

11.2 Solutions

Answer 1

$$\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} -12 \\ 1 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -8 \\ -1 \end{pmatrix}$$

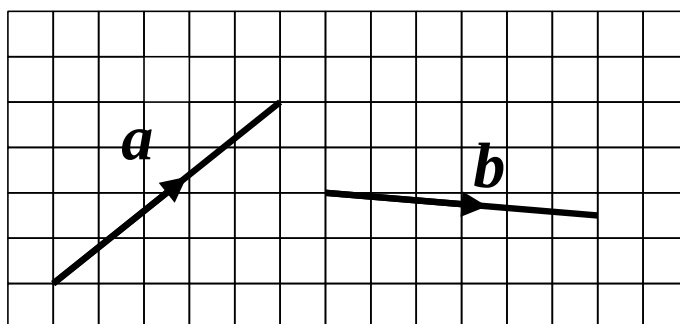
[3 marks]

Answer 2

$$\mathbf{d} = 4\mathbf{i} - 3\mathbf{j} \quad \mathbf{e} = -8\mathbf{i} + 4\mathbf{j} \quad \mathbf{f} = -5\mathbf{j}$$

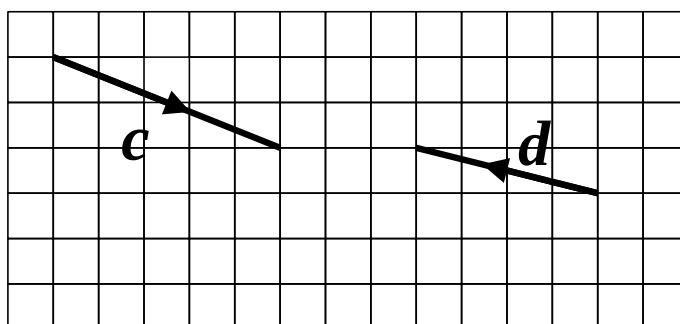
[3 marks]

Answer 3



[2 marks]

Answer 4



[2 marks]

Answer 5

(i) $|\mathbf{p}| = \sqrt{185}$ (13.6 but asked for in surd form)

[2 marks]

(ii) $360 - 54 = 306^\circ$

[2 marks]

Answer 6

$$-15 \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad 6 \begin{pmatrix} 25 \\ 7 \end{pmatrix} \quad -8 \begin{pmatrix} 10 \\ 3 \end{pmatrix} \quad 2 \begin{pmatrix} -25 \\ 6 \end{pmatrix} \quad 6 \begin{pmatrix} 10 \\ 3 \end{pmatrix}$$

The middle and the rightmost are parallel

[1 mark]

Answer 7

$$\begin{pmatrix} -21 \\ 36 \end{pmatrix} \quad \begin{pmatrix} 7 \\ 12 \end{pmatrix} \quad \begin{pmatrix} -18 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 12 \\ 7 \end{pmatrix} \quad \begin{pmatrix} -14 \\ -24 \end{pmatrix}$$

$$-21 \times 12 + 36 \times 7 = 0$$

[1 mark]

Answer 8

$$\begin{pmatrix} -2.19 \\ -6.01 \end{pmatrix}$$

[3 marks]

Answer 9

(i)

$$\begin{pmatrix} 1 \\ 48 \end{pmatrix}$$

(ii)

$$\begin{pmatrix} 24 \\ -8 \end{pmatrix}$$

[2, 2 marks]

Answer 10

$$\begin{aligned} \text{(a) } \quad \text{(i) } \quad \overrightarrow{PR} &= 3\mathbf{b} \\ \text{(ii) } \quad \overrightarrow{QR} &= -\mathbf{a} + 3\mathbf{b} \\ \text{(iii) } \quad \overrightarrow{PF} &= \overrightarrow{PQ} + \frac{1}{3}\overrightarrow{QR} = \frac{2}{3}\mathbf{a} + \mathbf{b} \end{aligned}$$

[3 marks]

$$\begin{aligned} \text{(b) } \quad \overrightarrow{EF} &= \overrightarrow{EP} + \overrightarrow{PF} \\ &= -\mathbf{b} + \frac{2}{3}\mathbf{a} + \mathbf{b} \\ &= \frac{2}{3}\mathbf{a} \\ &= \frac{2}{3}\overrightarrow{PQ} \quad \text{i.e. as required with } k = \frac{2}{3} \end{aligned}$$

[2 marks]

(c) They are parallel.
 EF is two thirds the length of PQ

[2 marks]

Answer 11

$$\begin{aligned} \text{(a) } \quad \text{(i) } \quad \overrightarrow{XY} &= 3\mathbf{z} & \text{(ii) } \quad \overrightarrow{OX} &= \mathbf{y} - 3\mathbf{z} \\ \text{(iii) } \quad \overrightarrow{XZ} &= 4\mathbf{z} - \mathbf{y} & \text{(iv) } \quad \overrightarrow{MZ} &= 2.5\mathbf{z} - \mathbf{y} \end{aligned}$$

[4 marks]

$$\begin{aligned} \text{(b) } \quad \overrightarrow{YZ} &= -\mathbf{y} + \mathbf{z} = \begin{pmatrix} -17 \\ 8 \end{pmatrix} + \begin{pmatrix} 11 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 8 \end{pmatrix} \\ &\therefore |\overrightarrow{YZ}| = 10 \end{aligned}$$

[2 marks]

$$\begin{aligned} \text{(c) } \quad \cos \theta &= \frac{11^2 + 353 - 10^2}{2 \times 11 \times \sqrt{353}} = 0.905 \\ &\therefore \theta = 25.2^\circ \end{aligned}$$

[2 marks]

Answer 12

(a) $\overrightarrow{MN} = -x$

(b) $\overrightarrow{AC} = x + y$

[2 marks]

(c) $\begin{aligned}\overrightarrow{PA} &= \overrightarrow{PC} + \overrightarrow{CA} \\ &= \frac{1}{2}x - y - x - y \\ &= -\frac{1}{2}x - 2y\end{aligned}$

[2 marks]

Answer 13

$$\overrightarrow{AB} = -3a + 6b \quad \text{and} \quad \overrightarrow{OC} = 3a + 4b$$

$$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$$

$$s\overrightarrow{OC} = \overrightarrow{OA} + t\overrightarrow{AB}$$

$$s(3a + 4b) = 3a + t(-3a + 6b)$$

$$3sa + 4sb = 3a - 3ta + 6tb$$

$$\therefore 3s = 3 - 3t \quad \text{and} \quad 4s = 6t$$

$$3s = 3 - 2s$$

$$5s = 3$$

$$s = \frac{3}{5} \Rightarrow t = \frac{2}{5}$$

$$\overrightarrow{ON} = s\overrightarrow{OC} = \frac{3}{5}(3a + 4b)$$

$$\overrightarrow{ON} = \frac{9}{5}a + \frac{12}{5}b$$

[5 marks]

Answer 14

$$\begin{aligned}\cos \theta &= \frac{3 \times 12 + 4 \times 5}{\sqrt{3^2 + 4^2} \times \sqrt{12^2 + 5^2}} \\ &= \frac{36 + 20}{5 \times 13} \\ &= \frac{56}{65} \\ &= 0.862 \\ \therefore \theta &= 30.5^\circ\end{aligned}$$

[3 marks]