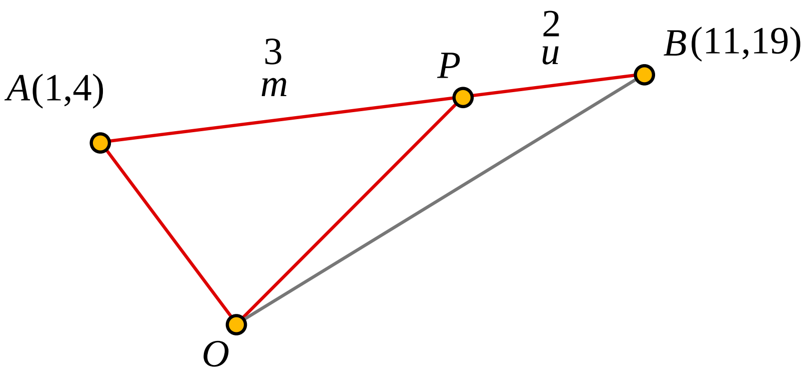
8.4 Answers

8.4.1 Solution to 8.2 Example (Teaching Video)



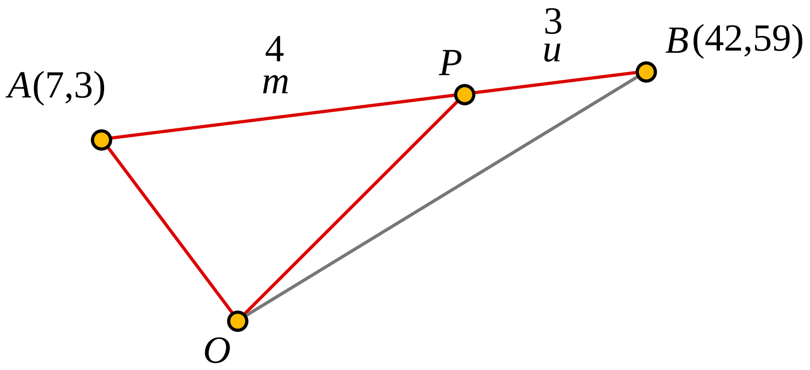
$$
\begin{aligned}
\overrightarrow{OP} &= \overrightarrow{OA} + \frac{m}{u+m}\overrightarrow{AB} \\
&= \overrightarrow{OA} + \frac{m}{u+m}\left(-\overrightarrow{OA} + \overrightarrow{OB}\right) \\
&= \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \frac{3}{2+3}\left(-\begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 11 \\ 19 \end{pmatrix}\right) \\
&= \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \frac{3}{5}\begin{pmatrix} 10 \\ 15 \end{pmatrix} \\
&= \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 6 \\ 9 \end{pmatrix} \\
&= \begin{pmatrix} 7 \\ 13 \end{pmatrix}
\end{aligned}
$$

$\therefore\ P\ (7,\ 13)$

[3 marks]

8.4.2 Solutions to 8.3 Exercise

Answer 1



$$\begin{aligned}
\overrightarrow{OP} &= \overrightarrow{OA} + \frac{m}{u+m}\overrightarrow{AB} \\
&= \overrightarrow{OA} + \frac{m}{u+m}\left(-\overrightarrow{OA} + \overrightarrow{OB}\right) \\
&= \begin{pmatrix} 7 \\ 3 \end{pmatrix} + \frac{4}{3+4}\left(-\begin{pmatrix} 7 \\ 3 \end{pmatrix} + \begin{pmatrix} 42 \\ 59 \end{pmatrix}\right) \\
&= \begin{pmatrix} 7 \\ 3 \end{pmatrix} + \frac{4}{7}\begin{pmatrix} 35 \\ 56 \end{pmatrix} \\
&= \begin{pmatrix} 7 \\ 3 \end{pmatrix} + \begin{pmatrix} 20 \\ 32 \end{pmatrix} \\
&= \begin{pmatrix} 27 \\ 35 \end{pmatrix}
\end{aligned}$$

$\therefore\ P\ (27,\ 35)$

[3 marks]

Answer 2

(i) $\overrightarrow{OA} = \overrightarrow{OC} + \overrightarrow{CB} + \overrightarrow{BA}$

$= \boldsymbol{c} + \boldsymbol{b} - 5\boldsymbol{c}$

$= \boldsymbol{b} - 4\boldsymbol{c}$

[1 mark]

(ii) $\overrightarrow{OP} = \overrightarrow{OA} + \dfrac{m}{u+m}\overrightarrow{AB}$

$= \boldsymbol{b} - 4\boldsymbol{c} + \dfrac{3}{2+3} \times 5\boldsymbol{c}$

$= \boldsymbol{b} - 4\boldsymbol{c} + \dfrac{3}{5} \times 5\boldsymbol{c}$

$= \boldsymbol{b} - 4\boldsymbol{c} + 3\boldsymbol{c}$

$= \boldsymbol{b} - \boldsymbol{c}$

[3 marks]

Answer 3

(i) $\overrightarrow{OP} = \overrightarrow{OA} + \dfrac{m}{u + m}\overrightarrow{AB}$

$= \boldsymbol{a} + \dfrac{3}{1+3}(-\boldsymbol{a} + \boldsymbol{b})$

$= \boldsymbol{a} + \dfrac{3}{4}(-\boldsymbol{a} + \boldsymbol{b})$

$= \boldsymbol{a} - \dfrac{3}{4}\boldsymbol{a} + \dfrac{3}{4}\boldsymbol{b}$

$= \dfrac{1}{4}\boldsymbol{a} + \dfrac{3}{4}\boldsymbol{b}$

[2 marks]

(ii) $\overrightarrow{CP} = \overrightarrow{CO} + \overrightarrow{OP}$

$= \dfrac{1}{2}\overrightarrow{AO} + \overrightarrow{OP}$

$= -\dfrac{1}{2}\overrightarrow{OA} + \overrightarrow{OP}$

$= -\dfrac{1}{2}\boldsymbol{a} + \dfrac{1}{4}\boldsymbol{a} + \dfrac{3}{4}\boldsymbol{b}$

$= -\dfrac{1}{4}\boldsymbol{a} + \dfrac{3}{4}\boldsymbol{b}$

[1 mark]

(iii) $\overrightarrow{PD} = \overrightarrow{PO} + \overrightarrow{OD}$

$= -\overrightarrow{OP} + \overrightarrow{OB} + \overrightarrow{BD}$

$= -\overrightarrow{OP} + \dfrac{3}{2}\overrightarrow{OB}$

$= -\left(\dfrac{1}{4}\boldsymbol{a} + \dfrac{3}{4}\boldsymbol{b}\right) + \dfrac{3}{2}\boldsymbol{b}$

$= -\dfrac{1}{4}\boldsymbol{a} + \dfrac{3}{4}\boldsymbol{b}$

(iv) As $\overrightarrow{CP} = \overrightarrow{PD}$, CP and PD are parallel.
They also have a common point, P, and so C, P and D
lie on the same straight line □

[1 mark]

(v) $CP : PD = 1 : 1$
P is the midpoint of CD

[1 mark]

Answer 4

(i) $\overrightarrow{OP} = \overrightarrow{OA} + \dfrac{m}{u + m}\overrightarrow{AB}$

$= 3\boldsymbol{a} + \dfrac{2}{1+2}\left(-3\boldsymbol{a} + \boldsymbol{b}\right)$

$= 3\boldsymbol{a} + \dfrac{2}{3}\left(-3\boldsymbol{a} + \boldsymbol{b}\right)$

$= 3\boldsymbol{a} - 2\boldsymbol{a} + \dfrac{2}{3}\boldsymbol{b}$

$= \boldsymbol{a} + \dfrac{2}{3}\boldsymbol{b}$

[3 marks]

(ii) $\overrightarrow{QP} = \overrightarrow{QO} + \overrightarrow{OP}$

$= -\boldsymbol{a} + \boldsymbol{a} + \dfrac{2}{3}\boldsymbol{b}$

$= \dfrac{2}{3}\boldsymbol{b}$

$= \dfrac{2}{3}\overrightarrow{OB}$ □

[2 marks]

(iii) *QP* is parallel to *OB*

QP is $\dfrac{2}{3}$ of the length of *OB*

[2 marks]

Answer 5

By the Ratio Theorem, $\overrightarrow{OP} = \overrightarrow{OA} + \dfrac{m}{u + m}\overrightarrow{AB}$

$= 2\boldsymbol{a} + \dfrac{5}{3+5}\left(-2\boldsymbol{a} + 2\boldsymbol{b}\right)$

$= 2\boldsymbol{a} + \dfrac{5}{8}\left(-2\boldsymbol{a} + 2\boldsymbol{b}\right)$

$= 2\boldsymbol{a} - \dfrac{5}{4}\boldsymbol{a} + \dfrac{5}{4}\boldsymbol{b}$

$= \dfrac{3}{4}\boldsymbol{a} + \dfrac{5}{4}\boldsymbol{b}$

$= \dfrac{1}{4}(3\boldsymbol{a} + 5\boldsymbol{b})$

$\therefore\ k = \dfrac{1}{4}$

[3 marks]

Answer 6

(a) **(i)** $\overrightarrow{BA} = \boldsymbol{a} - 2\boldsymbol{b}$

[1 mark]

(ii) By the Ratio Theorem,

$$\overrightarrow{MQ} = \overrightarrow{MB} + \frac{m}{u + m}\overrightarrow{BA}$$
$$= \boldsymbol{b} + \frac{1}{3}\left(-2\boldsymbol{b} + \boldsymbol{a}\right)$$
$$= \boldsymbol{b} - \frac{2}{3}\boldsymbol{b} + \frac{1}{3}\boldsymbol{a}$$
$$= \frac{1}{3}\boldsymbol{b} + \frac{1}{3}\boldsymbol{a}$$
$$= \frac{1}{3}(\boldsymbol{a} + \boldsymbol{b})$$

[2 marks]

(iii) By the Ratio Theorem,

$$\overrightarrow{OP} = \overrightarrow{OB} + \frac{m}{u + m}\overrightarrow{BA}$$
$$= 2\boldsymbol{b} + \frac{2}{3}(-2\boldsymbol{b} + \boldsymbol{a})$$
$$= 2\boldsymbol{b} - \frac{4}{3}\boldsymbol{b} + \frac{2}{3}\boldsymbol{a}$$
$$= \frac{2}{3}\boldsymbol{b} + \frac{2}{3}\boldsymbol{a}$$
$$= \frac{2}{3}(\boldsymbol{a} + \boldsymbol{b})$$

[2 marks]

(b) As $\overrightarrow{OP} = 2\overrightarrow{MQ}$, OP is parallel to MQ

$\therefore$ $OMQP$ is a trapezium

[2 marks]

Answer 7

(i) $\overrightarrow{CE} = \boldsymbol{a} + \boldsymbol{b}$

[1 mark]

(ii) $\overrightarrow{FE} = \overrightarrow{FC} + \overrightarrow{CE}$

$= \boldsymbol{a} - \boldsymbol{b} + \boldsymbol{a} + \boldsymbol{b}$ using part (i)

$= 2\,\boldsymbol{a}$

$\therefore\ \overrightarrow{FE} = 2\,\overrightarrow{CD}$ and so $\overrightarrow{FE}$ and $\overrightarrow{CD}$ are parallel ☐

[1 mark]

(iii) $\overrightarrow{FM} = \overrightarrow{FC} + \overrightarrow{CD} + \overrightarrow{DM}$

$$= \overrightarrow{FC} + \overrightarrow{CD} + \frac{1}{2}\overrightarrow{DE}$$

$$= \boldsymbol{a} - \boldsymbol{b} + \boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$$

$$= 2\,\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$$

[1 mark]

(iv) By the Ratio Theorem, using $FX : XM = 4 : 1$

$$\overrightarrow{EX} = \overrightarrow{EF} + \frac{m}{u + m}\overrightarrow{FM}$$

$$= -2\,\boldsymbol{a} + \frac{4}{1 + 4}\left(2\,\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}\right)$$

$$= -2\,\boldsymbol{a} + \frac{4}{5}\left(2\,\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}\right)$$

$$= -2\,\boldsymbol{a} + \frac{8}{5}\boldsymbol{a} - \frac{2}{5}\boldsymbol{b}$$

$$= -\frac{2}{5}\boldsymbol{a} - \frac{2}{5}\boldsymbol{b}$$

$$= -\frac{2}{5}\left(\boldsymbol{a} + \boldsymbol{b}\right)$$

$$= -\frac{2}{5}\overrightarrow{CE}$$

$\therefore$ *EX* is parallel with *CE* with a common point *E*

$\therefore$ *C*, *X* and *E* lie on the same straight line ☐

[3 marks]

Answer 8

By the Ratio Theorem,

$$\begin{aligned}\overrightarrow{OP} &= \overrightarrow{OA} + \frac{m}{u + m}\overrightarrow{AB} \\ &= \boldsymbol{a} + \frac{1}{3 + 1}\boldsymbol{c} \\ &= \boldsymbol{a} + \frac{1}{4}\boldsymbol{c}\end{aligned}$$

$$\begin{aligned}\overrightarrow{OQ} &= \frac{2}{3}\overrightarrow{OC} \\ &= \frac{2}{3}\boldsymbol{c}\end{aligned}$$

$$\begin{aligned}\overrightarrow{PQ} &= \overrightarrow{PO} + \overrightarrow{OQ} \\ &= -\boldsymbol{a} - \frac{1}{4}\boldsymbol{c} + \frac{2}{3}\boldsymbol{c} \\ &= -\boldsymbol{a} + \frac{5}{12}\boldsymbol{c}\end{aligned}$$

[4 marks]