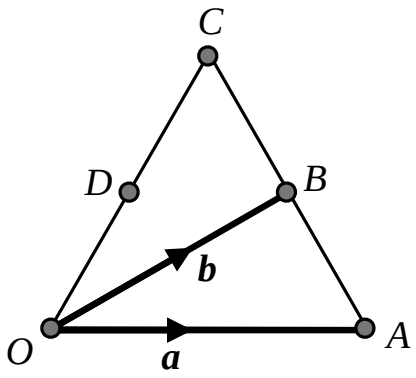
5.4 Answers

5.4.1 Solution to 5.2 Example (Teaching Video)



(i) $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC}$

$= -\boldsymbol{a} + \boldsymbol{b}$

(ii) $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AC}$

$= -\boldsymbol{a} + \boldsymbol{b}$

(iii) $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$

$= \boldsymbol{b} + (-\boldsymbol{a} + \boldsymbol{b})$

$= -\boldsymbol{a} + 2\boldsymbol{b}$

(iv) $\overrightarrow{OD} = \frac{1}{2}\overrightarrow{OC}$

$= \frac{1}{2}(-\boldsymbol{a} + 2\boldsymbol{b})$

$= -\frac{1}{2}\boldsymbol{a} + \boldsymbol{b}$

(v) $\overrightarrow{DB} = \overrightarrow{DO} + \overrightarrow{OB}$

$= -\overrightarrow{OD} + \overrightarrow{OB}$

$= \frac{1}{2}\boldsymbol{a} - \boldsymbol{b} + \boldsymbol{b}$

$= \frac{1}{2}\boldsymbol{a}$

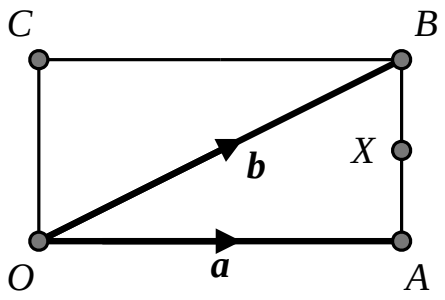
(vi) Told $\overrightarrow{DB} = k\overrightarrow{OA}$

$\frac{1}{2}\boldsymbol{a} = k\boldsymbol{a}$

$\therefore k = \frac{1}{2}$

5.4.2 Solutions (5.3 Exercise)

Answer 1



(a) **(i)** $\overrightarrow{CB} = \boldsymbol{a}$

(ii) $\overrightarrow{AB} = -\boldsymbol{a} + \boldsymbol{b}$

(iii) $\overrightarrow{AX} = \frac{1}{2}\overrightarrow{AB}$

$= \frac{1}{2}(-\boldsymbol{a} + \boldsymbol{b})$

$= -\frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

[3 marks]

(b) **(i)** $\overrightarrow{CX} = \overrightarrow{CB} - \frac{1}{2}\overrightarrow{AB}$

$= \boldsymbol{a} - \frac{1}{2}(-\boldsymbol{a} + \boldsymbol{b})$

$= \boldsymbol{a} + \frac{1}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$

$= \frac{3}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$

[2 marks]

(ii) $\overrightarrow{CX} = \overrightarrow{CB} - \overrightarrow{OB} + \overrightarrow{OA} + \overrightarrow{AX}$

$= \boldsymbol{a} - \boldsymbol{b} + \boldsymbol{a} - \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

$= \frac{3}{2}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$

[2 marks]

Answer 2

(a) **(i)** $\overrightarrow{DA} = \boldsymbol{a} + \boldsymbol{b}$

[1 mark]

(ii) $\overrightarrow{DX} = \frac{1}{2}\overrightarrow{DA} = \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

[1 mark]

(iii) $\overrightarrow{CD} = \boldsymbol{a} - \boldsymbol{b}$

[1 mark]

(iv) $\overrightarrow{YX} = \overrightarrow{YD} + \overrightarrow{DX}$

$= \frac{1}{2}\overrightarrow{CD} + \overrightarrow{DX}$

$= \frac{1}{2}(\boldsymbol{a} - \boldsymbol{b}) + \frac{1}{2}\boldsymbol{b} + \frac{1}{2}\boldsymbol{a}$

$= \boldsymbol{a}$

[1 mark]

(b) $\overrightarrow{YX} = \boldsymbol{a}$ and $\overrightarrow{CA} = 2\boldsymbol{a}$

$\therefore \overrightarrow{YX} = \frac{1}{2}\overrightarrow{CA}$ i.e. $\lambda = \frac{1}{2}$

[1 mark]

(c) $\boldsymbol{a}$ and $\boldsymbol{b}$ are mutually perpendicular

Use the Theorem of Pythagoras

$\Rightarrow |\boldsymbol{a} + \boldsymbol{b}| = \sqrt{274}$

$= 16.6$ cm

[1 mark]

Answer 3

(a) **(i)** $\overrightarrow{OY} = 2\boldsymbol{b}$ **(ii)** $\overrightarrow{AY} = -\boldsymbol{a} + 2\boldsymbol{b}$

[2 marks]

(b) $\overrightarrow{XB} = -\frac{1}{2}\boldsymbol{a} + \boldsymbol{b}$

[1 mark]

(c) $\overrightarrow{XB} = -\frac{1}{2}\boldsymbol{a} + \boldsymbol{b}$ and $\overrightarrow{AY} = -\boldsymbol{a} + 2\boldsymbol{b}$

$\therefore \overrightarrow{XB} = \frac{1}{2}\overrightarrow{AY}$ so $\overrightarrow{XB}$ is parallel to $\overrightarrow{AY}$

[1 mark]

(d) By Pythagoras, $|\overrightarrow{AY}| = \sqrt{(3+3)^2 - 3^2} = 3\sqrt{3}$

$= 5.20$ cm

[1 mark]

Answer 4

(a) **(i)** $\overrightarrow{DB} = \boldsymbol{a} + \boldsymbol{b}$ **(ii)** $\overrightarrow{DC} = \boldsymbol{b}$

(iii) $\overrightarrow{FC} = -2\boldsymbol{a} + 2\boldsymbol{b}$ **(iv)** $\overrightarrow{FD} = \boldsymbol{b} - 2\boldsymbol{a}$

[4 marks]

(b) $\overrightarrow{DX} = \overrightarrow{DA} + \overrightarrow{AX}$

$= \overrightarrow{DA} - \overrightarrow{XA}$

$= 2\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$

[1 mark]

(c) 7.45 cm

[2 marks]

Answer 5

(a) **(i)** $\overrightarrow{OP} = 2\boldsymbol{a}$ **(ii)** $\overrightarrow{PB} = -2\boldsymbol{a} + \boldsymbol{b}$

(iii) $\overrightarrow{OM} = \boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$ **(iv)** $\overrightarrow{AN} = -\frac{1}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$

(v) $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$

$= \boldsymbol{a} - \frac{1}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$

$= \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$

[1, 1, 1, 1, 2 marks]

(b) $\overrightarrow{OM} = \boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$ and $\overrightarrow{ON} = \frac{2}{3}\boldsymbol{a} + \frac{1}{3}\boldsymbol{b}$

$\overrightarrow{OM} = k\overrightarrow{ON}$ with $k = \frac{3}{2}$

[2 marks]

Answer 6

(a) **(i)** $\overrightarrow{AB} = -2\boldsymbol{a} + 2\boldsymbol{b}$ **(ii)** $\overrightarrow{BC} = -2\boldsymbol{b} + 2\boldsymbol{c}$

(iii) $\overrightarrow{PQ} = \overrightarrow{PA} + \overrightarrow{AQ}$

$= \boldsymbol{a} + \frac{1}{2}\overrightarrow{AB}$

$= \boldsymbol{a} - \boldsymbol{a} + \boldsymbol{b}$

$= \boldsymbol{b}$

(iv) $\overrightarrow{QR} = \overrightarrow{QB} + \overrightarrow{BR}$

$= \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC}$

$= -\boldsymbol{a} + \boldsymbol{b} - \boldsymbol{b} + \boldsymbol{c}$

$= -\boldsymbol{a} + \boldsymbol{c}$

(v) $\overrightarrow{PS} = \overrightarrow{PO} + \overrightarrow{OS}$

$= -\boldsymbol{a} + \boldsymbol{c}$

[1, 1, 1, 2, 1 marks]

(b) Same magnitude and direction

[1 mark]

(c) Parallelogram

[1 mark]

Answer 7

(i) $\overrightarrow{XM} = \frac{1}{2}\overrightarrow{XY}$ $\qquad$ $\overrightarrow{XZ} = \overrightarrow{XO} + \overrightarrow{OZ}$

$= \frac{1}{2}\overrightarrow{OZ}$ $\qquad$ $= \begin{pmatrix} -8 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

$= \frac{1}{2}\begin{pmatrix} -2 \\ 6 \end{pmatrix}$ $\qquad$ $= \begin{pmatrix} -10 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

[2 marks]

(ii) $\overrightarrow{ON} = v\overrightarrow{OM}$

$= v\left(\overrightarrow{OX} + \overrightarrow{XM}\right)$

$= v\left(\begin{pmatrix} 8 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 3 \end{pmatrix}\right)$

$= v\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

[2 marks]

(iii) $\overrightarrow{ON} = \overrightarrow{OX} + w\overrightarrow{XZ}$

$= \begin{pmatrix} 8 \\ 0 \end{pmatrix} + w\begin{pmatrix} -10 \\ 6 \end{pmatrix}$

[1 mark]

(iv) $v\begin{pmatrix} 7 \\ 3 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} + w\begin{pmatrix} -10 \\ 6 \end{pmatrix}$

$\left.\begin{matrix} 7v + 10w = 8 \\ 3v - 6w = 0 \end{matrix}\right\}$ $\qquad$ $\left.\begin{matrix} 21v + 30w = 24 \\ 21v - 42w = 0 \end{matrix}\right\}$ $\qquad 72w = 24$

$w = \frac{1}{3}$

and so $v = \frac{2}{3}$

[3 marks]

(v) N is a third of the way along XZ
N is two-thirds of the way along OM

[1 mark]