

Chapter 11

GCSE and A-Level Pure Mathematics Vectors I

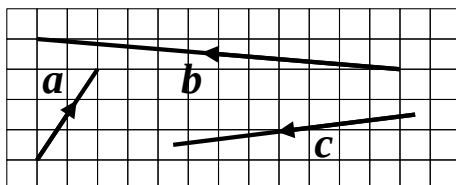
11.1 TEST

Any solution based entirely on graphical or numerical methods is not acceptable.

Make the method used clear.

Marks available : 50

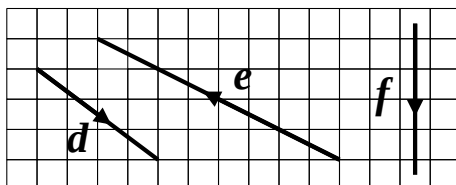
Question 1



Write the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in the form $\begin{pmatrix} p \\ q \end{pmatrix}$ where p and q are integers.

[3 marks]

Question 2



Write the vectors $\mathbf{d}$, $\mathbf{e}$ and $\mathbf{f}$ in the form :

$$p\mathbf{i} + q\mathbf{j}$$

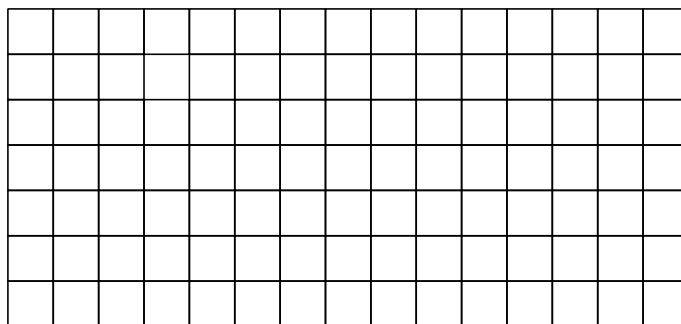
[3 marks]

Question 3

On the grid draw the following vectors, labelling each with its letter and an arrow.

$$\mathbf{a} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 6 \\ -0.5 \end{pmatrix}$$



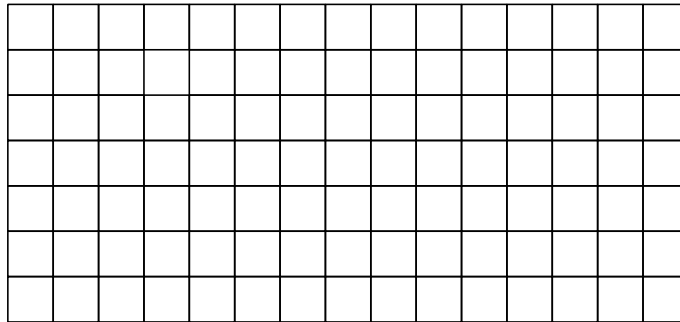
[2 marks]

Question 4

On the grid draw the following vectors, labelling each with its letter and an arrow.

$$\mathbf{c} = 5\mathbf{i} - 2\mathbf{j}$$

$$\mathbf{d} = -4\mathbf{i} + \mathbf{j}$$



[2 marks]

Question 5

(i) Find $|\mathbf{p}|$ giving the answer in surd form.

$$\mathbf{p} = \begin{pmatrix} 8 \\ -11 \end{pmatrix}$$

[2 marks]

(ii) Determine the direction in which the vector $\mathbf{p}$ acts.

[2 marks]

Question 6

Circle the two vectors that are parallel;

$$\begin{pmatrix} -60 \\ -45 \end{pmatrix}$$

$$\begin{pmatrix} 150 \\ 42 \end{pmatrix}$$

$$\begin{pmatrix} -80 \\ -24 \end{pmatrix}$$

$$\begin{pmatrix} -50 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} 60 \\ 18 \end{pmatrix}$$

[1 mark]

Question 7

Circle the two vectors that are perpendicular;

$$\begin{pmatrix} -21 \\ 36 \end{pmatrix} \quad \begin{pmatrix} 7 \\ 12 \end{pmatrix} \quad \begin{pmatrix} -18 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 12 \\ 7 \end{pmatrix} \quad \begin{pmatrix} -14 \\ -24 \end{pmatrix}$$

[1 mark]

Question 8

In polar form, a vector, $\mathbf{v}$, has a magnitude of 6.4, and direction 250°

Express $\mathbf{v}$ in rectangular form, that is, in the form

$$\mathbf{v} = \begin{pmatrix} p \\ q \end{pmatrix}$$

for some values of p and q which you should determine.

[3 marks]

Question 9

Given that;

$$\mathbf{f} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \quad \mathbf{g} = \begin{pmatrix} -7 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{h} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$

Calculate;

(i) $5\mathbf{f} + 2\mathbf{g}$

(ii) $\mathbf{h} - 3\mathbf{g} + \mathbf{f}$

[2, 2 marks]

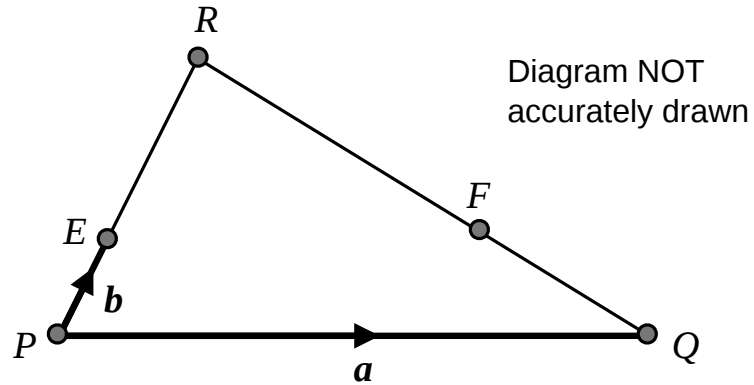
Question 10

GCSE Examination Question from May 2007, Paper 3H, Q16

PQR is a triangle.

E is the point on PR such that $PR = 3 PE$

F is the point on QR such that $QR = 3 QF$



$$\vec{PQ} = a \quad \vec{PE} = b$$

(a) Find, in terms of a and b ;

(i) $\vec{PR} =$

[1 mark]

(ii) $\vec{QR} =$

[1 mark]

(iii) $\vec{PF} =$

[1 mark]

(b) Show that $\vec{EF} = k \vec{PQ}$
where k is a rational number which you must determine

[2 marks]

(c) What does the part (b) result tell you about vectors $\vec{EF}$ and $\vec{PQ}$?

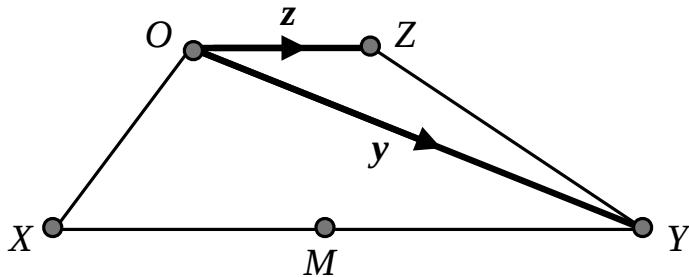
[2 marks]

Question 11

The diagram, which is not drawn to scale, shows a trapezium $OXYZ$

The point M is the mid-point of XY

Furthermore $\overrightarrow{OY} = \mathbf{y}$ $\overrightarrow{OZ} = \mathbf{z}$ $\overrightarrow{XY} = 3\overrightarrow{OZ}$



- (a) Express the following vectors in terms of $\mathbf{y}$ and $\mathbf{z}$;

(i) $\overrightarrow{XY} =$

(ii) $\overrightarrow{OX} =$

(iii) $\overrightarrow{XZ} =$

(iv) $\overrightarrow{MZ} =$

[4 marks]

- (b) Given that,

$$\mathbf{y} = \begin{pmatrix} 17 \\ -8 \end{pmatrix} \quad \text{and} \quad \mathbf{z} = \begin{pmatrix} 11 \\ 0 \end{pmatrix}$$

What is $\left| \overrightarrow{YZ} \right|$?

[2 marks]

- (c) Find the angle between the vectors $\mathbf{y}$ and $\mathbf{z}$

HINT : The Reversed Cosine Rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

[2 marks]

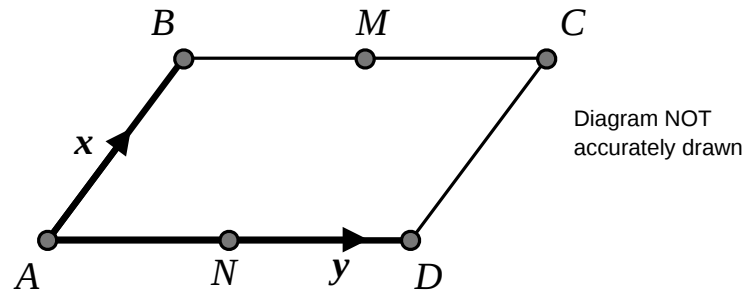
Question 12

GCSE Examination Question from June 2009, Paper 4H, Q18

The diagram shows a parallelogram, $ABCD$

M is the midpoint of BC

N is the midpoint of AD



$$\vec{AB} = x$$

$$\vec{AD} = y$$

Find, in terms of x and y , the vectors

(a) $\vec{MN}$

(b) $\vec{AC}$

[2 marks]

P is the point such that $\vec{CP} = y - \frac{1}{2}x$

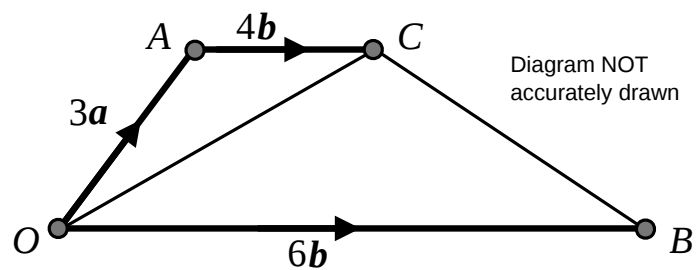
- (c) Find, in terms of x and y , the vector $\vec{PA}$
Simplify your answer as much as possible

[2 marks]

Question 13

GCSE Examination Question from January 2020, Paper 2HR, Q26

The diagram shows trapezium $OACB$



$$\vec{OA} = 3\mathbf{a} \quad \vec{OB} = 6\mathbf{b} \quad \vec{AC} = 4\mathbf{b}$$

N is the point on OC such that ANB is a straight line

Find $\vec{ON}$ as a simplified expression in terms of $\mathbf{a}$ and $\mathbf{b}$

[5 marks]

Question 14

#VectorFasinatingFact

Here is another fascinating fact about vectors !

Given two vectors, $\mathbf{X}$ and $\mathbf{Y}$, such that

$$\mathbf{X} = \begin{pmatrix} a \\ b \end{pmatrix} \quad \text{and} \quad \mathbf{Y} = \begin{pmatrix} c \\ d \end{pmatrix}$$

Then the angle, θ , between the vectors is given by;

$$\cos \theta = \frac{ac + bd}{\sqrt{a^2 + b^2} \sqrt{c^2 + d^2}}$$

Demonstrate how to use this result to find the angle between the following two vectors;

$$\mathbf{P} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$$

[3 marks]