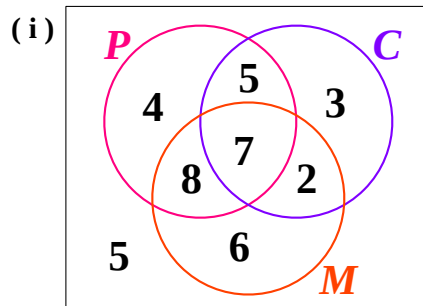
6.2 Exercise

Marks Available : 35

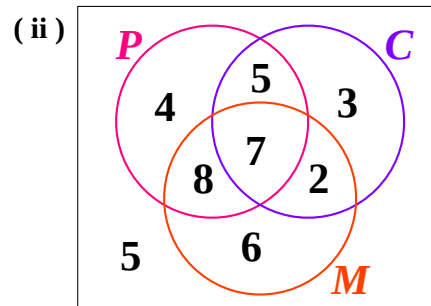
Question 1

This continues the example featured at the start of the lesson and in the teaching video with a further set of four questions about the college students.

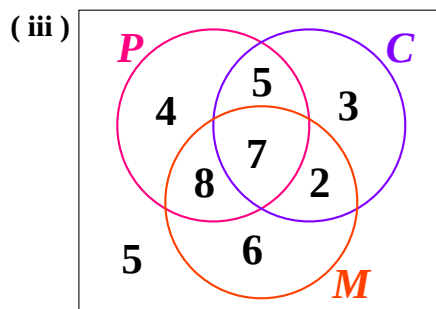
In each of the following Venn Diagram, shade in the region specified and hence give the total number of sixth form students in that region.



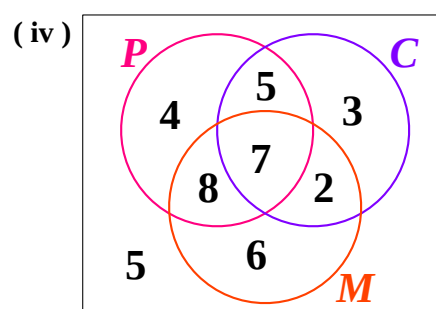
$$n(C) =$$



$$n(P \cap M) =$$



$$n(P \cap M \cap C') =$$

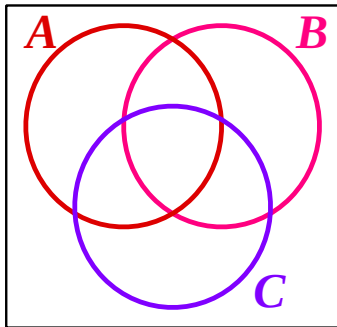


$$n(P \cap M)' =$$

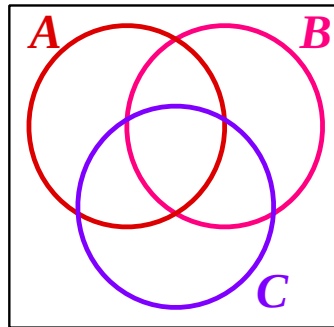
[8 marks]

Question 2

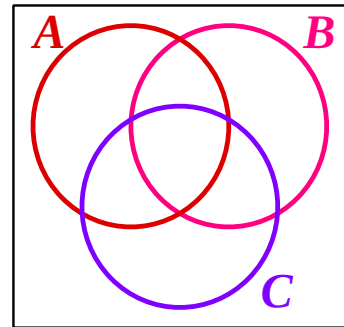
On the Venn Diagrams below, shade the region specified;



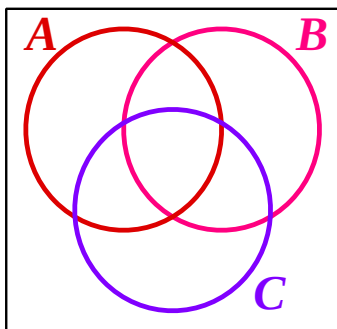
Shade : A



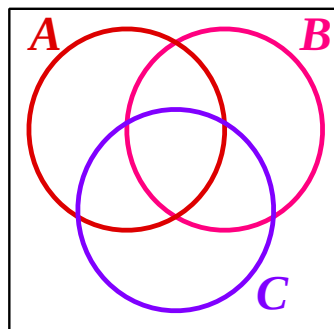
Shade : $A \cap B$



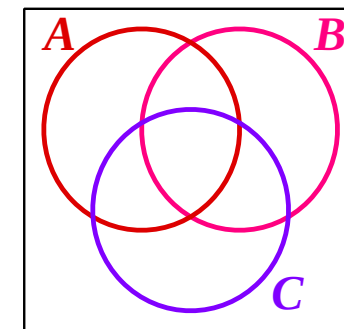
Shade : $A \cap B \cap C$



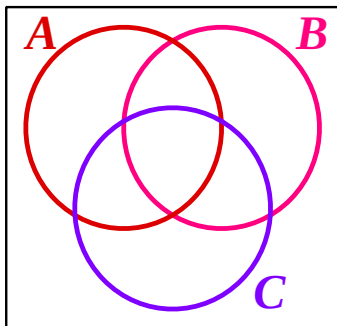
Shade : B



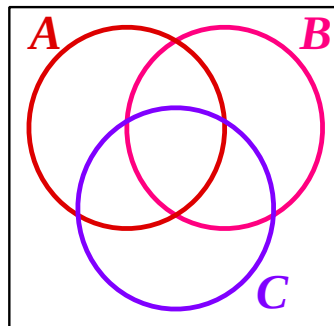
Shade : $B \cap C$



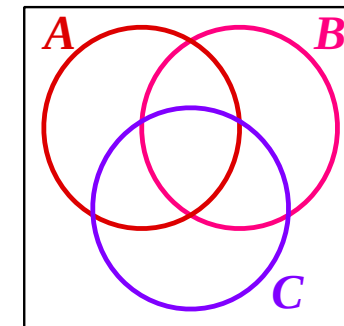
Shade : $B \cap C \cap A'$



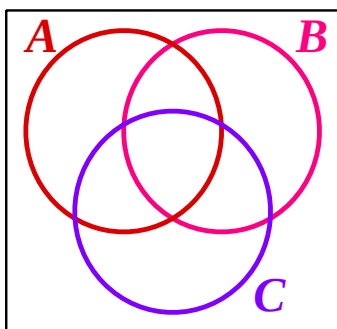
Shade : $A \cap C$



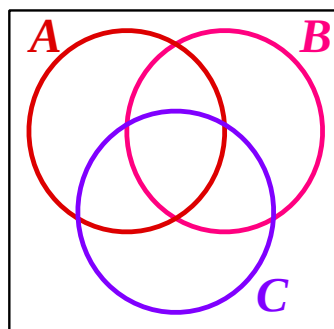
Shade : $A \cap C \cap B$



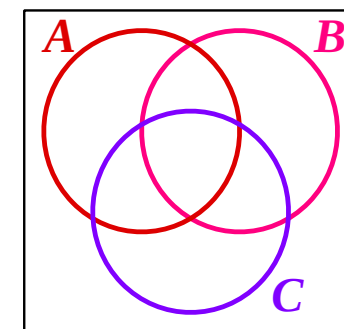
Shade : $(A \cap C \cap B)'$



Shade : C'



Shade : $C' \cap B$



Shade : $C' \cap B \cap A'$

[12 marks]

Question 3

Let T , M and F be the following sets;

$T = \{ \text{The first five triangular numbers} \}$

$M = \{ \text{The first five multiples of 3} \}$

$F = \{ \text{The factors of 15} \}$

(a) List the elements of sets T , M and F below,

(i) $T = \{ \quad, \quad, \quad, \quad, \quad \}$

[1 mark]

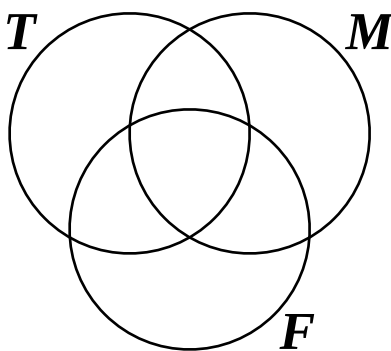
(ii) $M = \{ \quad, \quad, \quad, \quad, \quad \}$

[1 mark]

(iii) $F = \{ \quad, \quad, \quad, \quad \}$

[1 mark]

(b) Show the relationship between T , M and F on a Venn Diagram



[5 marks]

(c) List the elements of the following,

(i) $T \cap M = \{ \quad, \quad, \quad \}$

[1 mark]

(ii) $T \cap F = \{ \quad, \quad, \quad \}$

[1 mark]

(iii) $M \cap F = \{ \quad, \quad \}$

[1 mark]

(iv) $T \cap M \cap F = \{ \quad, \quad \}$

[1 mark]

(v) $T \cap M \cap F' = \{ \quad \}$

[1 mark]

(vi) $T \cap M' \cap F = \{ \quad \}$

[1 mark]

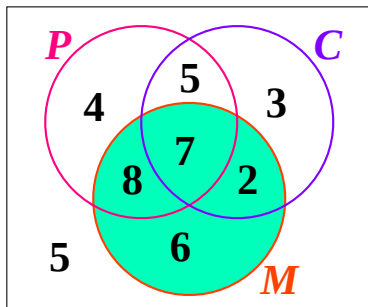
(vii) What does the statement $T' \cap M \cap F = \emptyset$ tell you ?

[1 mark]

6.3 Answers

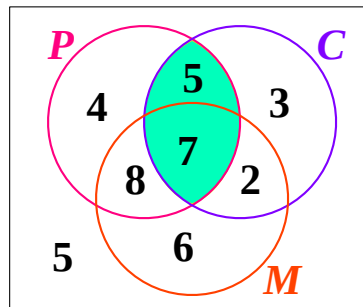
6.3.1 Solution to 6.1 Example (Teaching Video)

(i)



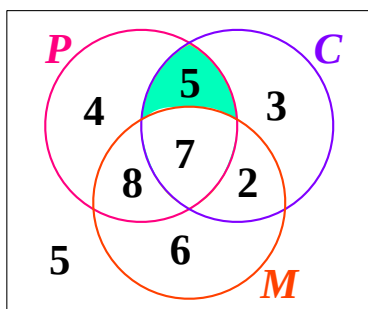
$$n(M) = 23$$

(ii)



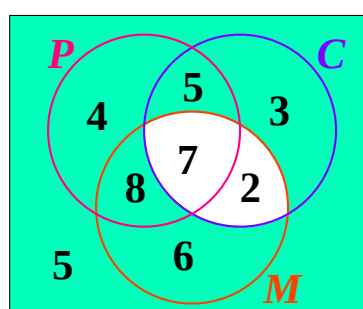
$$n(P \cap C) = 12$$

(iii)



$$n(P \cap C \cap M') = 5$$

(iv)



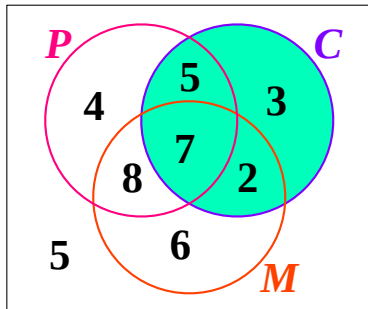
$$n(C \cap M)' = 31$$

[8 marks]

6.3.2 Solution to 6.2 Exercise

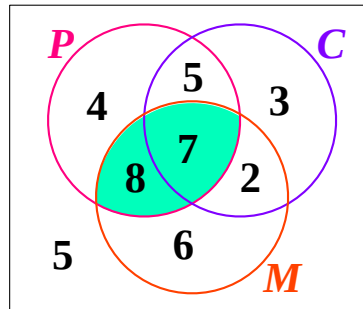
Answer 1

(i)



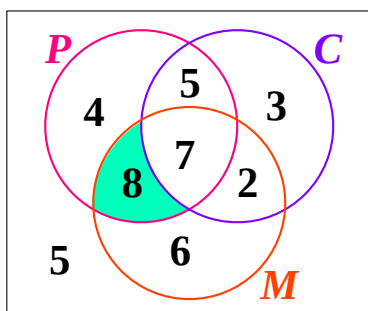
$$n(C) = 17$$

(ii)



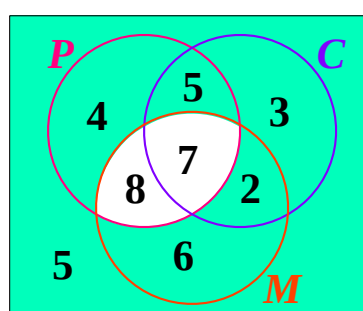
$$n(P \cap M) = 15$$

(iii)



$$n(P \cap M \cap C') = 8$$

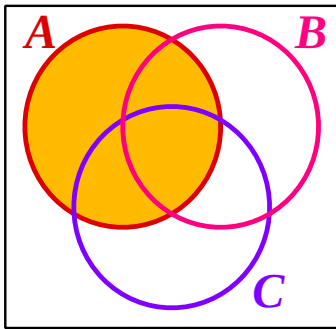
(iv)



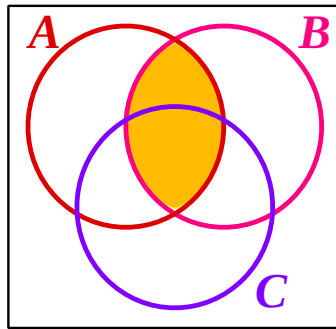
$$n(P \cap M)' = 25$$

[8 marks]

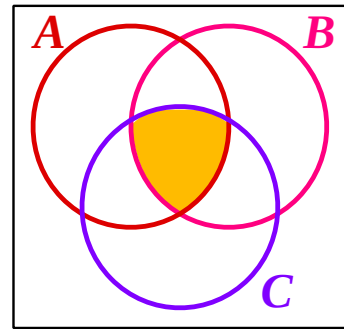
Answer 2



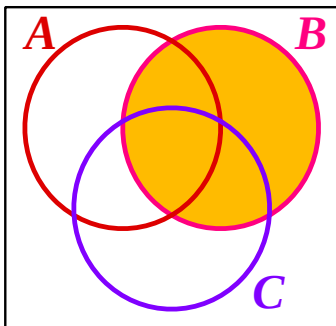
Shade : A



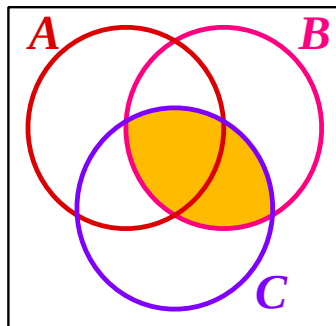
Shade : $A \cap B$



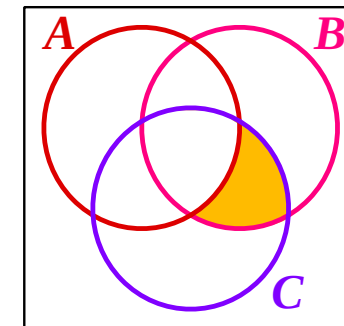
Shade : $A \cap B \cap C$



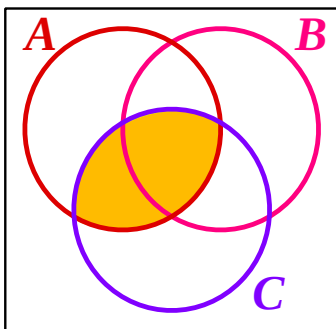
Shade : B



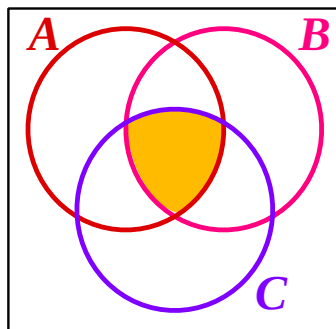
Shade : $B \cap C$



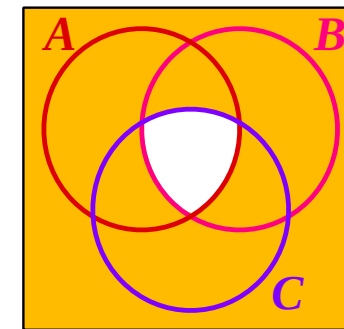
Shade : $B \cap C \cap A'$



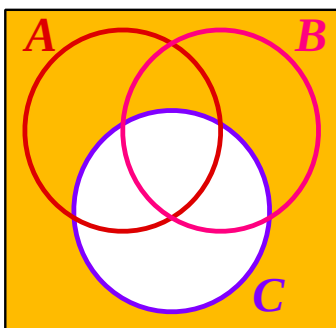
Shade : $A \cap C$



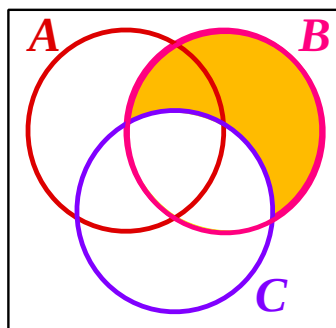
Shade : $A \cap C \cap B$



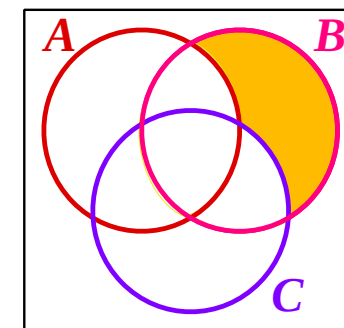
Shade : $(A \cap C \cap B)'$



Shade : C'



Shade : $C' \cap B$



Shade : $C' \cap B \cap A'$

[12 marks]

Answer 3

(a) (i) $T = \{ 1, 3, 6, 10, 15 \}$

[1 mark]

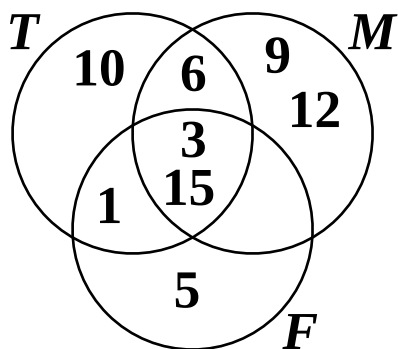
(ii) $M = \{ 3, 6, 9, 12, 15 \}$

[1 mark]

(iii) $F = \{ 1, 3, 5, 15 \}$

[1 mark]

(b)



[5 marks]

(c) (i) $T \cap M = \{ 3, 6, 15 \}$

[1 mark]

(ii) $T \cap F = \{ 1, 3, 15 \}$

[1 mark]

(iii) $M \cap F = \{ 3, 15 \}$

[1 mark]

(iv) $T \cap M \cap F = \{ 3, 15 \}$

[1 mark]

(v) $T \cap M \cap F' = \{ 6 \}$

[1 mark]

(vi) $T \cap M' \cap F = \{ 1 \}$

[1 mark]

(vii) $T' \cap M \cap F = \emptyset$ is observing the region that contains no elements on the Venn Diagram

[1 mark]

7.1 Revision

Marks Available : 74

Question 1

Here are two descriptions of a set F ;

- $F = \{ \text{The factors of 14} \}$
- $F = \{ 1, 2, 7, 14 \}$

In a similar manner, give another description of the following sets

(i) $S = \{ \text{Snapchat, Instagram, Tumblr, Twitter, FaceBook, ...} \}$

[1 mark]

(ii) $C = \{ \text{Playing pieces in the game of Chess} \}$

[1 mark]

(iii) $O = \{ \text{Odd Numbers} \}$

[1 mark]

(iv) $M = \{ 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, ... \}$

[1 mark]

Question 2

Let $\Delta = \{ \text{Triangular Numbers} \}$ i.e. $\Delta = \{ 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, ... \}$

For each of the following, decide if the given statement is TRUE or FALSE.

(i) $2 \in \Delta$ (vi) $136 \in \Delta$ (ii) $25 \notin \Delta$ (vii) $1 \notin \Delta$ (iii) $36 \in \Delta$ (viii) $40000.5 \in \Delta$ (iv) $91 \notin \Delta$ (ix) $\sqrt{100} \in \Delta$ (v) $120 \notin \Delta$ (x) $\pi \in \Delta$

[10 marks]

This next question is going to ask you whether certain numbers are in the sequence 2, 6, 10, 14, 18, 22, or not.

Each term is four more than the term before.

Here is a clever trick that lets you see what the numbers in this sequence can end in.

You write out quite a lot of the sequence but try to arrange it so that a pattern in the endings becomes obvious.

2	6	10	14	18
22	26	30	34	38
42	46	50	54	58
62	66	70	74	78
82	...			

Now try the question:

Question 3

Let $C = \{ \text{The sequence of numbers that begins 2, 6, 10, 14, 18, 22, ...} \}$

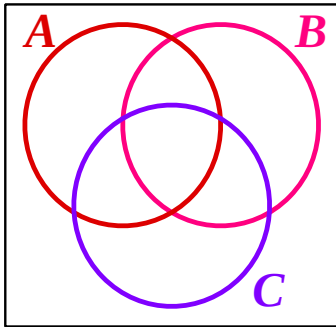
For each of the following, decide if the given statement is TRUE or FALSE.

- | | |
|----------------------|----------------------|
| (i) $14 \in C$ | (vi) $387 \in C$ |
| (ii) $8 \notin C$ | (vii) $90 \in C$ |
| (iii) $23 \in C$ | (viii) $590 \in C$ |
| (iv) $30 \notin C$ | (ix) $102 \in C$ |
| (v) $20 \in C$ | (x) $524 \notin C$ |

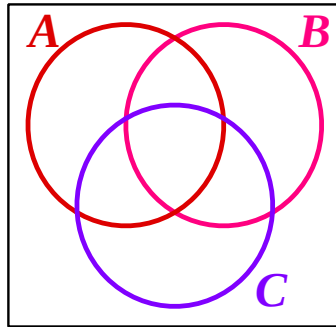
[10 marks]

Question 4

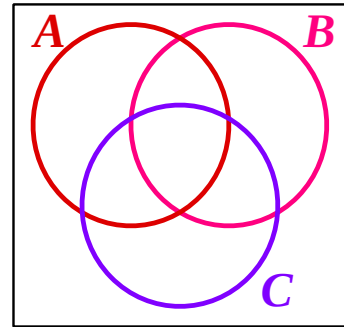
On the Venn Diagrams below, shade the region specified;



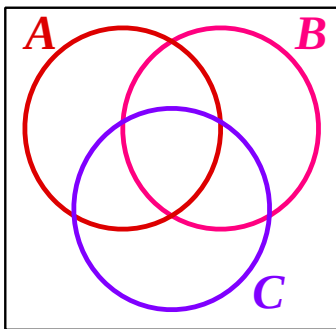
Shade : C



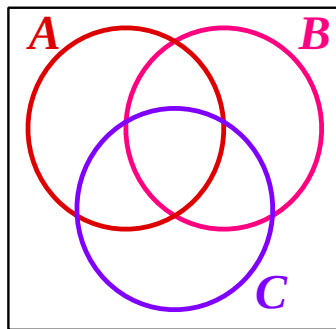
Shade : $C \cap B$



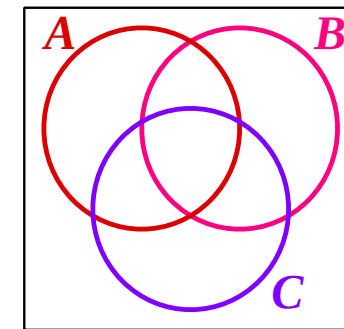
Shade : $C \cap B \cap A$



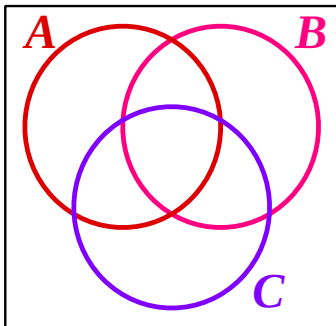
Shade : B'



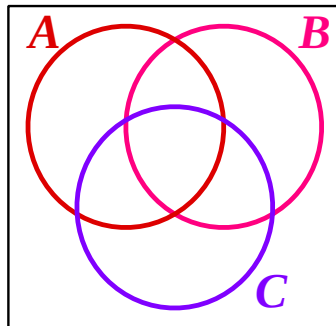
Shade : $B' \cap C$



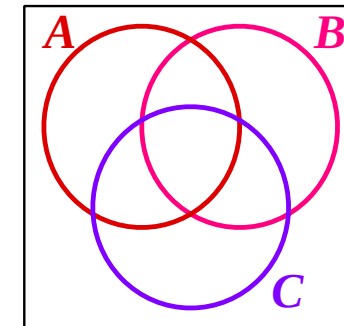
Shade : $B' \cap C \cap A$



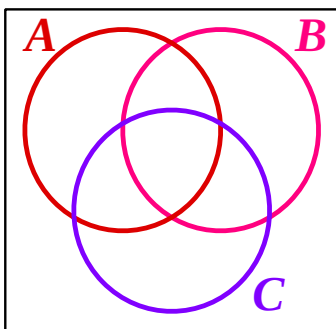
Shade : A



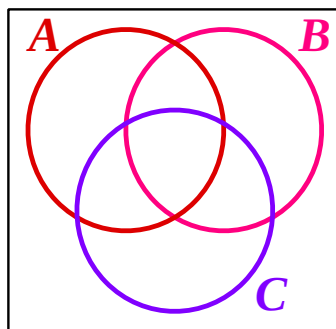
Shade : $A \cap C$



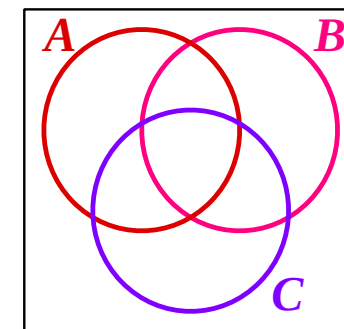
Shade : $(A \cap C)'$



Shade : $B' \cap C'$



Shade : $B' \cap C' \cap A$



Shade : $C' \cap B' \cap A'$

[12 marks]

Question 5

Let S , M and F be the following sets;

$S = \{ \text{The first five square numbers} \}$

$M = \{ \text{The first eight multiples of 4} \}$

$F = \{ \text{The factors of 32} \}$

(a) List the elements of sets S , M and F below,

(i) $S = \{ _, _, _, _, _ \}$

[1 mark]

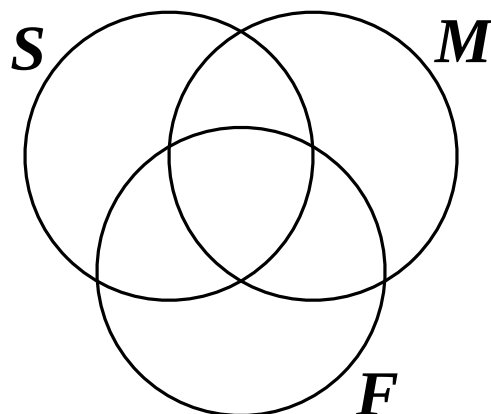
(ii) $M = \{ _, _, _, _, _, _, _, _ \}$

[1 mark]

(iii) $F = \{ _, _, _, _, _, _ \}$

[1 mark]

(b) Show the relationship between S , M and F on a Venn Diagram



[5 marks]

(c) (i) $S \cap M = \{ _, _ \}$

[1 mark]

(ii) $S \cap F = \{ _, _, _ \}$

[1 mark]

(iii) $M \cap F = \{ _, _, _, _ \}$

[1 mark]

(iv) $S \cap M \cap F = \{ _, _ \}$

[1 mark]

(v) $S \cap F \cap M' = \{ _ \}$

[1 mark]

(vi) $F \cap S' \cap M = \{ _, _ \}$

[1 mark]

(vii) What does the statement $S \cap M \cap F' = \emptyset$ tell you ?

[1 mark]

Question 6



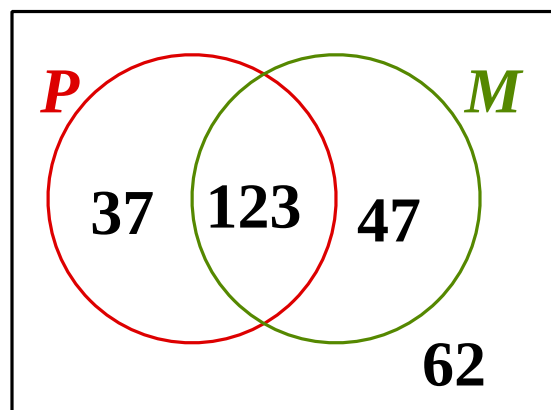
The Year 9 pupils in a school book club were asked if they enjoyed reading the Harry Potter books.

They were also asked if they enjoyed reading the His Dark Materials books.

The Venn Diagram provides a summary of their replies

where • P is the hoop containing those who enjoyed the Harry Potter books

and • M is the hoop containing those who enjoyed the His Dark Materials books



- (i) How many pupils were asked ? [1 mark]
- (ii) How many pupils **only** enjoyed reading the Harry Potter books ? [1 mark]
- (iii) How many pupils enjoyed reading the Harry Potter books ? [1 mark]
- (iv) How many pupils didn't enjoy reading the Harry Potter books ? [1 mark]
- (v) How many pupils enjoyed Harry Potter or His Dark Materials but not both ? [1 mark]

Question 7

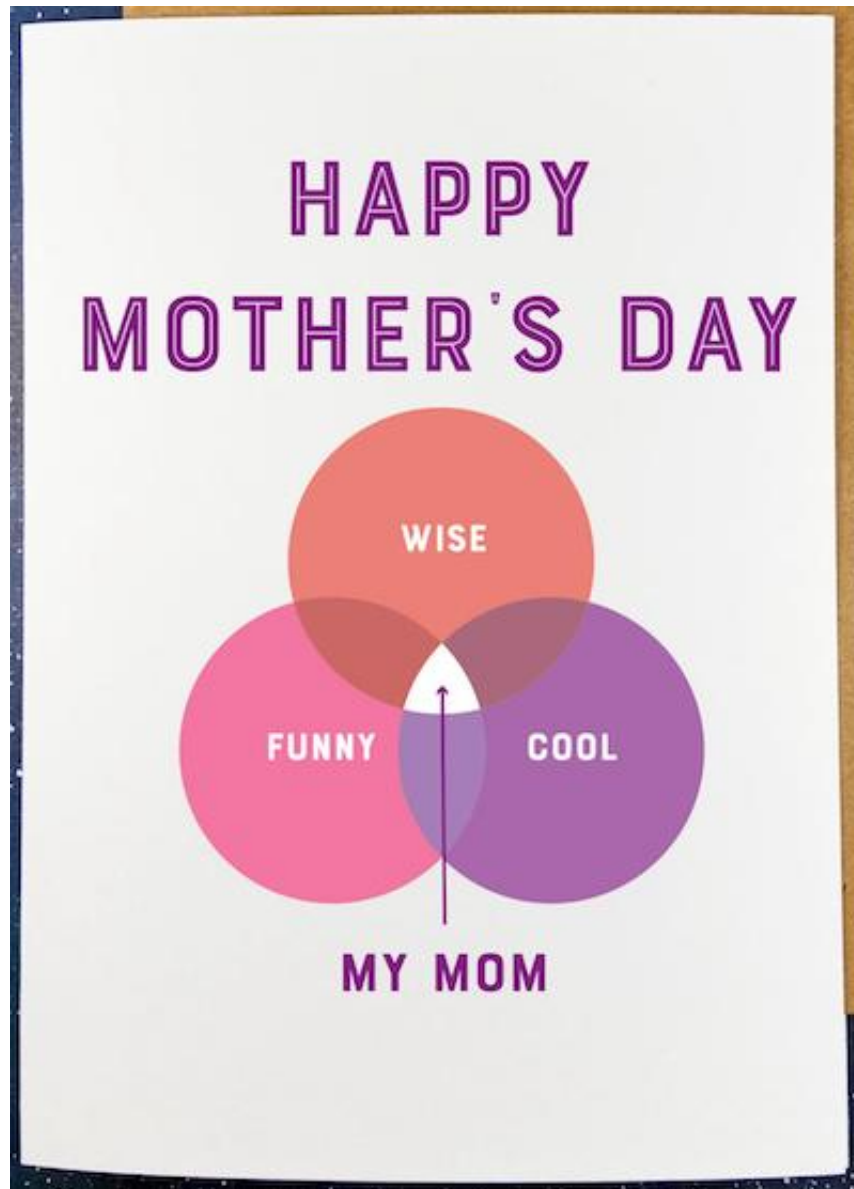
In this question, we are only working with the positive integers, $\{1, 2, 3, 4, 5, 6, 7, \dots\}$

Let set O be the set of odd numbers; $O = \{1, 3, 5, 7, 9, \dots\}$

Describe, in words, the set O'

[2 marks]

Question 8



Explain what message this “Happy Mother's Day” card is conveying. ?

[2 marks]

Question 9

In this question, we are only working with the positive integers, $\{1, 2, 3, 4, 5, 6, 7, \dots\}$

Let $S = \{ \text{The multiples of 7} \}$

and $E = \{ \text{The multiples of 8} \}$

(a) List the first few elements of sets S and E below,

(i) $S = \{ _, _, _, _, _, _, _, _, \dots \}$

(ii) $E = \{ _, _, _, _, _, _, _, _, \dots \}$

[2 marks]

(b) For each of the following, decide if the given statement is TRUE or FALSE

(i) $12 \notin E$

(vi) $S \cap E = \{ \text{Multiples of 56} \}$

(ii) $14 \in S$

(vii) $S' = E'$

(iii) $56 \in S \cap E$

(viii) $112 \in S' \cap E$

(iv) $560 \in S \cap E$

(ix) $7^9 \in S$

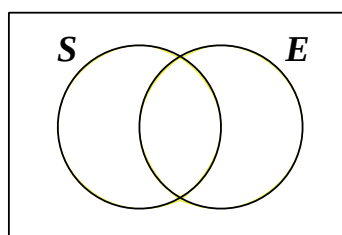
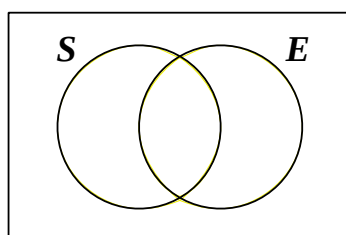
(v) $21 \in (S \cap E)'$

(x) $S' \cap E' = (S \cap E)'$

[10 marks]

(c) (i) On the first Venn diagrams below shade in $S' \cap E'$

(ii) On the second Venn diagram below shade in $(S \cap E)'$



[2 marks]

7.2 Answers

Answer 1

(i) $S = \{ \text{Social Media Applications} \}$

[1 mark]

(ii) $R = \{ \text{Rook, Knight, Bishop, King, Queen, Pawn} \}$

[1 mark]

(iii) $O = \{ 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, \dots \}$

[1 mark]

(iv) $M = \{ \text{Multiples of 5} \}$

[1 mark]

Answer 2

(i) F

(vi) T

(ii) T

(vii) F

(iii) T

(viii) F

(iv) F

(ix) T

(v) F

(x) F

[10 marks]

Answer 3

(i) T

(vi) F

(ii) T

(vii) T

(iii) F

(viii) T

(iv) F

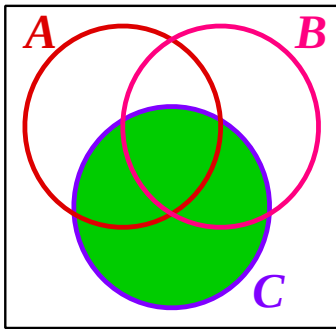
(ix) T

(v) F

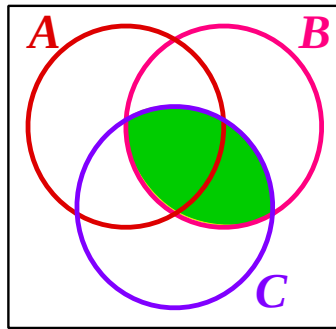
(x) T

[10 marks]

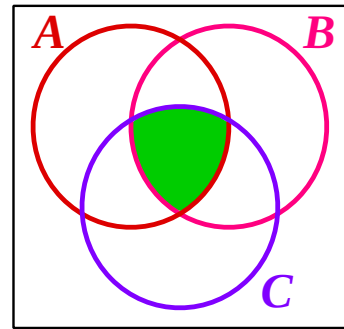
Answer 4



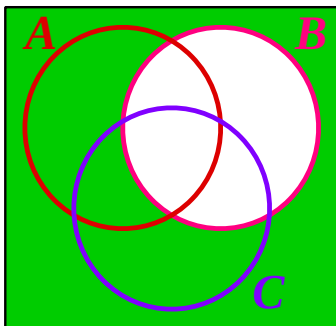
Shade : C



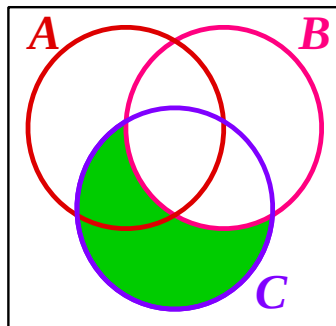
Shade : $C \cap B$



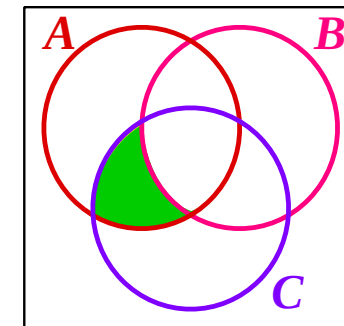
Shade : $C \cap B \cap A$



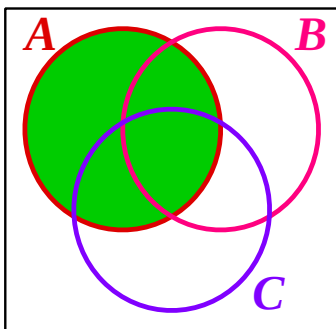
Shade : B'



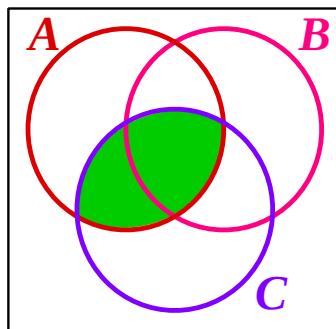
Shade : $B' \cap C$



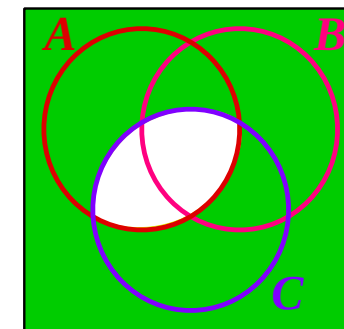
Shade : $B' \cap C \cap A$



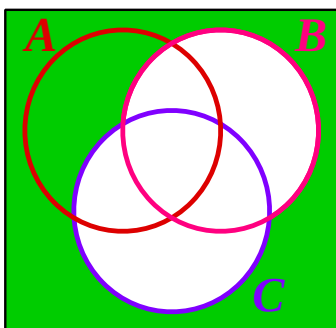
Shade : A



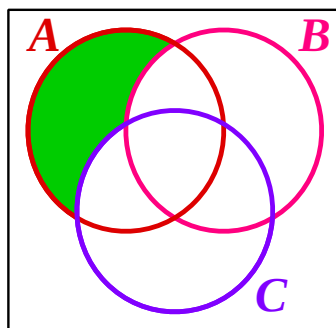
Shade : $A \cap C$



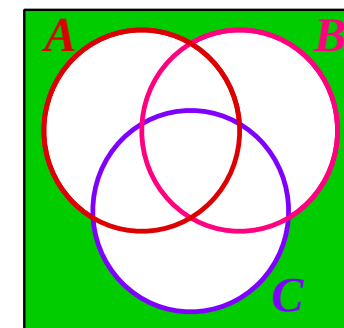
Shade : $(A \cap C)'$



Shade : $B' \cap C'$



Shade : $B' \cap C' \cap A$



Shade : $C' \cap B' \cap A$

[12 marks]

Answer 5

(a) (i) $S = \{ 1, 4, 9, 16, 25 \}$

[1 mark]

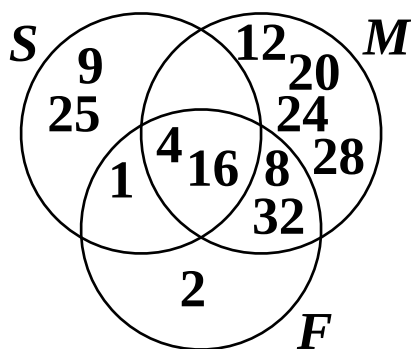
(ii) $M = \{ 4, 8, 12, 16, 20, 24, 28, 32 \}$

[1 mark]

(iii) $F = \{ 1, 2, 4, 8, 16, 32 \}$

[1 mark]

(b)



[5 marks]

(c) (i) $S \cap M = \{ 4, 16 \}$

[1 mark]

(ii) $S \cap F = \{ 1, 4, 16 \}$

[1 mark]

(iii) $M \cap F = \{ 4, 8, 16, 32 \}$

[1 mark]

(iv) $S \cap M \cap F = \{ 4, 16 \}$

[1 mark]

(v) $S \cap F \cap M' = \{ 1 \}$

[1 mark]

(vi) $F \cap S' \cap M = \{ 8, 32 \}$

[1 mark]

(vii) $S \cap M \cap F' = \emptyset$ is observing a region that contains no elements on the Venn Diagram

[1 mark]

Answer 6

(i) 269

(ii) 37

(iii) 160

(iv) 109

(v) 84

[5 marks]

Answer 7

O' is the set of even numbers

[2 marks]

Answer 8

That the mother to whom the card is sent is wise, funny and cool

[2 marks]

Answer 9

(a) (i) $S = \{ 7, 14, 21, 28, 35, 42, 49, 56, \dots \}$

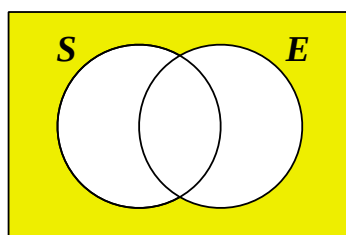
 (ii) $E = \{ 8, 16, 24, 32, 40, 48, 56, 64, \dots \}$

[2 marks]

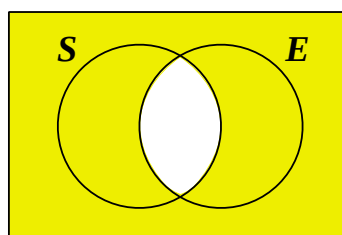
- | | | | | |
|-------|---------|---|----------|---|
| (b) | (i) | T | (vi) | T |
| | (ii) | T | (vii) | F |
| | (iii) | T | (viii) | F |
| | (iv) | T | (ix) | T |
| | (v) | T | (x) | F |

[10 marks]

(c)



$$S' \cap E'$$



$$(S \cap E)'$$

$$= S' \cup E' \text{ using Boolean Algebra}$$

[2 mark]

8.1 Test

Marks Available : 70

Question 1

Here are two descriptions of a set F;

- $F = \{ \text{The factors of } 22 \}$
- $F = \{ 1, 2, 11, 22 \}$

In a similar manner, give another description of the following sets

(i) $T = \{ \text{Earl Grey, Darjeeling, Assam, Lapsang Souchong, ...} \}$

[1 mark]

(ii) $S = \{ \text{The suits in a pack of playing cards} \}$

[1 mark]

(iii) $E = \{ \text{Even numbers} \}$

[1 mark]

(iv) $M = \{ 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, ... \}$

[1 mark]

Question 2

Let $P = \{ \text{Prime Numbers} \}$ i.e. $P = \{ 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ... \}$

For each of the following, decide if the given statement is TRUE or FALSE.

(i) $1 \in P$ (vi) $936 \in P$ (ii) $25 \notin P$ (vii) $70.2 \notin P$ (iii) $40 \in P$ (viii) $417^2 \in P$ (iv) $17 \notin P$ (ix) $\sqrt{121} \in P$ (v) $1220 \notin P$ (x) $\pi \in P$

[10 marks]

This next question is going to ask you whether certain numbers are in the sequence 1, 5, 9, 13, 17, 21, or not.

Each term is four more than the term before.

Here is a clever trick that lets you see what the numbers in this sequence can end in.

You write out quite a lot of the sequence but try to arrange it so that a pattern in the endings becomes obvious.

1	5	9	13	17
21	25	29	33	37
41	45	49	53	57
61	65	69	73	77
81	...			

Now try the question:

Question 3

Let $C = \{ \text{The sequence of numbers that begins } 1, 5, 9, 13, 17, 21, \dots \}$

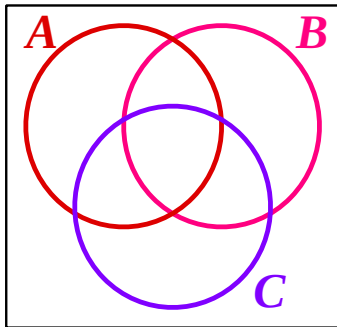
For each of the following, decide if the given statement is TRUE or FALSE.

- | | | | |
|---------|---------------|----------|----------------|
| (i) | $13 \in C$ | (vi) | $386 \in C$ |
| (ii) | $7 \notin C$ | (vii) | $89 \in C$ |
| (iii) | $22 \in C$ | (viii) | $589 \in C$ |
| (iv) | $29 \notin C$ | (ix) | $101 \in C$ |
| (v) | $19 \in C$ | (x) | $523 \notin C$ |

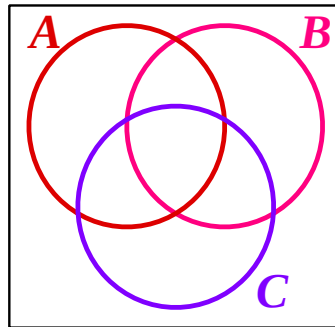
[10 marks]

Question 4

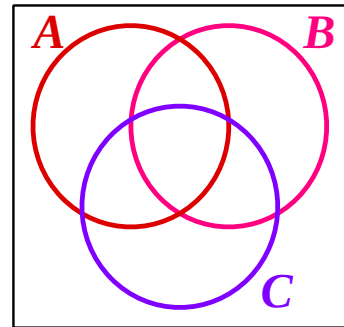
On the Venn Diagrams below, shade the region(s) specified;



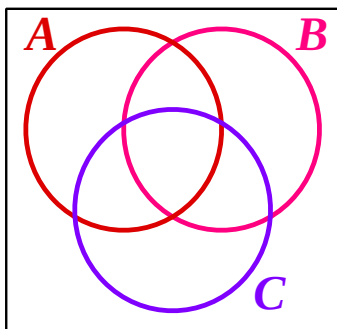
Shade : A



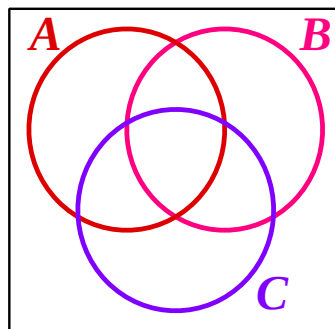
Shade : $A \cap B$



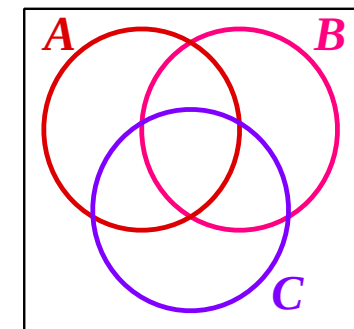
Shade : $A \cap B \cap C$



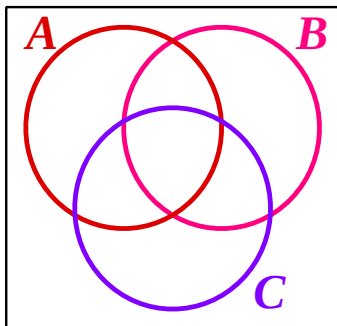
Shade : B



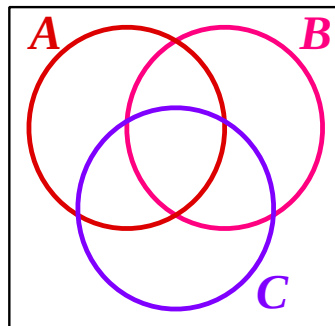
Shade : $B \cap C$



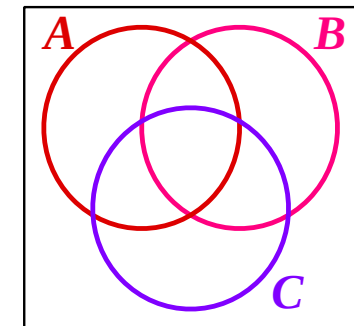
Shade : $B \cap C \cap A'$



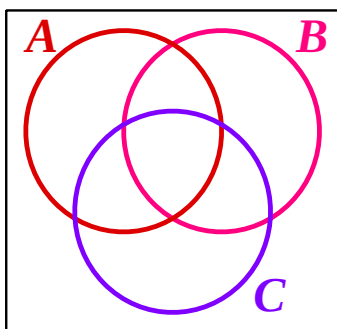
Shade : $A \cap C$



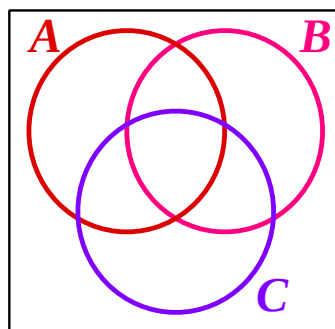
Shade : $A \cap C \cap B$



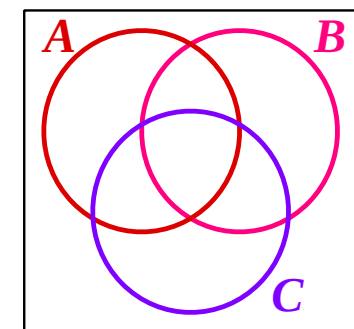
Shade : $(A \cap C \cap B)'$



Shade : C'



Shade : $C' \cap B$



Shade : $C' \cap B \cap A'$

[12 marks]

Question 5

Let $F = \{ \text{Factors of 6} \}$
 $P = \{ \text{Primes less than 10} \}$
 $T = \{ \text{The first four triangular numbers} \}$

(a) List the elements of sets F , P and T below,

$F = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

[1 mark]

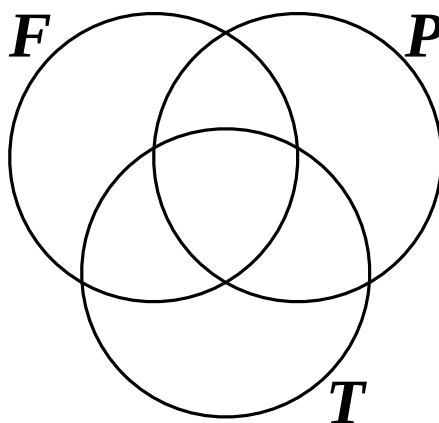
$P = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

[1 mark]

$T = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

[1 mark]

(b) Show the relationship between F , P and T on a Venn Diagram.



[5 marks]

(c) List all elements, that are in the following intersections.

(i) $F \cap P = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

[1 mark]

(ii) $F \cap T = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

[1 mark]

(iii) $P \cap T = \{ \underline{\hspace{1cm}} \}$

[1 mark]

(iv) $F \cap P \cap T = \{ \underline{\hspace{1cm}} \}$

[1 mark]

(v) $F \cap P \cap T' = \{ \underline{\hspace{1cm}} \}$

[1 mark]

(vi) $F \cap P' \cap T = \{ \underline{\hspace{1cm}}, \underline{\hspace{1cm}} \}$

[1 mark]

(vii) What does the statement $F' \cap P \cap T = \emptyset$ tell you ?

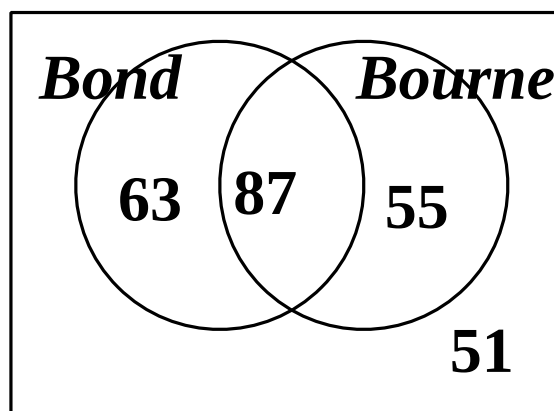
[1 mark]

Question 6



All the members of a film club were asked if they liked James **Bond** films.
They were also asked if they liked Jason **Bourne** films.

The Venn Diagram provides a summary of their replies.



- (i) How many members of the film club replied ?[1 mark]
- (ii) How many pupils liked James **Bond** only ?[1 mark]
- (iii) How many pupils liked James **Bond** ?[1 mark]
- (iv) How many pupils don't like James **Bond** ?[1 mark]
- (v) How many pupils like James **Bond** or Jason **Bourne** but not both ?[1 mark]

Question 7

In this question, we are only working with the positive integers, $\{1, 2, 3, 4, 5, 6, 7, \dots\}$

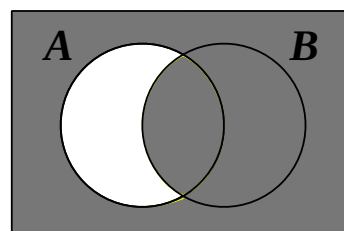
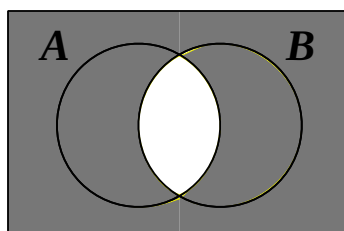
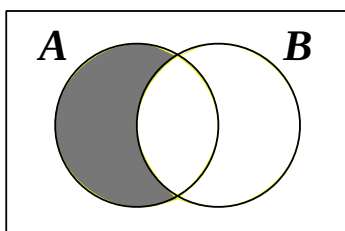
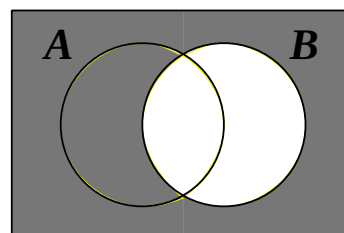
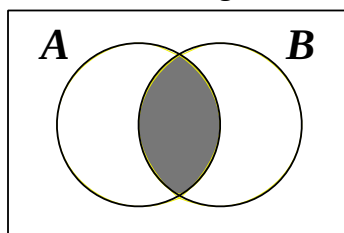
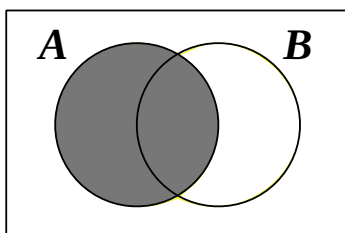
Let the set O be the set $\{ \text{Odd numbers} \}$ and set P be the set $\{ \text{Prime numbers} \}$.

List the six numbers less than 30 that are in $O \cap P'$

[2 marks]

Question 8

Below each Venn diagram describe the region shaded.



[6 marks]

Question 9

If $A = \{ \text{The multiples of 2} \}$ and $B = \{ \text{The multiples of 5} \}$
then describe, with a single set theory symbol, the set $A \cap B$

[2 marks]

Question 10

Let A be the set $\{ \text{Positive integer powers of 2} \}$

and B be the set $\{ \text{Positive integer powers of 3} \}$

Set A can be described like this : $A = \{ 2^1, 2^2, 2^3, 2^4, \dots \}$

(i) Describe set B in a similar manner.

[2 marks]

(ii) Let C be the set $\{ \text{Positive integer powers of 6} \}$.

Is set C to be found within the set $(A \cup B)$?

[2 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk

8.2 Answers

Answer 1

(i) $S = \{ \text{Types of Tea} \}$

[1 mark]

(ii) $R = \{ \text{Spades, Clubs, Hearts, Diamonds} \}$

[1 mark]

(iii) $O = \{ 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, \dots \}$

[1 mark]

(iv) $M = \{ \text{Multiples of 6} \}$

[1 mark]

Answer 2

(i) F

(vi) F

(ii) T

(vii) T

(iii) F

(viii) F

(iv) F

(ix) T

(v) T

(x) F

[10 marks]

Answer 3

(i) T

(vi) F

(ii) T

(vii) T

(iii) F

(viii) T

(iv) F

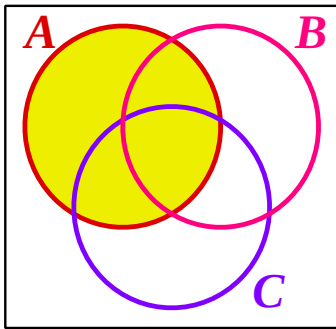
(ix) T

(v) F

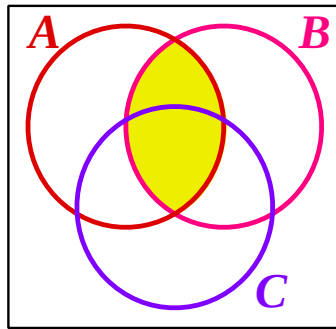
(x) T

[10 marks]

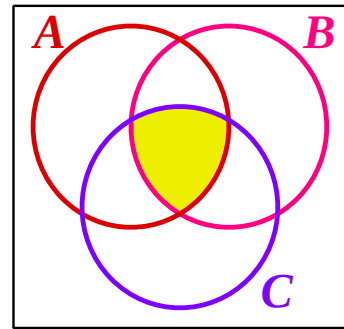
Answer 4



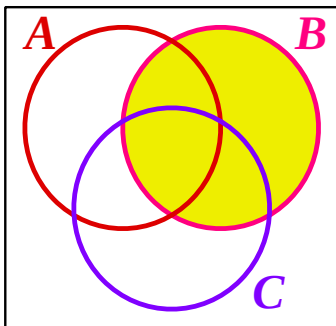
Shade : A



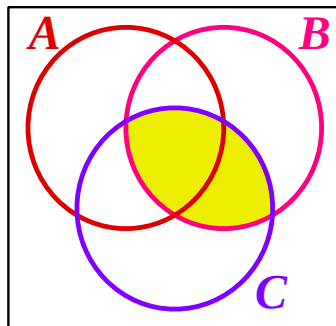
Shade : $A \cap B$



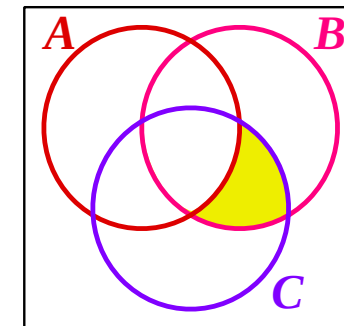
Shade : $A \cap B \cap C$



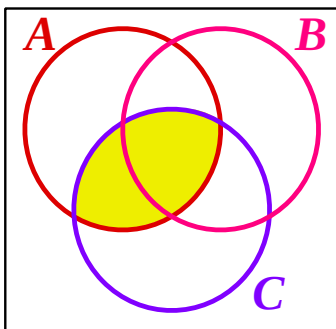
Shade : B



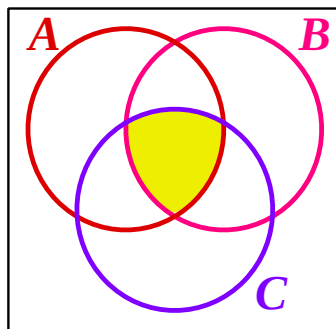
Shade : $B \cap C$



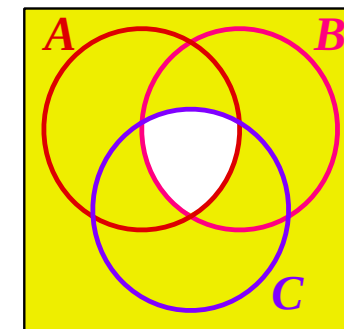
Shade : $B \cap C \cap A'$



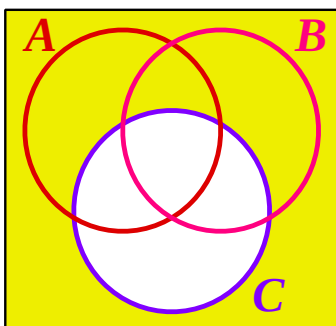
Shade : $A \cap C$



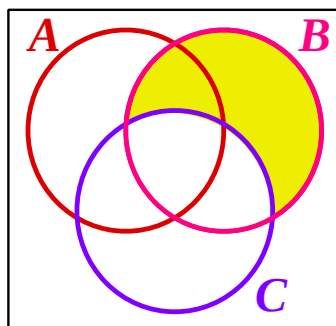
Shade : $A \cap C \cap B$



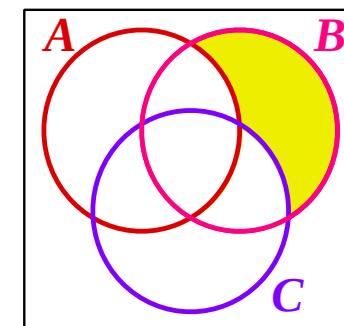
Shade : $(A \cap C \cap B)'$



Shade : C'



Shade : $C' \cap B$



Shade : $C' \cap B \cap A'$

[12 marks]

Answer 5

(a) (i) $F = \{ 1, 2, 3, 6 \}$

[1 mark]

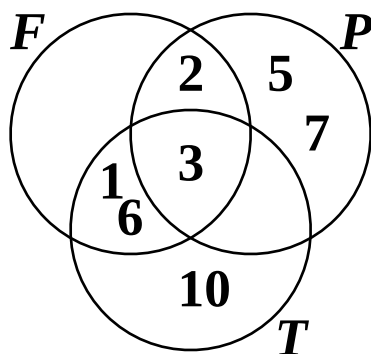
(ii) $P = \{ 2, 3, 5, 7 \}$

[1 mark]

(iii) $T = \{ 1, 3, 6, 10 \}$

[1 mark]

(b)



[5 marks]

(c) (i) $F \cap P = \{ 2, 3 \}$

[1 mark]

(ii) $F \cap T = \{ 1, 3, 6 \}$

[1 mark]

(iii) $P \cap T = \{ 3 \}$

[1 mark]

(iv) $F \cap P \cap T = \{ 3 \}$

[1 mark]

(v) $F \cap P \cap T' = \{ 2 \}$

[1 mark]

(vi) $F \cap P' \cap T = \{ 1, 6 \}$

[1 mark]

(vii) $F' \cap P \cap T = \emptyset$ is observing a region that contains no elements on the Venn Diagram

[1 mark]

Answer 6

(i) 256

(ii) 63

(iii) 150

(iv) 106

(v) 118

[5 marks]

Answer 7

Odd number that are not prime, less than 30

$$O \cap P' = \{ 1, 9, 15, 21, 25, 27 \}$$

[2 marks]

Answer 8

(i) A

(ii) $A \cap B$

(iii) B'

(iv) $A \cap B'$

(v) $(A \cap B)'$

(vi) $(A \cap B')'$

[6 marks]

Answer 9

$$A \cap B = \emptyset$$

[2 marks]

Answer 10

(i) $B = \{ 3^1, 3^2, 3^3, 3^4, \dots \}$

[2 marks]

(ii) No! $C \not\subset (A \cap B)$

[2 marks]