6.4 Answers

6.4.1 Solutions to 6.2 A Recent Examination Question (Teaching Video)

$$\begin{aligned}
\overrightarrow{AM} &= \overrightarrow{AP} + \overrightarrow{PM} \\
\therefore\ \overrightarrow{PM} &= \overrightarrow{AM} - \overrightarrow{AP} \\
&= \overrightarrow{AB} + \overrightarrow{BM} - \overrightarrow{AP} \\
&= \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} - \overrightarrow{AP} \\
&= 4\boldsymbol{a} + \frac{1}{2}(-4\boldsymbol{a} + 2\boldsymbol{b}) - \left(\frac{3}{2}\boldsymbol{a} + \frac{3}{4}\boldsymbol{b}\right) \\
&= 4\boldsymbol{a} - 2\boldsymbol{a} + \boldsymbol{b} - \frac{3}{2}\boldsymbol{a} - \frac{3}{4}\boldsymbol{b} \\
&= \frac{1}{2}\boldsymbol{a} + \frac{1}{4}\boldsymbol{b}
\end{aligned}$$

$$\begin{aligned}
AP &: PM \\
\frac{3}{2}\boldsymbol{a} + \frac{3}{4}\boldsymbol{b} &: \frac{1}{2}\boldsymbol{a} + \frac{1}{4}\boldsymbol{b} \\
3\left(\frac{1}{2}\boldsymbol{a} + \frac{1}{4}\boldsymbol{b}\right) &: \left(\frac{1}{2}\boldsymbol{a} + \frac{1}{4}\boldsymbol{b}\right) \\
3 &: 1
\end{aligned}$$

[3 marks]

6.4.2 Solutions to 6.3 Exercise

Answer 1

$$
\begin{aligned}
\overrightarrow{AM} &= \overrightarrow{AP} + \overrightarrow{PM} \\
\therefore\ \overrightarrow{PM} &= \overrightarrow{AM} - \overrightarrow{AP} \\
&= \overrightarrow{AB} + \overrightarrow{BM} - \overrightarrow{AP} \\
&= \overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} - \overrightarrow{AP} \\
&= 5\boldsymbol{a} + \frac{1}{2}(-5\boldsymbol{a} + 3\boldsymbol{b}) - \left(\frac{5}{6}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}\right) \\
&= 5\boldsymbol{a} - \frac{5}{2}\boldsymbol{a} + \frac{3}{2}\boldsymbol{b} - \frac{5}{6}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b} \\
&= \frac{5}{2}\boldsymbol{a} + \frac{3}{2}\boldsymbol{b} - \frac{5}{6}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b} \\
&= \frac{15}{6}\boldsymbol{a} + \frac{3}{2}\boldsymbol{b} - \frac{5}{6}\boldsymbol{a} - \frac{1}{2}\boldsymbol{b} \\
&= \frac{10}{6}\boldsymbol{a} + \boldsymbol{b}
\end{aligned}
$$

$$
\begin{aligned}
AP &: PM \\
\frac{5}{6}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} &: \frac{10}{6}\boldsymbol{a} + \boldsymbol{b} \\
\left(\frac{5}{6}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}\right) &: 2\left(\frac{5}{6}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}\right) \\
1 &: 2
\end{aligned}
$$

[3 marks]

Answer 2

(i)

$$AX : XB = 1 : 2$$

$$\Rightarrow \overrightarrow{XB} = 2\overrightarrow{AX}$$

$$\overrightarrow{AB} = \overrightarrow{AX} + \overrightarrow{XB}$$

$$= \overrightarrow{AX} + 2\overrightarrow{AX}$$

$$-3\boldsymbol{a} + 6\boldsymbol{b} = 3\overrightarrow{AX}$$

$$\overrightarrow{AX} = -\boldsymbol{a} + 2\boldsymbol{b}$$

(ii)

$$\overrightarrow{XB} = 2\overrightarrow{AX}$$

$$= -2\boldsymbol{a} + 4\boldsymbol{b}$$

$$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$$

$$= 3\boldsymbol{a} - \boldsymbol{a} + 2\boldsymbol{b}$$

$$= 2\boldsymbol{a} + 2\boldsymbol{b}$$

$$\overrightarrow{XY} = \overrightarrow{XB} + \overrightarrow{BY}$$

$$= -2\boldsymbol{a} + 4\boldsymbol{b} + 5\boldsymbol{a} - \boldsymbol{b}$$

$$= 3\boldsymbol{a} + 3\boldsymbol{b}$$

$$OX : XY$$

$$2\boldsymbol{a} + 2\boldsymbol{b} : 3\boldsymbol{a} + 3\boldsymbol{b}$$

$$2 : 3$$

[4 marks]

Answer 3

$$\overrightarrow{XD} = \overrightarrow{XA} + \overrightarrow{AD} \quad \Rightarrow \quad \overrightarrow{AD} = \overrightarrow{XD} - \overrightarrow{XA}$$

$$= \overrightarrow{XD} - \frac{1}{2}\overrightarrow{CA}$$

$$= 3\boldsymbol{a} - \frac{1}{2}\boldsymbol{c} - \frac{1}{2}(-\boldsymbol{c} + \boldsymbol{a})$$

$$= 3\boldsymbol{a} - \frac{1}{2}\boldsymbol{c} + \frac{1}{2}\boldsymbol{c} - \frac{1}{2}\boldsymbol{a}$$

$$= \frac{5}{2}\boldsymbol{a}$$

$$OA : AD$$

$$\boldsymbol{a} : \frac{5}{2}\boldsymbol{a}$$

$$2 : 5$$

[4 marks]

Answer 4

$$\begin{aligned}
\overrightarrow{PM} &= \overrightarrow{PA} + \overrightarrow{AM} \\
&= \frac{1}{3}\overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} \\
&= \frac{1}{3} \times 6\boldsymbol{a} + \frac{1}{2}(-6\boldsymbol{a} + 4\boldsymbol{b}) \\
&= 2\boldsymbol{a} - 3\boldsymbol{a} + 2\boldsymbol{b} \\
&= -\boldsymbol{a} + 2\boldsymbol{b}
\end{aligned}$$

$$\begin{aligned}
\overrightarrow{MC} &= \overrightarrow{MB} + \overrightarrow{BC} \\
&= \frac{1}{2}\overrightarrow{AB} + 4\boldsymbol{b} \\
&= \frac{1}{2}(-6\boldsymbol{a} + 4\boldsymbol{b}) + 4\boldsymbol{b} \\
&= -3\boldsymbol{a} + 2\boldsymbol{b} + 4\boldsymbol{b} \\
&= -3\boldsymbol{a} + 6\boldsymbol{b} \\
&= 3(-\boldsymbol{a} + 2\boldsymbol{b})
\end{aligned}$$

Thus,

$$\overrightarrow{PM} = \frac{1}{3}\overrightarrow{MC}$$

$\therefore$ $\overrightarrow{PM}$ and $\overrightarrow{MC}$ are parallel and, as they have the point M is common,

PMC is a straight line ☐

[5 marks]

Answer 5

(i) $\overrightarrow{BX} = \overrightarrow{BO} + \overrightarrow{OX}$

$= -3\boldsymbol{b} + 2\boldsymbol{a}$ as $\overrightarrow{AX} = \boldsymbol{a}$

$$BM : MX = 1 : 4$$
$$\Rightarrow \overrightarrow{MX} = 4\overrightarrow{BM}$$

$$\overrightarrow{BX} = \overrightarrow{BM} + \overrightarrow{MX}$$
$$= \overrightarrow{BM} + 4\overrightarrow{BM}$$
$$-3\boldsymbol{b} + 2\boldsymbol{a} = 5\overrightarrow{BM}$$
$$\overrightarrow{BM} = \frac{2}{5}\boldsymbol{a} - \frac{3}{5}\boldsymbol{b} \qquad \square$$

[2 marks]

(ii)

$$\overrightarrow{YM} = \overrightarrow{YB} + \overrightarrow{BM}$$
$$= -\boldsymbol{b} + \frac{2}{5}\boldsymbol{a} - \frac{3}{5}\boldsymbol{b}$$
$$= \frac{2}{5}\boldsymbol{a} - \frac{8}{5}\boldsymbol{b}$$

$$\overrightarrow{MA} = \overrightarrow{MX} + \overrightarrow{XA}$$
$$= 4\overrightarrow{BM} + \overrightarrow{XA}$$
$$= \frac{8}{5}\boldsymbol{a} - \frac{12}{5}\boldsymbol{b} - \boldsymbol{a}$$
$$= \frac{3}{5}\boldsymbol{a} - \frac{12}{5}\boldsymbol{b}$$

$$YM : MA$$
$$\frac{2}{5}\boldsymbol{a} - \frac{8}{5}\boldsymbol{b} : \frac{3}{5}\boldsymbol{a} - \frac{12}{5}\boldsymbol{b}$$
$$2\left(\frac{1}{5}\boldsymbol{a} - \frac{4}{5}\boldsymbol{b}\right) : 3\left(\frac{1}{5}\boldsymbol{a} - \frac{4}{5}\boldsymbol{b}\right)$$
$$2 : 3$$

[4 marks]

Answer 6

(a)

$$\overrightarrow{CD} = \overrightarrow{CB} + \overrightarrow{BA} + \overrightarrow{AD}$$
$$= -\boldsymbol{c} - \boldsymbol{b} + 3\,\boldsymbol{c}$$
$$= -\boldsymbol{b} + 2\,\boldsymbol{c}$$

[2 marks]

(b)

$$AP : PC = 2 : 1$$
$$\Rightarrow \overrightarrow{AP} = 2\,\overrightarrow{PC}$$

$$\overrightarrow{AC} = \overrightarrow{AP} + \overrightarrow{PC}$$
$$\boldsymbol{b} + \boldsymbol{c} = 2\,\overrightarrow{PC} + \overrightarrow{PC}$$
$$\overrightarrow{PC} = \frac{1}{3}\boldsymbol{b} + \frac{1}{3}\boldsymbol{c} \qquad \therefore\ \overrightarrow{AP} = \frac{2}{3}\boldsymbol{b} + \frac{2}{3}\boldsymbol{c}$$

$$\overrightarrow{BP} = \overrightarrow{BA} + \overrightarrow{AP}$$
$$= -\boldsymbol{b} + \frac{2}{3}\boldsymbol{b} + \frac{2}{3}\boldsymbol{c}$$
$$= -\frac{1}{3}\boldsymbol{b} + \frac{2}{3}\boldsymbol{c}$$

$$\overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$$
$$= -\boldsymbol{b} + 3\,\boldsymbol{c}$$

As $\overrightarrow{BP}$ and $\overrightarrow{BD}$ are not multiples of each other BPD is not a straight line

[4 marks]

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