7.4 Answers

7.4.1 Solution to 7.2 Example (Teaching Video)

(i)
$$\boldsymbol{s} = 6\boldsymbol{A} + 4\boldsymbol{B} = 6\begin{pmatrix}2\\5\end{pmatrix} + 4\begin{pmatrix}7\\3\end{pmatrix}$$
$$= \begin{pmatrix}12\\30\end{pmatrix} + \begin{pmatrix}28\\12\end{pmatrix}$$
$$= \begin{pmatrix}40\\42\end{pmatrix}$$
$$= 2\begin{pmatrix}20\\21\end{pmatrix}$$

[1 mark]

(ii)
$$\boldsymbol{t} = 9\boldsymbol{A} + 6\boldsymbol{B} = 9\begin{pmatrix}2\\5\end{pmatrix} + 6\begin{pmatrix}7\\3\end{pmatrix}$$
$$= \begin{pmatrix}18\\45\end{pmatrix} + \begin{pmatrix}42\\18\end{pmatrix}$$
$$= \begin{pmatrix}60\\63\end{pmatrix}$$
$$= 3\begin{pmatrix}20\\21\end{pmatrix}$$

[1 mark]

(iii) Parts (i) and (ii) show that $\boldsymbol{s}$ and $\boldsymbol{t}$ are parallel.

[1 mark]

(iv) $\boldsymbol{s} = 6\boldsymbol{A} + 4\boldsymbol{B} = 2(3\boldsymbol{A} + 2\boldsymbol{B})$ $\qquad$ $\boldsymbol{t} = 9\boldsymbol{A} + 6\boldsymbol{B} = 3(3\boldsymbol{A} + 2\boldsymbol{B})$

The $3\boldsymbol{A} + 2\boldsymbol{B}$ is both expressions shows that $\boldsymbol{s}$ and $\boldsymbol{t}$ are parallel regardless of the values of $\boldsymbol{A}$ and $\boldsymbol{B}$.

[2 marks]

7.4.2 Solutions to 7.3 Exercise

Answer 1

(i) $3\begin{pmatrix}9\\19\end{pmatrix}$ $\qquad$ **(ii)** $5\begin{pmatrix}9\\19\end{pmatrix}$

[1, 1 mark]

(iii) Parts (i) and (ii) show that $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel.

[1 mark]

(iv) $\boldsymbol{a} = 9\boldsymbol{E} + 3\boldsymbol{Z} = 3(3\boldsymbol{E} + \boldsymbol{Z})$ $\qquad$ $\boldsymbol{b} = 15\boldsymbol{E} + 5\boldsymbol{Z} = 5(3\boldsymbol{E} + \boldsymbol{Z})$

The $3\boldsymbol{E} + \boldsymbol{Z}$ in both expressions shows that $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel regardless of the values of $\boldsymbol{E}$ and $\boldsymbol{Z}$.

[2 marks]

Answer 2

(i) $4\begin{pmatrix} 4 \\ -5 \end{pmatrix}$ **(ii)** $3\begin{pmatrix} 4 \\ -5 \end{pmatrix}$

[1, 1 mark]

(iii) $\boldsymbol{p} = 8\boldsymbol{E} - 20\boldsymbol{T} = 4(2\boldsymbol{E} - 5\boldsymbol{T})$ $\boldsymbol{h} = 6\boldsymbol{E} - 15\boldsymbol{T} = 3(2\boldsymbol{E} - 5\boldsymbol{T})$

The $2\boldsymbol{E} - 5\boldsymbol{T}$ in both expressions shows that $\boldsymbol{p}$ and $\boldsymbol{h}$ are parallel regardless of the values of $\boldsymbol{E}$ and $\boldsymbol{T}$.

[2 marks]

Answer 3

(i) $\begin{pmatrix} -15 \\ 9 \end{pmatrix} = 3\begin{pmatrix} -5 \\ 3 \end{pmatrix}$ $\therefore \boldsymbol{X} = 3\boldsymbol{Y}$ $\therefore$ parallel

[1 mark]

(ii) Not parallel

[1 mark]

(iii) $\begin{pmatrix} 8 \\ -4 \\ 6 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} 12 \\ -6 \\ 9 \end{pmatrix}$ $\therefore \boldsymbol{X} = \frac{2}{3}\boldsymbol{Y}$ $\therefore$ parallel

[1 mark]

(iv) Not parallel

[1 mark]

(v) $\begin{pmatrix} 9 \\ -6 \end{pmatrix} = \frac{3}{4}\begin{pmatrix} 12 \\ -8 \end{pmatrix}$ $\therefore \boldsymbol{X} = \frac{3}{4}\boldsymbol{Y}$ $\therefore$ parallel

[1 mark]

(vi) Not parallel

[1 mark]

(vii) $(3\boldsymbol{i} - \boldsymbol{j}) = -\frac{1}{2}(-6\boldsymbol{i} + 2\boldsymbol{j})$ $\therefore \boldsymbol{X} = -\frac{1}{2}\boldsymbol{Y}$ $\therefore$ parallel

[1 mark]

(viii) $(4\boldsymbol{a} - 12\boldsymbol{b}) = \frac{2}{3}(6\boldsymbol{a} - 18\boldsymbol{b})$ $\therefore \boldsymbol{X} = \frac{2}{3}\boldsymbol{Y}$ $\therefore$ parallel

[1 mark]

(ix) $15\boldsymbol{a} = \frac{15}{16}\,16\boldsymbol{a}$ $\therefore \boldsymbol{X} = \frac{15}{16}\boldsymbol{Y}$ $\therefore$ parallel

[1 mark]

(x) $\boldsymbol{i} - \boldsymbol{j} = -1(-\boldsymbol{i} + \boldsymbol{j})$ $\therefore \boldsymbol{X} = -\boldsymbol{Y}$ $\therefore$ parallel

[1 mark]

(xi) Not parallel

[1 mark]

(xii) Not parallel

[1 mark]

Answer 4

(i) Yes. They are perpendicular

(ii) Yes. They are perpendicular

(iii) No. They are NOT perpendicular

(iv) $w = 16.5$

[1, 1, 1, 2 marks]

Answer 5

(i) By the Theorem of Pythagoras

(ii) $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

[1, 2, 2 marks]

Answer 6

(i) $\boldsymbol{v} = 9 (3 \boldsymbol{p} + 2 \boldsymbol{q})$ **(ii)** $\boldsymbol{w} = 13 (3 \boldsymbol{p} + 2 \boldsymbol{q})$

(iii) The $3 \boldsymbol{p} + 2 \boldsymbol{q}$ in both expressions shows that $\boldsymbol{v}$ and $\boldsymbol{w}$ are parallel regardless of the values of $\boldsymbol{p}$ and $\boldsymbol{q}$.

[1, 1, 2 marks]

Answer 7

(i) $\boldsymbol{r} = 9 (\boldsymbol{p} - \boldsymbol{q})$

(ii) $\boldsymbol{s} = 14 (\boldsymbol{p} - \boldsymbol{q})$

(iii) $\boldsymbol{t} = -2 \boldsymbol{p} - 9 \boldsymbol{q}$

[1, 1, 1 mark]

(iv) The $\boldsymbol{p} - \boldsymbol{q}$ in the expressions shows that $\boldsymbol{r}$ and $\boldsymbol{s}$ are parallel regardless of the values of $\boldsymbol{p}$ and $\boldsymbol{q}$.

[2 marks]