9.4 Answers

9.4.1 Solution to 9.2 Example (Teaching Video)

By the Ratio Theorem, $\overrightarrow{OP} = \overrightarrow{OA} + \frac{m}{u+m}\overrightarrow{AB}$

$$= \boldsymbol{a} + \frac{4}{7}(-\boldsymbol{a} + b)$$

$$= \frac{3}{7}\boldsymbol{a} + \frac{4}{7}\boldsymbol{b}$$

$$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$$

$$s\left(\frac{3}{7}\boldsymbol{a} + \frac{4}{7}\boldsymbol{b}\right) = \boldsymbol{a} + t\,\boldsymbol{b}$$

$$\frac{3s}{7}\boldsymbol{a} + \frac{4s}{7}\boldsymbol{b} = \boldsymbol{a} + t\,\boldsymbol{b}$$

$$\therefore \frac{3s}{7}\boldsymbol{a} = 1 \Rightarrow s = \frac{7}{3}$$

$$\text{Also, } \frac{4s}{7} = t \Rightarrow t = \frac{4}{7} \times \frac{7}{3}$$

$$t = \frac{4}{3}$$

$$\overrightarrow{OX} = s\left(\frac{3}{7}\boldsymbol{a} + \frac{4}{7}\boldsymbol{b}\right)$$

$$= \frac{7}{3}\left(\frac{3}{7}\boldsymbol{a} + \frac{4}{7}\boldsymbol{b}\right)$$

$$= \boldsymbol{a} + \frac{4}{3}\boldsymbol{b}$$

[5 marks]

9.4.2 Solutions to 9.3 Exercise

Answer 1

(i) By the Ratio Theorem, $\overrightarrow{OM} = \overrightarrow{OA} + \frac{m}{u+m}\overrightarrow{AB}$

$$= \boldsymbol{a} + \frac{1}{2}\left(-\boldsymbol{a} + \boldsymbol{b}\right)$$

$$= \frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$$

[1 mark]

(ii) $\overrightarrow{OP} = \frac{4}{7}\overrightarrow{OM} = \frac{4}{7}\left(\frac{1}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}\right)$

$$= \frac{2}{7}\boldsymbol{a} + \frac{2}{7}\boldsymbol{b}$$

[1 mark]

(iii) $\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP} = -\boldsymbol{a} + \frac{2}{7}\boldsymbol{a} + \frac{2}{7}\boldsymbol{b}$

$$= -\frac{5}{7}\boldsymbol{a} + \frac{2}{7}\boldsymbol{b}$$

[1 mark]

(iv)

$$\overrightarrow{AN} = \overrightarrow{AO} + \overrightarrow{ON}$$

$$s\overrightarrow{AP} = -\overrightarrow{OA} + t\overrightarrow{OB}$$

$$s\left(-\frac{5}{7}\boldsymbol{a} + \frac{2}{7}\boldsymbol{b}\right) = -\boldsymbol{a} + t\boldsymbol{b}$$

$$-\frac{5s}{7}\boldsymbol{a} + \frac{2s}{7}\boldsymbol{b} = -\boldsymbol{a} + t\boldsymbol{b}$$

$$\therefore\ -\frac{5s}{7} = -1 \Rightarrow s = \frac{7}{5}$$

$$\text{Also, } \frac{2s}{7} = t \Rightarrow t = \frac{2}{7} \times \frac{7}{5}$$

$$t = \frac{2}{5}$$

$$\overrightarrow{ON} = t\boldsymbol{b}$$

$$= \frac{2}{5}\boldsymbol{b}$$

$$\therefore\ ON : NB = 2 : 3$$

[4 marks]

Answer 2

(i) $\overrightarrow{AB} = -\boldsymbol{a} + \boldsymbol{b}$

(ii) $\overrightarrow{NM} = -3\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

(iii)
$$\overrightarrow{AP} = \overrightarrow{AN} + \overrightarrow{NP}$$
$$s(-\boldsymbol{a} + \boldsymbol{b}) = 2\boldsymbol{a} + t\left(-3\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}\right)$$
$$-s\boldsymbol{a} + s\boldsymbol{b} = 2\boldsymbol{a} - 3t\boldsymbol{a} + \frac{t}{2}\boldsymbol{b}$$
$$\therefore \quad -s = 2 - 3t \quad \text{and} \quad s = \frac{t}{2}$$
$$-\frac{t}{2} = 2 - 3t$$
$$-t = 4 - 6t$$
$$5t = 4$$
$$t = \frac{4}{5} \Rightarrow s = \frac{2}{5}$$
$$\therefore AP : PB = 2 : 3$$

[5 marks]

Answer 3

$$\overrightarrow{AY} = -\boldsymbol{a} + \frac{2}{3}\boldsymbol{c} \text{ and } \overrightarrow{OB} = \boldsymbol{a} + \boldsymbol{c}$$
$$\overrightarrow{AX} = \overrightarrow{AO} + \overrightarrow{OX}$$
$$s\overrightarrow{AY} = -\overrightarrow{OA} + t\overrightarrow{OB}$$
$$s(-\boldsymbol{a} + \frac{2}{3}\boldsymbol{c}) = -\boldsymbol{a} + t(\boldsymbol{a} + \boldsymbol{c})$$
$$-s\boldsymbol{a} + \frac{2s}{3}\boldsymbol{c} = -\boldsymbol{a} + t\boldsymbol{a} + t\boldsymbol{c}$$
$$\therefore \quad -s = -1 + t \quad \text{and} \quad \frac{2s}{3} = t$$
$$-s = -1 + \frac{2s}{3}$$
$$-3s = -3 + 2s$$
$$5s = 3$$
$$s = \frac{3}{5} \Rightarrow t = \frac{2}{5}$$
$$\therefore AX : XY = 3 : 2$$

[6 marks]

Answer 4

$$\overrightarrow{OP} = \boldsymbol{a} + \frac{3}{4} \times 2\,\boldsymbol{c}$$

$$= \boldsymbol{a} + \frac{3}{2}\boldsymbol{c}$$

$$\overrightarrow{AC} = -\boldsymbol{a} + \boldsymbol{c}$$

$$\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ}$$

$$s\overrightarrow{AC} = -\overrightarrow{OA} + t\overrightarrow{OP}$$

$$s(-\boldsymbol{a} + \boldsymbol{c}) = -\boldsymbol{a} + t\left(\boldsymbol{a} + \frac{3}{2}\boldsymbol{c}\right)$$

$$-s\,\boldsymbol{a} + s\,\boldsymbol{c} = -\boldsymbol{a} + t\,\boldsymbol{a} + \frac{3t}{2}\boldsymbol{c}$$

$$\therefore \quad -s = -1 + t \qquad \text{and} \qquad s = \frac{3t}{2}$$

$$-\frac{3t}{2} = -1 + t$$

$$-3t = -2 + 2t$$

$$5t = 2$$

$$t = \frac{2}{5} \Rightarrow s = \frac{3}{5}$$

$$\overrightarrow{AQ} = s\overrightarrow{AC}$$

$$= \frac{3}{5}\overrightarrow{AC}$$

$$\therefore AQ : QC = 3 : 2$$

[5 marks]

Answer 5

(i) $\overrightarrow{BP} = \boldsymbol{a} - \boldsymbol{b}$ and $\overrightarrow{AB} = -3\boldsymbol{a} + \boldsymbol{b}$

[1 mark]

(ii) $\overrightarrow{MB} = \frac{1}{2}\overrightarrow{AB} = -\frac{3}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

[1 mark]

(iii) $\overrightarrow{MX} = \overrightarrow{MB} + \overrightarrow{BX}$

$= \overrightarrow{MB} + k\overrightarrow{BP}$

$= -\frac{3}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} + k(\boldsymbol{a} - \boldsymbol{b})$

[3 marks]

(iv) $\overrightarrow{MX} = -\frac{3}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b} + k(\boldsymbol{a} - \boldsymbol{b})$

$= \left(-\frac{3}{2} + k\right)\boldsymbol{a} + \left(\frac{1}{2} - k\right)\boldsymbol{b}$

If MX is parallel to BO the $\boldsymbol{a}$ component of $\overrightarrow{MX}$ must be zero.

$$\therefore -\frac{3}{2} + k = 0$$

$$k = \frac{3}{2}$$

[2 marks]