

GCSE Mathematics

PROPORTIONALITY



On The Cover

The yacht pictured, a Vertue, is an example of a boat with a displacement hull. It moves through the water by pushing the water to either side. Notice that, because it's moving fairly fast, a large bow wave is being created.

A displacement hull has a speed beyond which it cannot pass through water. As the hull speed approaches the theoretical maximum the energy from the sails makes the bow wave bigger rather than move the hull faster.

A displacement hull's maximum speed through the water, V_{max} is directly proportional to the square root of its waterline length, LWL .

The Vertue has a maximum theoretical speed of 6.3 knots. Its LWL - Length Water Line - is 22 feet.

Lesson 1

Proportionality : GCSE

1.1 Direct Proportion

The topic of proportionality is about how mathematicians describe equations.

For GCSE the equations will typically involve two variables, say x and y .

In their mathematical description all of the following equations would invoke the words *directly proportional to*;

$$y = 6x \qquad y = 5\sqrt{x} \qquad y = 0.4x^2$$

and the numbers 6, 5 and 0.4 are each a *constant of the proportionality*.

In full we'd say;

$$y = 6x$$

y is directly proportional to x

The constant of the proportionality is 6

$$y = 5\sqrt{x}$$

y is directly proportional to the square root of x

The constant of the proportionality is 5

$$y = 0.4x^2$$

y is directly proportional to the square of x

The constant of the proportionality is 0.4

Example

Given that,

$$y = 9\sqrt{x}$$

- (i)** Tick which applies ☐ y is directly proportional to square root x
☐ y is inversely proportional to square root x
- (ii)** What is the constant of the proportionality ?
- (iii)** When $x = 49$, what is y ?
- (iv)** If the value of x was increased, would the value of y increase or decrease ?

1.2 Inverse Proportion

In their mathematical description all of the following equations would invoke the words *inversely proportional to*;

$$y = \frac{8}{x} \qquad y = \frac{9}{\sqrt{x}} \qquad y = \frac{3}{x^2}$$

and the numbers 8, 9 and 3 are each a *constant of the proportionality*.

In full we'd say;

$$y = \frac{8}{x} \quad \text{or} \quad y = 8 \times \frac{1}{x}$$

y is inversely proportional to x

The constant of the proportionality is 8

$$y = \frac{9}{\sqrt{x}} \quad \text{or} \quad y = 9 \times \frac{1}{\sqrt{x}}$$

y is inversely proportional to the square root of x

The constant of the proportionality is 9

$$y = \frac{3}{x^2} \quad \text{or} \quad y = 3 \times \frac{1}{x^2}$$

y is inversely proportional to the square of x

The constant of the proportionality is 3

Example

Given that,

$$y = \frac{20}{\sqrt{x}}$$

- (i) Tick which applies ☐ y is directly proportional to square root x
 ☐ y is inversely proportional to square root x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 16$, what is y ?
- (iv) If the value of x was increased, would the value of y increase or decrease ?

1.3 Exercise

Question 1

$$y = 7\sqrt{x}$$

- (i) Tick which applies ☐ y is directly proportional to the square root of x
 ☐ y is inversely proportional to the square root of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 9$, what is y ?
- (iv) When $x = 25$, what is y ?
- (v) If the value of x was increased, would the value of y increase or decrease ?

Question 2

$$y = \frac{36}{\sqrt{x}}$$

- (i) Tick which applies
- ☐ y is directly proportional to the square root of x
☐ y is inversely proportional to the square root of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 9$, what is y ?
- (iv) When $x = 100$, what is y ?
- (v) If the value of x was increased, would the value of y increase or decrease ?

Question 3

$$y = 0.5 x^2$$

- (i) Tick which applies ☐ y is directly proportional to the square of x
 ☐ y is inversely proportional to the square of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 4$, what is y ?
- (iv) When $x = 6$, what is y ?
- (v) If the value of x was increased, would the value of y increase or decrease ?

Question 4

$$y = 100 \times \frac{1}{x^2}$$

- (i) Tick which applies ☐ y is directly proportional to the square of x
 ☐ y is inversely proportional to the square of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 2$, what is y ?
- (iv) When $x = 5$, what is y ?
- (iv) If the value of x was increased, would the value of y increase or decrease ?

Question 5

$$y = 8x$$

- (i) Tick which applies ☐ y is directly proportional to x
 ☐ y is inversely proportional to x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 6$, what is y ?
- (iv) When $x = 0.5$, what is y ?
- (v) When $y = 32$, what is x ?

Question 6

$$y = \frac{6}{x}$$

- (i) Tick which applies ☐ y is directly proportional to x
 ☐ y is inversely proportional to x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 2$, what is y ?
- (iv) When $x = 12$, what is y ?
- (v) When $y = 2$, what is x ?

Question 7

Write down the equation described by,

y is directly proportional to the square root of x

The constant of the proportionality is 0.4

Question 8

Write down the equation described by,

y is inversely proportional to the square of x

The constant of the proportionality is 12

Question 9

$$y = \frac{5m^2\sqrt{x}}{z}$$

- (i)** Tick which three apply
- ☐ y is directly proportional to square of m
 - ☐ y is inversely proportional to square of m
 - ☐ y is directly proportional to the square root of x
 - ☐ y is inversely proportional to the square root of x
 - ☐ y is directly proportional to z
 - ☐ y is inversely proportional to z
- (ii)** What is the constant of the proportionality ?
- (iii)** When $m = 3$, $x = 100$, and $z = 50$ what is y ?

Question 10

$$y = \frac{2\sqrt{p}q}{v^2}$$

- (i)** Tick which three apply
- ☐ y is directly proportional to square root of p
 - ☐ y is inversely proportional to square root of p
 - ☐ y is directly proportional to q
 - ☐ y is inversely proportional to q
 - ☐ y is directly proportional to the square of v
 - ☐ y is inversely proportional to the square of v
- (ii)** What is the constant of the proportionality ?
- (iii)** When $p = 25$, $q = 9$, and $v = 2$ what is y ?

Question 11

Write down the one equation described by,

y is directly proportional to the square of *m*

y is inversely proportional to w

y is directly proportional to the square root of *s*

The constant of the proportionality is 6

Question 12

Write down the equation described by,

y is directly proportional to the cube of x

The constant of the proportionality is 5

When x is 10, what is y ?

Question 13

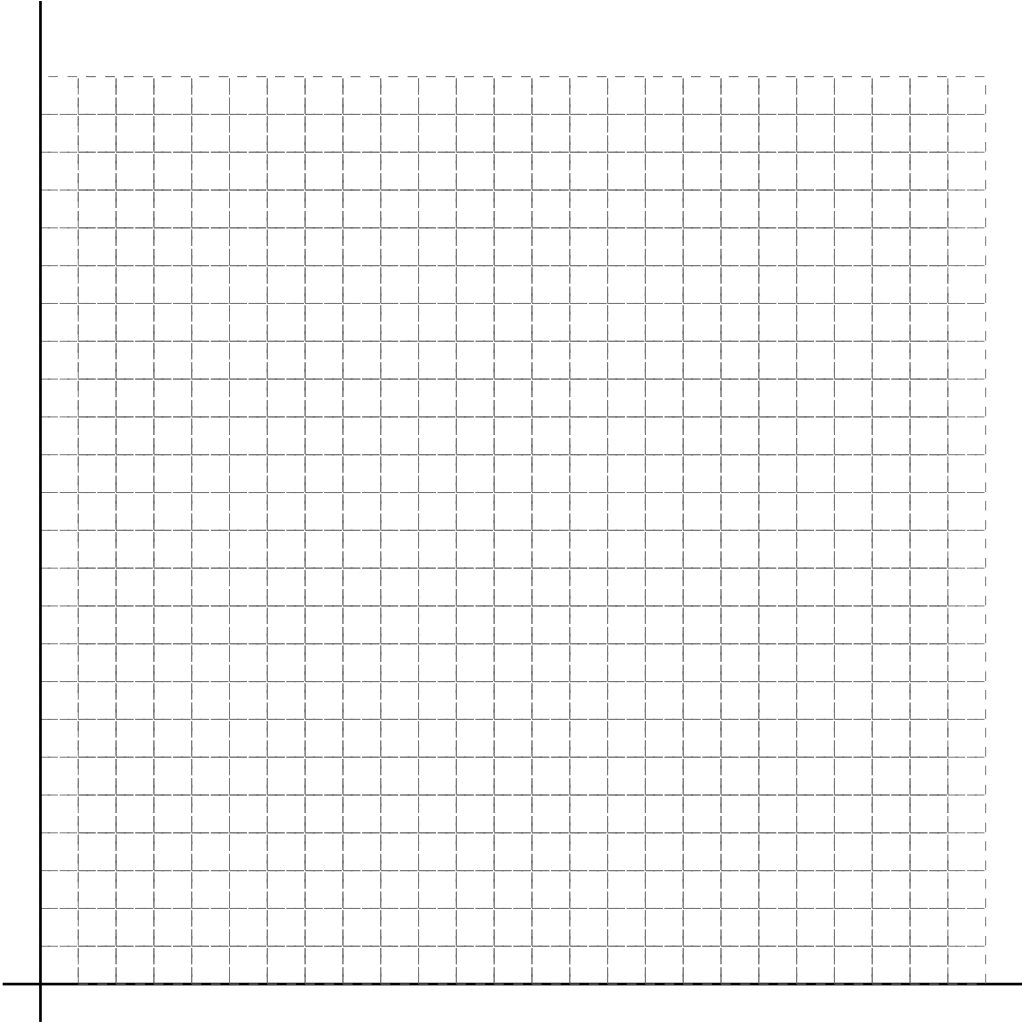
This question is about plotting a graph of direct proportion for the equation

$$y = 0.5 \sqrt{x}$$

Complete the following table,

x	0	1	4	9	16	25
y						2.5

Plot the graph of the equation.



This direct proportion with a square root is sometimes described "diminishing returns". Explain this.

1.4 Answers

1.4.1 Solutions (1.1 Examples)

- (i) Tick ☐ y is directly proportional to square root x
- (ii) 9
- (iii) $y = 63$
- (iv) INCREASE "More is More"

1.4.2 Solutions (1.2 Examples)

- (i) Tick ☐ y is inversely proportional to the square root of x .
- (ii) 20
- (iii) $y = 5$
- (iv) DECREASE "More is Less"

1.4.3 Solutions (1.3 Exercise)

Answer 1

- (i) Tick ☐ y is directly proportional to square root x
- (ii) 7
- (iii) $y = 21$
- (iv) $y = 35$
- (v) INCREASE "More is More"

Answer 2

- (i) Tick ☐ y is inversely proportional to the square root of x .
- (ii) 36
- (iii) $y = 12$
- (iv) $y = 3.6$
- (v) DECREASE "More is Less"

Answer 3

- (i) Tick ☐ y is directly proportional to the square of x
- (ii) 0.5
- (iii) $y = 8$
- (iv) $y = 18$
- (v) INCREASE "More is More"

Answer 4

- (i) Tick ☐ y is inversely proportional to the square of x .
- (ii) 100
- (iii) $y = 25$
- (iv) $y = 4$
- (v) DECREASE "More is Less"

Answer 5

- (i) Tick ☐ y is directly proportional to x
 (ii) 8
 (iii) $y = 48$
 (iv) $y = 4$
 (v) $x = 4$

Answer 6

- (i) Tick ☐ y is inversely proportional to x .
 (ii) 6
 (iii) $y = 3$
 (iv) $y = 0.5$
 (v) $x = 3$

Answer 7

$$y = 0.4\sqrt{x}$$

Answer 8

$$y = \frac{12}{x^2}$$

Answer 9

- (i) Tick ☐ y is directly proportional to square of m
 ☐ y is directly proportional to the square root of x
 ☐ y is inversely proportional to z
 (ii) 5
 (iii) $y = 9$

Answer 10

- (i) Tick ☐ y is directly proportional to square root of p
 ☐ y is directly proportional to q
 ☐ y is inversely proportional to the square of v
 (ii) 2
 (iii) $y = 22.5$

Answer 11

$$y = \frac{6m^2\sqrt{s}}{w}$$

Answer 12

$$y = 5x^3$$

$$y = 5000$$

Lesson 2

Proportionality : GCSE

2.1 Direct and Inverse Proportion Exercise

Question 1

$$y = 10\sqrt{x}$$

- (i) Tick which applies ☐ y is directly proportional to the square root of x
 ☐ y is inversely proportional to the square root of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 81$, what is y ?
- (iv) When $x = 121$, what is y ?
- (v) If the value of x was increased, would the value of y increase or decrease ?

Question 2

$$y = \frac{16}{\sqrt{x}}$$

- (i) Tick which applies
 ☐ y is directly proportional to the square root of x
 ☐ y is inversely proportional to the square root of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 16$, what is y ?
- (iv) When $x = 64$, what is y ?
- (v) If the value of x was increased, would the value of y increase or decrease ?

Question 3

$$y = \frac{2}{3}x^2$$

- (i) Tick which applies ☐ y is directly proportional to the square of x
 ☐ y is inversely proportional to the square of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 3$, what is y ?
- (iv) When $x = 6$, what is y ?
- (v) If the value of x was increased, would the value of y increase or decrease ?

Question 4

$$y = 12 \times \frac{1}{x^2}$$

- (i) Tick which applies ☐ y is directly proportional to the square of x
 ☐ y is inversely proportional to the square of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 1$, what is y ?
- (iv) When $x = 2$, what is y ?
- (iv) If the value of x was increased, would the value of y increase or decrease ?

Question 5

$$y = 9x$$

- (i) Tick which applies ☐ y is directly proportional to x
 ☐ y is inversely proportional to x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 5$, what is y ?
- (iv) When $x = 14$, what is y ?
- (v) When $y = 45$, what is x ?

Question 6

$$y = \frac{24}{x}$$

- (i) Tick which applies ☐ y is directly proportional to x
 ☐ y is inversely proportional to x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 8$, what is y ?
- (iv) When $x = 48$, what is y ?
- (v) When $y = 6$, what is x ?

Question 7

Write down the equation described by,

y is directly proportional to the square root of x

The constant of the proportionality is 7

Question 8

Write down the equation described by,

y is inversely proportional to the square of x

The constant of the proportionality is 18

Question 9

$$y = \frac{4 m^2 x}{\sqrt{z}}$$

- (i) Tick which applies
- ☐ y is directly proportional to square of m
 - ☐ y is inversely proportional to square of m
 - ☐ y is directly proportional to x
 - ☐ y is inversely proportional to x
 - ☐ y is directly proportional to the square root of z
 - ☐ y is inversely proportional to the square root of z
- (ii) What is the constant of the proportionality ?
- (iii) When $m = 5$, $x = 7$, and $z = 100$ what is y ?

Question 10

$$y = \frac{10 \sqrt{c} w}{z^2}$$

- (i) Tick which applies
- ☐ y is directly proportional to square root of c
 - ☐ y is inversely proportional to square root of c
 - ☐ y is directly proportional to w
 - ☐ y is inversely proportional to w
 - ☐ y is directly proportional to the square of z
 - ☐ y is inversely proportional to the square of z
- (ii) What is the constant of the proportionality ?
- (iii) When $c = 9$, $w = 100$, and $z = 5$ what is y ?

Question 11

Write down the one equation described by,

y is directly proportional to the square of v

y is inversely proportional to n

y is directly proportional to the square root of a

The constant of the proportionality is 7

Question 12

Write down the equation described by,

y is directly proportional to the cube of x

The constant of the proportionality is 5

When x is 3, what is y ?

Question 13

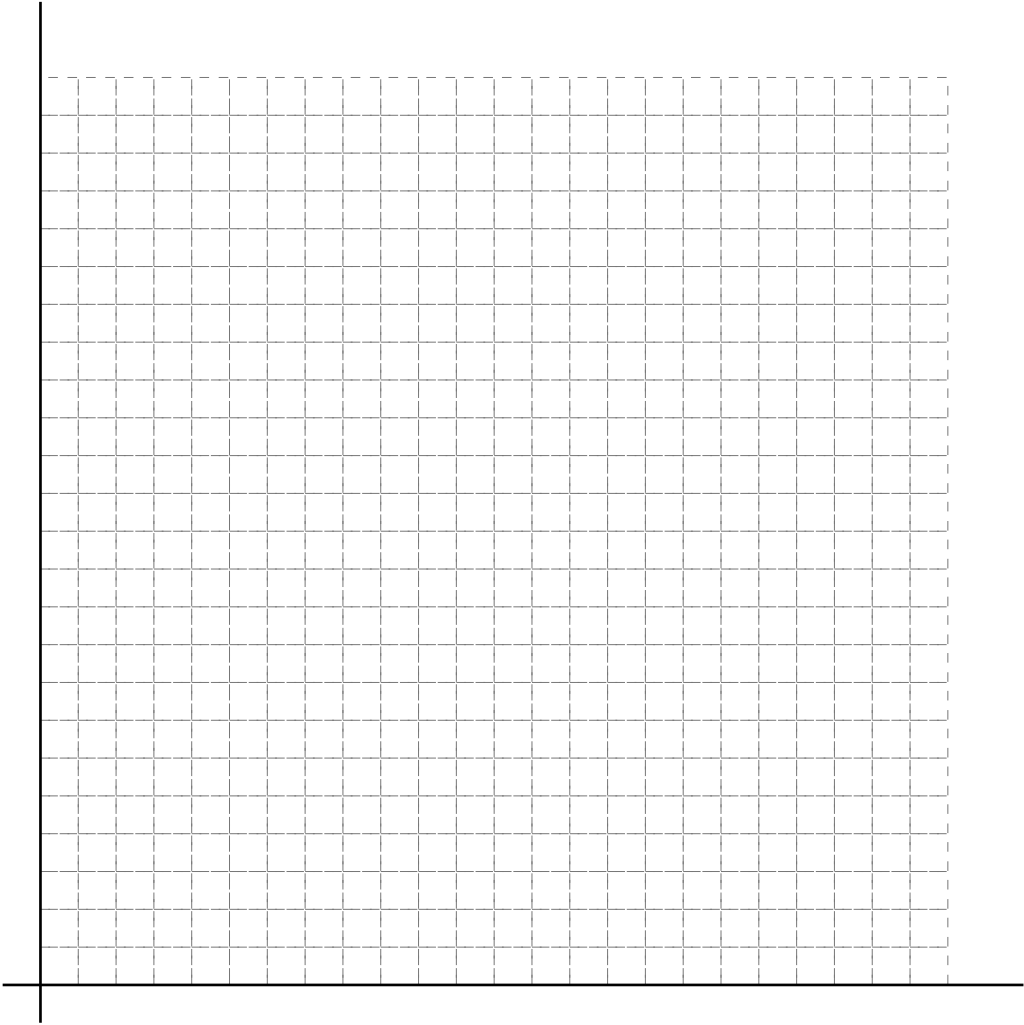
This question is about plotting a graph of inverse proportion for the equation

$$y = \frac{12}{x}$$

Complete the following table,

x	0.5	1	2	3	4	6	8	12	24
y						2			

Plot the graph of the equation.



Inverse proportion is sometimes described as "more is less".
Explain this.

2.2 Answers

Answer 1

- (i) Tick ☐ y is directly proportional to square root x
- (ii) 10
- (iii) $y = 90$
- (iv) $y = 110$
- (v) INCREASE "More is More"

Answer 2

- (i) Tick ☐ y is inversely proportional to the square root of x .
- (ii) 16
- (iii) $y = 4$
- (iv) $y = 2$
- (v) DECREASE "More is Less"

Answer 3

- (i) Tick ☐ y is directly proportional to the square of x
- (ii) $\frac{2}{3}$
- (iii) $y = 6$
- (iv) $y = 24$
- (v) INCREASE "More is More"

Answer 4

- (i) Tick ☐ y is inversely proportional to the square of x .
- (ii) 12
- (iii) $y = 12$
- (iv) $y = 3$
- (v) DECREASE "More is Less"

Answer 5

- (i) Tick ☐ y is directly proportional to x
- (ii) 9
- (iii) $y = 45$
- (iv) $y = 126$
- (v) $x = 5$

Answer 6

- (i) Tick ☐ y is inversely proportional to x .
- (ii) 24
- (iii) $y = 3$
- (iv) $y = 0.5$
- (v) $x = 4$

Answer 7

$$y = 7\sqrt{x}$$

Answer 8

$$y = \frac{18}{x^2}$$

Answer 9

- (i) Tick ☐ y is directly proportional to square of m
 ☐ y is directly proportional to x
 ☐ y is inversely proportional to the square root of z
- (ii) 4
- (iii) $y = 70$

Answer 10

- (i) Tick ☐ y is directly proportional to square root of c
 ☐ y is directly proportional to w
 ☐ y is inversely proportional to the square of z
- (ii) 10
- (iii) $y = 120$

Answer 11

$$y = \frac{7v^2\sqrt{a}}{n}$$

Answer 12

$$y = 5x^3$$

$$y = 135$$

Question 1

Given that,

$$y = \frac{3}{4} \sqrt{x}$$

- (i) Tick which applies
- ☐ y is directly proportional to square root x
- ☐ y is inversely proportional to square root x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 64$, what is y ?
- (iv) If the value of x was increased, would the value of y increase or decrease ?

Question 2

Given that,

$$y = \frac{27}{2\sqrt{x}}$$

- (i)** Tick which applies ☐ y is directly proportional to square root x
☐ y is inversely proportional to square root x
- (ii)** What is the constant of the proportionality ?
- (iii)** When $x = 9$, what is y ?
- (iv)** If the value of x was increased, would the value of y increase or decrease ?

Question 3

Write down the equation described by,

y is directly proportional to the square of x

The constant of the proportionality is $\frac{4}{5}$

Question 4

Write down the equation described by,

y is inversely proportional to the square root of x

The constant of the proportionality is $\frac{2}{3}$

Question 5

$$y = \frac{1}{2} a b \sin C$$

- (i) Tick which applies ☐ y is directly proportional to a
 ☐ y is inversely proportional to a
 ☐ y is directly proportional to b
 ☐ y is inversely proportional to b
 ☐ y is directly proportional to the sine of c
 ☐ y is inversely proportional to the sine of c
- (ii) What is the constant of the proportionality ?
- (iii) When $a = 36$, $b = 25$, and $C = 30^\circ$ what is y ?

Question 6

$$A = \pi r^2$$

- (i) Tick which applies ☐ A is directly proportional to the square of r
 ☐ A is inversely proportional to the square of r
- (ii) What is the constant of the proportionality ?
- (iii) When $r = 5$, what is A ?
 Give your answer to 3 significant figures.
- (iv) When $r = 1.4$, what is A ?
 Give your answer to 3 significant figures.
- (v) When $A = 100$, what is r ?

Question 7

$$T = \frac{D}{S}$$

- (i) Tick which applies ☐ T is directly proportional to D
 ☐ T is inversely proportional to D
 ☐ T is directly proportional to S
 ☐ T is inversely proportional to S
- (ii) What is the constant of the proportionality ?
- (iii) When $D = 5.6$ and $S = 55.9$ what is T ?
- (iv) When $T = 28.3$ and $D = 783$ what is S ?

Question 8

A large hole is to be dug in the ground to make a fish pond.

It is thought that eight men would take 6 days to dig the hole.

- (i) How long would ten men take to dig the hole ?
- (ii) Is this an example of direct proportion or inverse proportion ?

2.4 Answers (To 2.3 Homework)

Answer 1

- (i) Tick ☐ y is directly proportional to square root x
(ii) $\frac{3}{4}$
(iii) $y = 6$ (iv) Increase

Answer 2

- (i) Tick ☐ y is inversely proportional to square root x
(ii) $\frac{27}{2}$ or 13.5
(iii) $y = 4.5$ (iv) Decrease

Answer 3

$$y = \frac{4x^2}{5}$$

Answer 4

$$y = \frac{2}{3\sqrt{x}}$$

Answer 5

- (i) Tick ☐ y is directly proportional to a
 Tick ☐ y is directly proportional to b
 Tick ☐ y is directly proportional to $\sin C$
(ii) $\frac{1}{2}$
(iii) $y = 225$

Answer 6

- (i) Tick ☐ y is directly proportional to square of r
(ii) π
(iii) $A = 78.5$
(iv) $A = 6.16$
(v) $r = 5.64$

Answer 7

- (i) Tick ☐ T is directly proportional to D
 Tick ☐ T is inversely proportional to S
(ii) 1
(iii) $T = 0.100$
(iv) $S = 27.7$

Answer 8

- (i) 4.8 days (4 days 19 hours 12 minutes)
(ii) Inverse

Lesson 3

Proportionality : GCSE

3.1 The Constant of a Proportionality

3.2 Exercise

Question 1

$$y = 8\sqrt{x}$$

- (i) Tick which applies
- ☐ y is directly proportional to the square root of x
☐ y is inversely proportional to the square root of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 25$, what is y ?
- (iv) When $x = 400$, what is y ?
- (v) If the value of x was increased, would the value of y increase or decrease ?

Question 2

$$y = \frac{24}{\sqrt{x}}$$

- (i) Tick which applies ☐ y is directly proportional to the square root of x
 ☐ y is inversely proportional to the square root of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 64$, what is y ?
- (iv) When $x = 100$, what is y ?
- (v) If the value of x was increased, would the value of y increase or decrease ?

Question 3

- (i) Write down the equation described by,
y is directly proportional to the square root of x
The constant of the proportionality is 15

(ii) When $x = 64$, what is y ?

(iii) When $x = 144$, what is y ?

Question 4

- (i) Write down the equation described by,
y is inversely proportional to the square root of x
The constant of the proportionality is 21

(ii) When $x = 9$, what is y ?

(iii) When $x = 49$, what is y ?

Question 5

- (i) Write down the equation described by,
y is directly proportional to x
The constant of the proportionality is k

(ii) If $x = 9$ when y is 18, what is the value of k ?

(iii) Rewrite your part (i) answer with k replaced with its constant fixed value.

(iv) When x is 5, what is y ?

Question 6

- (i) Write down the equation described by,
y is directly proportional to the square root of x
The constant of the proportionality is k
- (ii) If $x = 9$ when y is 30, what is the value of k ?
- (iii) Rewrite your part (i) answer with k replaced with its constant fixed value.
- (iv) When x is 25, what is y ?

Question 7

- (i) Write down the equation described by,
y is directly proportional to the square of x
The constant of the proportionality is k
- (ii) If $x = 4$ when y is 64, what is the value of k ?
- (iii) Rewrite your part (i) answer with k replaced with its constant fixed value.
- (iv) When x is 10, what is y ?

Question 8

The formula for the area of a circle is $A = \pi r^2$.

- (i) Tick which applies ☐ A is directly proportional to the square of r
☐ A is inversely proportional to the square of r
- (ii) What is the constant of the proportionality ?

Question 9

- (i) Write down the equation described by,

y is inversely proportional to x

The constant of the proportionality is k

- (ii) If $x = 3$ when y is 5, what is the value of k ?
- (iii) Rewrite your part (i) answer with k replaced with its constant fixed value.
- (iv) When x is 15, what is y ?

Question 10

- (i) Write down the equation described by,

y is inversely proportional to the square root of x

The constant of the proportionality is k

- (ii) If $x = 9$ when y is 12, what is the value of k ?
- (iii) Rewrite your part (i) answer with k replaced with its constant fixed value.
- (iv) When x is 16, what is y ?

Question 11

Einstein's famous formula is $E = m c^2$.

- (i) Tick which applies ☐ E is directly proportional to m
☐ E is inversely proportional to m
- (ii) The constant of the proportionality is c^2 .
Out of interest, do you know what the fixed number, c^2 , is ?

Question 12

- (i) Write down the equation described by,

y is inversely proportional to the square of x

The constant of the proportionality is k

- (ii) If $x = 3$ when y is 2, what is the value of k ?

- (iii) Rewrite your part (i) answer with k replaced with its constant fixed value.

- (iv) When x is 1, what is y ?

Question 13

- (i) Write down the equation described by,

C is directly proportional to r

The constant of the proportionality is 2π

- (ii) You should recognise this formula.
What does this formula do ?

Question 14

- (i) Write down the equation described by,

A is directly proportional to b

A is directly proportional to h

The constant of the proportionality is $\frac{1}{2}$

- (ii) You should recognise this formula.
What does this formula do ?

Question 15

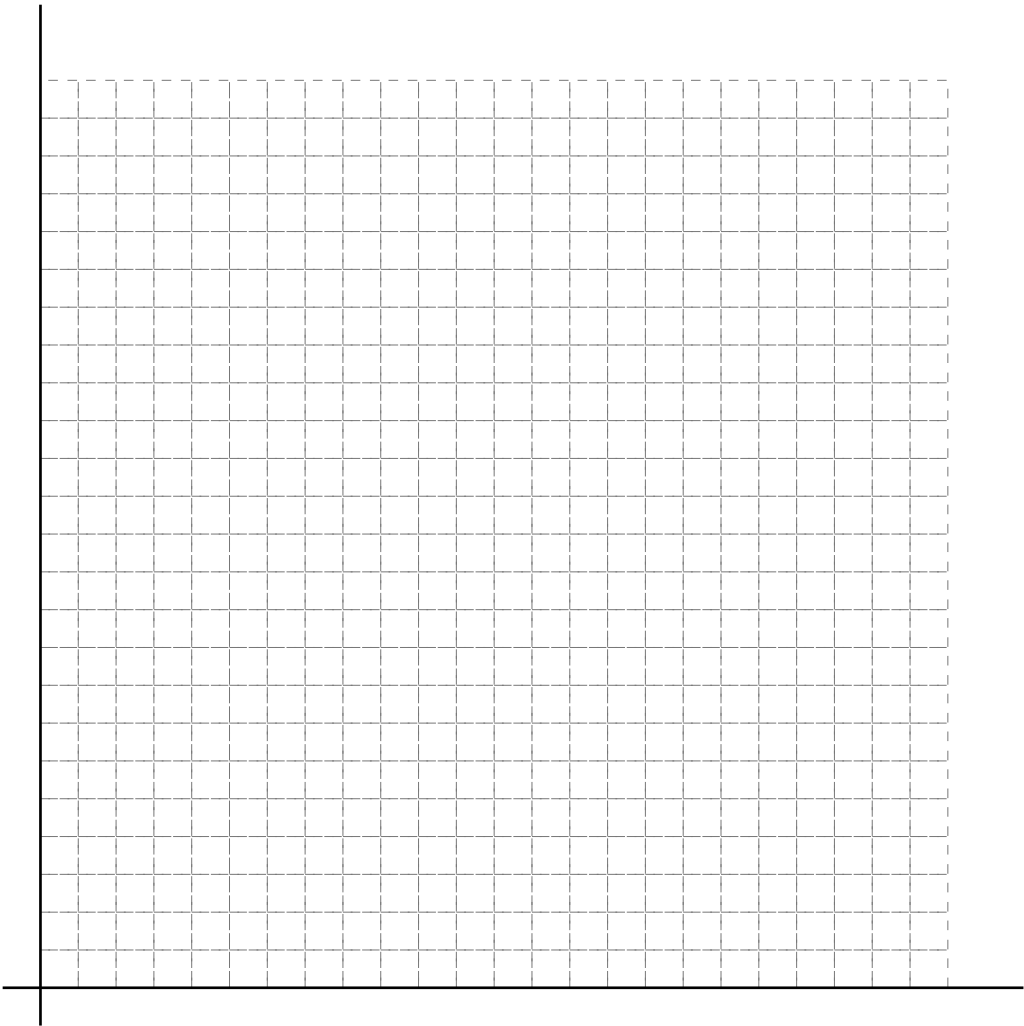
This question is about plotting a graph of inverse proportion for the equation

$$y = \frac{24}{\sqrt{x}}$$

Complete the following table,

x	1	4	9	16
y				

Plot the graph of the equation.



3.3 Answers

3.3.1 Solutions (3.2 Exercise)

Answer 1

- (i) Tick $\square$ y is directly proportional to square root of x
- (ii) 8
- (iii) $y = 40$
- (iv) $y = 160$
- (v) INCREASE "More is More"

Answer 2

- (i) Tick $\square$ y is inversely proportional to the square root of x .
- (ii) 24
- (iii) $y = 3$
- (iv) $y = 2.4$
- (v) DECREASE "More is Less"

Answer 3

- (i)
$$y = 15\sqrt{x}$$
- (ii) $y = 120$
- (iii) $y = 180$

Answer 4

- (i)
$$y = \frac{21}{\sqrt{x}}$$
- (ii) $y = 7$
- (iii) $y = 3$

Answer 5

- (i) $y = kx$
- (ii) $k = 2$
- (iii) $y = 2x$
- (iv) $y = 10$

Answer 6

- (i) $y = k\sqrt{x}$
- (ii) $k = 10$
- (iii) $y = 10\sqrt{x}$
- (iv) $y = 50$

Answer 7

- (i) $y = kx^2$
- (ii) $k = 4$
- (iii) $y = 4x^2$
- (iv) $y = 400$

Answer 8

- (i) Tick *directly*
- (ii) π

Answer 9**(i)**

$$y = \frac{k}{x}$$

(ii) $k = 15$ **(iii)**

$$y = \frac{15}{x}$$

(iv) $y = 1$ **Answer 10****(i)**

$$y = \frac{k}{\sqrt{x}}$$

(ii) $k = 36$ **(iii)**

$$y = \frac{36}{\sqrt{x}}$$

(iv) $y = 9$ **Answer 11****(i)** Tick *directly***(ii)** $c = 3 \times 10^8$ m/s c is the speed of light**Answer 12****(i)**

$$y = \frac{k}{x^2}$$

(ii) $k = 18$ **(iii)**

$$y = \frac{18}{x^2}$$

(iv) $y = 18$ **Answer 13****(i)** $C = 2 \pi r$ **(ii)** Formula for circumference of a circle**Answer 14****(i)****(ii)** Formula for area of a triangle

$$A = \frac{1}{2} b h$$

Lesson 4

Proportionality : GCSE

4.1 The proportionality symbol

Proportion questions often get asked in an abbreviated form.

Part of the abbreviation involves the symbol $\propto$ which means *proportional to*.

Example 1

If $C \propto \frac{1}{R}$ and $C = 12$ when $R = 4$ write a formula connecting C and R .

Translation

(i) Write down the equation described by,

C is inversely proportional to R

The constant of the proportionality is k

(ii) Use $C = 12$ when R is 4, to work out the value of k .

(iii) Rewrite your part (i) answer with k replaced with its constant fixed value.

So, to answer this example, answer the translation !

Example 2

If $T \propto AX$ and $T = 30$ when $A = 2$ and $X = 3$ write a formula connecting T , A and X .

4.2 Exercise

Question 1

If $Q \propto t$ and $Q = 20$ when $t = 4$ write a formula connecting Q and t .

Question 2

If $y \propto \frac{1}{x}$ and $y = 6$ when $x = 2$ write a formula connecting y and x .

Question 3

If $w \propto z^2$ and $w = 45$ when $z = 3$ write a formula connecting w and z .

Question 4

If $F \propto \sqrt{G}$ and $F = 18$ when $G = 81$ write a formula connecting F and G .

Question 5

If $R \propto \frac{1}{P^2}$ and $R = 4$ when $P = 4$ write a formula connecting R and P .

Question 6

If $L \propto \frac{1}{\sqrt{M}}$ and $L = 12$ when $M = 25$ write a formula connecting L and M .

Question 7

If $C \propto D^3$ and $C = 81$ when $D = 3$ write a formula connecting C and D .

Question 8

*This question tries to confuse by using the letter K as one of the variables. *CARE**

If $H \propto KC$ and $H = 75$ when $K = 3$ and $C = 5$ write a formula connecting H , K and C .

Question 9

If $I \propto \frac{J}{Q}$ and $I = 18$ when $J = 24$ and $Q = 8$ write a formula connecting I , J and Q .

Question 10

$T \propto \frac{1}{WA}$ and $T = 60$ when $W = 4$ and $A = 3$. Write a formula connecting T , W and A .

Question 11

$E \propto \sqrt{FG}$ and $E = 18$ when $F = 3$ and $G = 12$. Write a formula relating E , F and G .

Question 12

$s \propto \frac{t^2}{c}$ and $s = 10$ when $t = 8$ and $c = 32$. Write a formula connecting s , t and c .

Question 13

$X \propto \frac{R\sqrt{A}}{Y}$ and $X = 200$ when $R = 8$, $A = 25$ and $Y = 4$.

Write a formula connecting X , R , A and Y .

Question 14

T is inversely proportional to m .

$T = 6$ when $m = 2$.

Write a formula of the form $T = \frac{k}{m}$ in which is placed the appropriate k value.

Question 15

$$R \propto \frac{1}{Q}$$

If $R = 32$ when $Q = 4$ determine, k , the constant of the proportionality.

Hence write out a formula in terms of R , Q , and the numerical value of k .

Question 16

W is inversely proportional to x .

$W = 5$ when $x = 12$.

Write a formula of the form $W = \frac{k}{x}$ in which is placed the appropriate k value.

Use your formula to fill in the empty entries in the following table;

W	5			10	
x	12	24	1		3

Question 17

While doing underwater tests in one part of an ocean, a team of scientists noticed that the temperature, t , in $^{\circ}\text{C}$ was inversely proportional to the depth, d , in kilometres.

When the temperature was 3°C the depth was 1.5 km.

Plot a graph showing how temperature varies with depth.

* Put depth on the x -axis and have $0 \leq d \leq 10$

* Put temperature on the y -axis and have $0 \leq t \leq 5$

4.3 Answers

4.3.1 Solutions (4.1 The proportionality symbol)

Answer Example 1

$$C = \frac{48}{R}$$

Answer Example 2

$$T = 5AX$$

4.3.2 Solutions (4.2 Exercise)

Answer 1

$$Q = 5t$$

Answer 2

$$y = \frac{12}{x}$$

Answer 3

$$w = 5z^2$$

Answer 4

$$F = 2\sqrt{G}$$

Answer 5

$$R = \frac{64}{P^2}$$

Answer 6

$$L = \frac{60}{\sqrt{M}}$$

Answer 7

$$C = 3D^3$$

Answer 8

$$H = 5KC$$

Answer 9

$$I = \frac{6J}{Q}$$

Answer 10

$$T = \frac{720}{WA}$$

Answer 11

$$E = 3\sqrt{FG}$$

Answer 12

$$s = \frac{5t^2}{c}$$

Answer 13

$$X = \frac{20R\sqrt{A}}{Y}$$

Answer 14

$$T = \frac{12}{m}$$

Answer 15

$$R = \frac{128}{Q}$$

Answer 16

$$W = \frac{60}{x}$$

W	5	2.5	60	10	20
x	12	24	1	6	3

Answer 17

$$T = \frac{4.5}{D}$$

Lesson 5

Proportionality : GCSE

5.1 Practice

Example 1 (Answer over the page)

- (i) Express " B is **directly** proportional to the square of V " as a proportionality.

- (ii) If $B = 36$ when $V = 3$, find the constant of the proportionality.

- (iii) Write down a formula relating B and V .

- (iv) Use your formula to determine the value of B when $V = 5$.

Example 2 (Answer over the page)

- (i) Express " d is **inversely** proportional to the square root of m " as a proportionality.

- (ii) If $d = 20$ when $m = 16$ find the constant of the proportionality.

- (iii) Write down a formula relating d and m .

- (iv) Use your formula to determine the value of d when $m = 100$.

Now check your answers with those over the page...

Answer for Example 1

(i) $B \propto V^2$

(ii) $k = 4$

(iii) $B = 4 V^2$

(iv) $B = 100$

Answer for Example 2

(i) $d \propto \frac{1}{\sqrt{m}}$

(ii) $k = 80$

(iii) $d = \frac{80}{\sqrt{m}}$

(iv) $d = 8$

How did you do ?

- ☐ I done real good. A* here I come.
- ☐ Messed up but I understand it now.
- ☐ Is there any chance of moving down a set or two ?

5.2 Exercise

Question 1

(i) Express "T is **directly** proportional to the square of U" as a proportionality.

(ii) If $T = 28$ when $U = 2$ find the constant of the proportionality.

(iii) Write down a formula relating T and U .

(iv) Use your formula to determine the value of T when $U = 3$.

(v) Use your formula to determine the value of T when $U = 5$.

Question 2

- (i) Express "Z is **directly** proportional to Y" as a proportionality.
- (ii) If $Z = 210$ when $Y = 3$ find the constant of the proportionality.
- (iii) Write down a formula relating Z and Y.
- (iv) Use your formula to determine the value of Z when:
 - (a) $Y = 2$
 - (b) $Y = 8$
 - (c) $Y = 20$

Question 3

- (i) Express "A is **inversely** proportional to the square root of Q" as a proportionality.
- (ii) If $A = 100$ when $Q = 36$ find the constant of the proportionality.
- (iii) Write down a formula relating A and Q.
- (iv) Use your formula to determine the value of A when $Q = 100$.
- (v) Use your formula to determine the value of A when $Q = 400$.

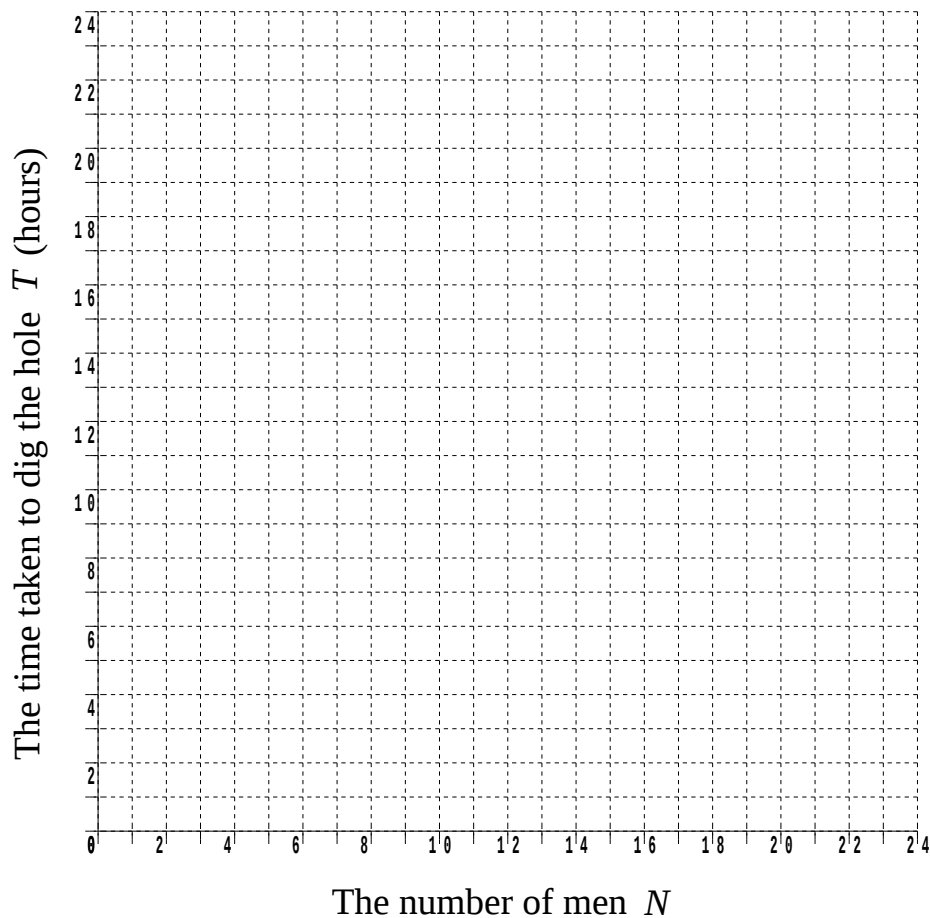
Question 4

- (i) Express " T is **inversely** proportional to N " as a proportionality.
- (ii) It is estimated that it will take 4 hours for 6 men to dig a hole.
By letting $T = 4$ when $N = 6$ find the constant of the proportionality.
- (iii) Write down a formula relating T and N .

- (iv) Use your formula to complete the following table:

N	1	2	3	4	6	8	12	24
T								

- (v) Use your table to plot a graph of inverse proportion for "men digging hole"



Question 5

(i) Express " M is **inversely** proportional to the square of S " as a proportionality.

(ii) If $M = 4$ when $S = 5$, find the constant of proportionality.

(iii) Write down a formula relating M and S .

(iv) Use your formula to find the value of M when:

(a) $S = 2$

(b) $S = 10$

(c) $S = 0.1$

Question 6

$$P \propto \frac{Q}{R}$$

(i) If $P = 8$ when $Q = 2$ and $R = 5$, find the constant of proportionality.

(ii) Write down a formula relating P , Q and R .

(iii) Use your formula to find the value of P when:

(a) $Q = 3$ and $R = 15$

(b) $Q = 7$ and $R = 200$.

Question 7

$$M \propto \frac{\sqrt{F}}{I}$$

- (i) If $M = 400$ when $F = 16$ and $I = 0.1$, find the constant of proportionality.
- (ii) Write down a formula relating M , F and I .
- (iii) Use your formula to find the value of M when:
 - (a) $F = 121$ and $I = 2$
 - (b) $F = 0.04$ and $I = 4$.

Question 8

$$Q \propto M H^2$$

- (i) If $Q = 0.25$ when $M = 40$ and $H = 0.1$, find the constant of proportionality.
- (ii) Write down a formula relating Q , M and H .
- (iii) Use your formula to find the value of Q when:
 - (a) $M = 100$ and $H = 8$
 - (b) $M = 4$ and $H = 15$.

Answer 1

(i) $T \propto U^2$ (ii) 7 (iii) $T = 7U^2$ (iv) 63 (v) 175

Answer 2

[illegible]

Answer 3

(i)	(ii)	(iii)	(iv)	(v)
$A \propto \frac{1}{\sqrt{Q}}$	600	$A = \frac{600}{\sqrt{Q}}$	60	30

Answer 4

Answer 4

(i)

$$T \propto \frac{1}{N}$$

(ii)

$$24$$

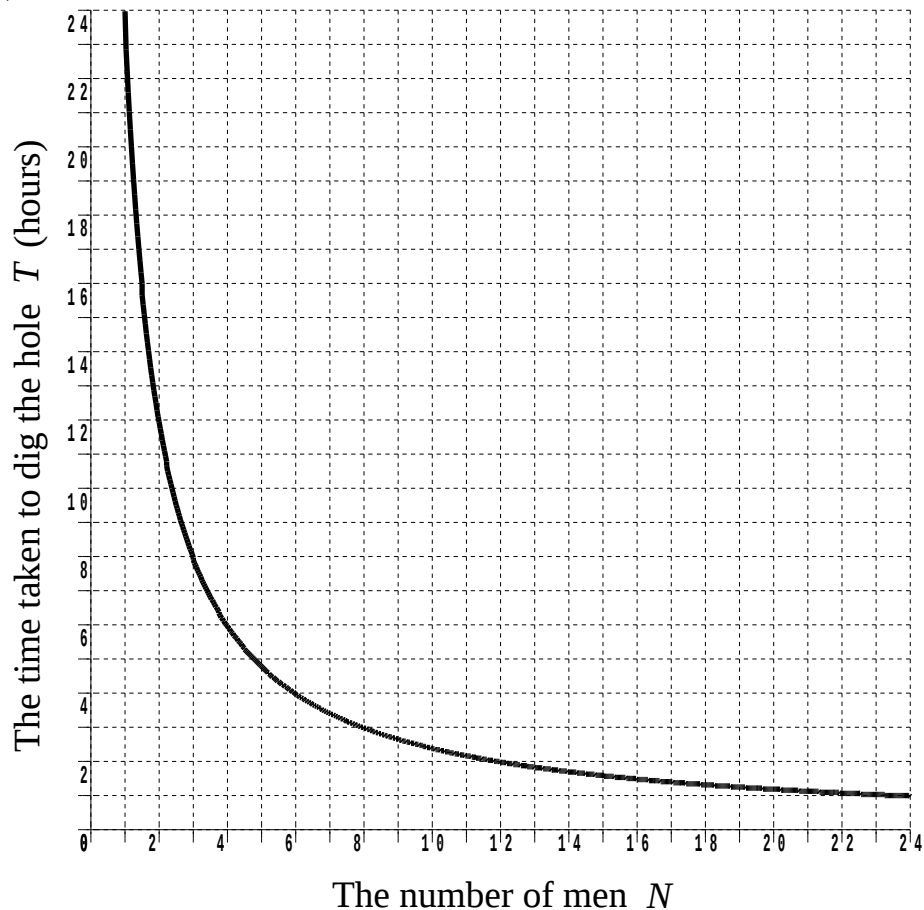
(iii)

$$T = \frac{24}{N}$$

(iv)

N	1	2	3	4	6	8	12	24
T	24	12	8	6	4	3	2	1

(v)



Answer 5

(i)	(ii)	(iii)
$M \propto \frac{1}{S^2}$	100	$M = \frac{100}{S^2}$
(iv)	(a)	25
	(b)	1
	(c)	1000

Answer 6

$$P = \frac{kQ}{R}$$

(i)	20
(ii)	
	$P = \frac{20Q}{R}$
(iii)	(a) 4
	(b) 0.7

Answer 7

$$M = \frac{k\sqrt{F}}{I}$$

(i)	10
(ii)	
	$M = \frac{10\sqrt{F}}{I}$
(iii)	(a) 55
	(b) 0.5

Answer 8

$$Q = kMH^2$$

(i)	
	$k = \frac{5}{8} \text{ or } 0.625$
(ii)	
	$Q = \frac{5MH^2}{8} \text{ or } Q = 0.625MH^2$
(iii)	(a) 4000
	(b) 562.5

Lesson 6

Proportionality : GCSE

6.1 Exam Questions

Question 1

Paper 3H, 7th June 2010

The light intensity, E , at a surface is inversely proportional to the square of the distance, r , of the surface from the light source.

$E = 4$ when $r = 50$

(a) Express E in terms of r .

[3 marks]

(b) Calculate the value of E when $r = 20$

[1 mark]

(c) Calculate the value of r when $E = 1600$

[2 marks]

Question 2

Paper 3H, 5th November 2009

t is proportional to the square root of d .

$t = 12$ when $d = 4$

(a) Find a formula for t in terms of d .

[3 marks]

(b) Calculate the value of t when $d = 9$

[2 marks]

Question 3

Specimen Paper, 2003

The distance, d km, it is possible to see on a clear day is proportional to the square root of the height, h m, above sea level.

Standing on a pier, 4 m above sea level, it is possible to see a distance of 10 km.

(a) Find a formula for d in terms of h .

[2 marks]

(b) Standing on top of the cliffs I can see a distance of 35 km.
How high are the cliffs ?

[2 marks]

Question 4

Paper 4H, 9th May 2006

A ball is dropped from a tower.

After t seconds, the ball has fallen a distance x metres.

x is directly proportional to t^2 .

When $t = 2$, $x = 19.6$

(a) Find an equation connecting x and t .

[3 marks]

(b) Find the value of x when $t = 3$

[2 marks]

(c) Find how long the ball takes to fall 10 m

[3 marks]

Question 5

Paper 3H, 15th May 2008

The distance, d kilometres, of the horizon from a person is directly proportional to the square root of the person's height, h metres, above sea level.

When $h = 225$, $d = 54$

(a) Find a formula for d in terms of h .

[3 marks]

(b) Calculate the distance of the horizon from a person whose height above sea level is 64 metres.

[1 mark]

(c) Calculate the height above sea level of a person, when the distance of the horizon is 61.2 kilometres.

[2 marks]

Question 6

Paper 4H, 16th November 2010

P is inversely proportional to V .

$P = 18$ when $V = 24$

(a) Express P on terms of V .

[3 marks]

(b) Find the positive value of V when $P = 3V$.

[2 marks]

Question 7

Paper 3H, 17th May 2007

The frequency, f kilohertz, of a radio wave is inversely proportional to its wavelength w metres.

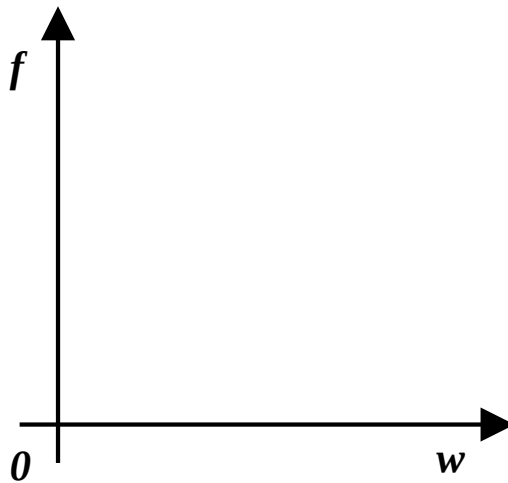
When $w = 200$, $f = 1500$

(a) (i) Express f in terms of w .

[3 marks]

(ii) On the axes, sketch the graph of f against w .

[1 mark]



(b) The wavelength of a radio wave is 1250 m.
Calculate its frequency.

[2 marks]

Question 8

Paper 3H, 5th November 2007

A wind turbine generates a power of P kilowatts when the wind speed is w m/s.

P is proportional to w^3 .

$P = 300$ when $w = 12$.

(a) Find a formula for P in terms of w .

[3 marks]

(b) Calculate the value of P when $w = 7.5$
Give your answer correct to 3 significant figures.

[2 marks]

(c) When the wind speed is x m/s, the wind turbine generates twice as much power as it does when the wind speed is 10 m/s.
Calculate the value of x .
Give your answer correct to 3 significant figures.

[4 marks]

Question 9

Paper 3H, 11th January 2012

Two small magnets attract each other with a force, F newtons.

F is inversely proportional to the square of the distance, d cm, between them.

When $d = 2$, $F = 12$.

(a) Express F in terms of d .

[3 marks]

(b) Calculate the value of F when $d = 5$

[1 mark]

(c) Calculate the value of d when $F = 3$

[2 marks]

Question 10

Paper 4H, 9th June 2016, Q13

M is directly proportional to p^3

$M = 128$ when $p = 8$

(a) Find a formula for M in terms of p

[3 marks]

(b) Find the value of M when $p = 5$

[1 mark]

6.2 Solutions to 6.1 Exercise

Answer 1

(a)	(b)	(c)
$E = \frac{10\,000}{r^2}$	25	2.5

Answer 2

(a)	(b)
$t = 6\sqrt{d}$	18

Answer 3

(a)	(b)
$d = 5\sqrt{h}$	49

Answer 4

(a)	(b)	(c)
$x = 4.9t^2$	44.1 m	$\frac{10}{7}$ or 1.4 seconds

Answer 5

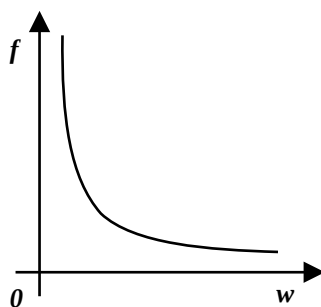
(a)	(b)	(c)
$d = 3.6\sqrt{h}$	28.8 km	289 m

Answer 6

(a)	(b)
$P = \frac{432}{V}$	12

Answer 7

(a)	(i)
	$f = \frac{300\,000}{w}$
	(ii)



(b)	240
-----	-----

Answer 8**(a)**

$$P = \frac{25 w^3}{144} \text{ or } P = 0.17361 w^3$$

(b) 73.2 kilowatts**(c)** 12.6 m/s**Answer 9****(a)**

$$F = \frac{48}{d^2}$$

(b) 1.92 newtons**(c)** 4 cm**Answer 10****(a)**

$$M = 0.25 p^3$$

(b) 31.25

Lesson 7

Proportionality : GCSE

7.1 Practical Proportionality

Calculator needed !



Example

The yacht pictured, a Vertue, is an example of a boat with a displacement hull. It moves through the water by pushing the water to either side. Notice that, because it's moving fairly fast, a large bow wave is being created.

A displacement hull has a speed beyond which it cannot pass through water. As the hull speed approaches the theoretical maximum the energy from the sails makes the bow wave bigger rather than move the hull faster.

A displacement hull's maximum speed through the water, V_{max} is directly proportional to the square root of its waterline length, LWL .

The Vertue has a maximum theoretical speed of 6.3 knots.

Its LWL - Length Water Line - is 22 feet.

(i) Write " V_{max} is **directly** proportional to the square root of LWL " in symbols.

(ii) If $V_{max} = 6.3$ when $LWL = 22$, find the constant of the proportionality.

(iii) Write down a formula relating V_{max} and LWL .

(iv) Use your formula to complete this table.

LWL	0	100	400	900	1200
V_{max}					

(v) On the graph paper on the following page,

(a) Plot the values from the table, above.

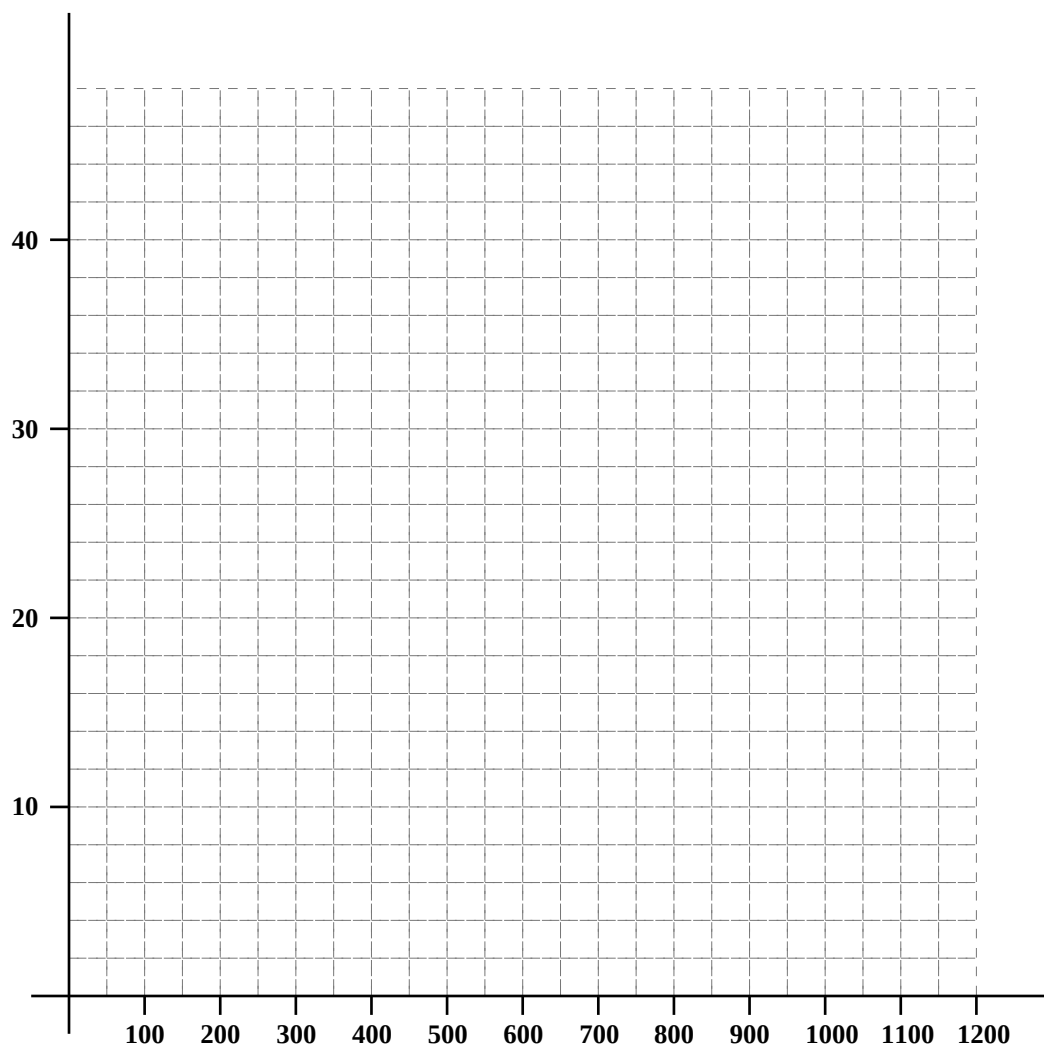
(b) Place the formula next to the curve plotted.

(c) Head your graph with the words
"A graph of Maximum Theoretical Speed against Waterline
Length for a Displacement Hull".

(d) Add the following words to the x-axis.
"Waterline Length in feet"

(e) Add the following words to the y-axis
"Speed in knots"

For homework you will look up the maximum speeds and waterline lengths of some famous ships to see how well real life matches what is theoretically possible.



7.2 Exercise

Question 1

The cost, C , of buying pencils is ***directly proportional*** to the number, N , of pencils bought.

(i) Express this statement as a proportionality.

(ii) If £4 buys 20 pencils, find the constant of the proportionality.

(iii) Write down a formula relating C and N .

(iv) Use your formula to complete this table.

N	0	10	40	90	120
C					

(v) On the graph paper on the following page,

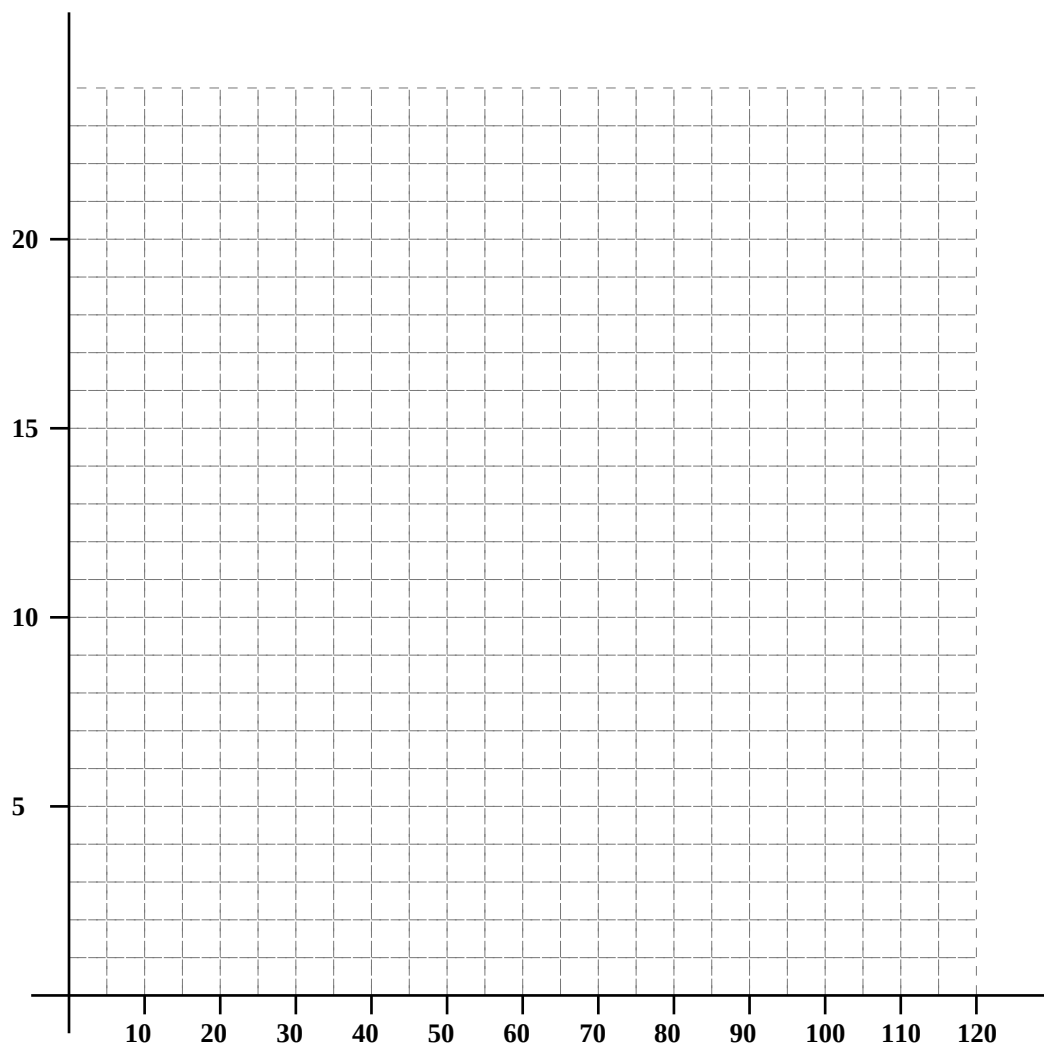
(a) Plot the values from the table, above.

(b) Place the formula next to the curve plotted.

(c) Head your graph with the words
"A graph of Cost against Number of Pencils".

(d) Add the following words to the x -axis.
"Number of Pencils"

(e) Add the following words to the y -axis
"Cost in £"



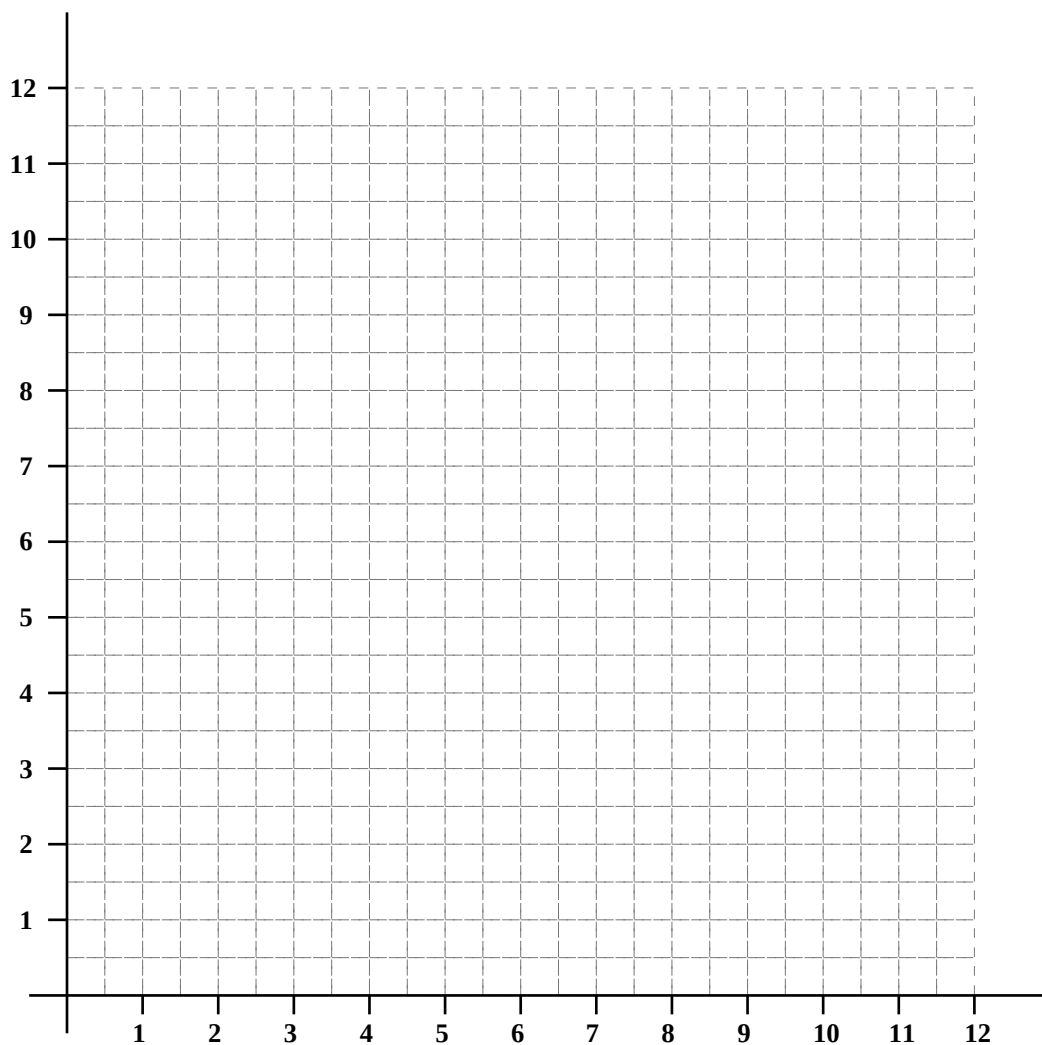
Question 2

The amount of paint, p , needed to paint a circle is *directly proportional to the square* of the circle's radius, r .

- (i) Express this statement as a proportionality.
- (ii) If 3 litres of paint is needed to paint a circle of radius 6 metres, find the constant of the proportionality.
- (iii) Write down a formula relating p and r .
- (iv) Use your formula to complete this table.

r	0	2	4	6	8	10	12
p							

- (v) On the graph paper on the following page,
- (a) Plot the values from the table, above.
- (b) Place the formula next to the curve plotted.
- (c) Head your graph with the words
"A graph of Amount of paint against Circle radius".
- (d) Add the following words to the x -axis.
"Circle radius in metres"
- (e) Add the following words to the y -axis
"Amount of paint in litres"



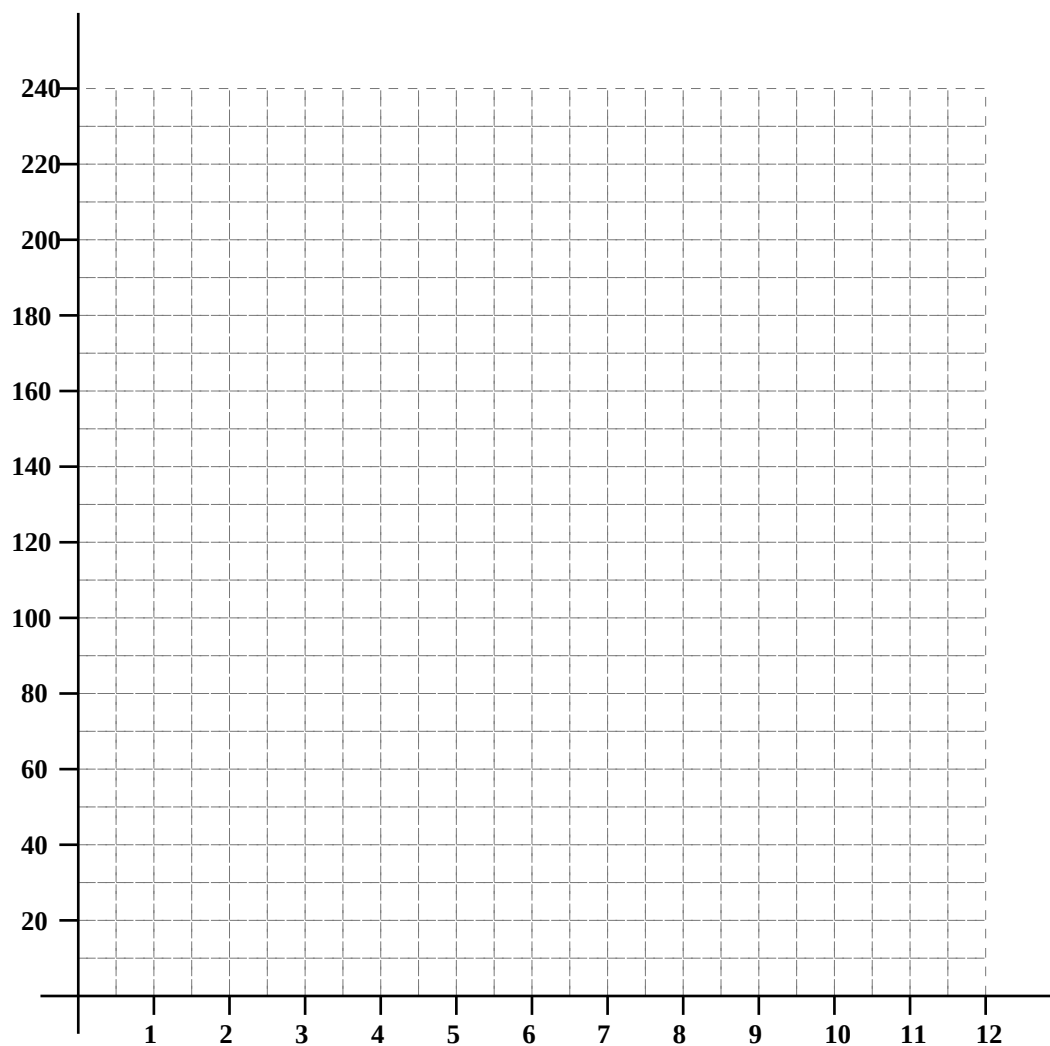
Question 3

The apparent power of a light bulb, P , is ***inversely proportional to the square of the distance*** of the viewer's eye from the light bulb, D .

- (i) Express this statement as a proportionality.
- (ii) If a light bulb appears to have a power of 60 watts from a distance of 4 metres find the constant of the proportionality.
- (iii) Write down a formula relating P and D .
- (iv) Use your formula to complete this table.

D	2	4	6	8	10	12
P						

- (v) On the graph paper on the following page,
- (a) Plot the values from the table, above.
- (b) Place the formula next to the curve plotted.
- (c) Head your graph with the words
"A graph of Distance from light bulb against Apparent power".
- (d) Add the following words to the x -axis.
"Distance from light bulb in metres"
- (e) Add the following words to the y -axis
"Apparent power in watts"



7.3 Homework : Theory Vs Practice

Provided is the document “Ship Specification for Chapter 7”
(Taken from the internet)

On this you will find;

- (i) The Queen Mary
In 1936 you could not fly to America.
The fastest way to get there was by an ocean passenger liner and these competed with each other to offer the fastest crossing of the Atlantic Ocean.
The prize for being fastest was 'the blue ribbon'.
- (ii) The USS Nimitz
I've been on this in 1978, when it visited Edinburgh.
It's one of the biggest and oldest American aircraft carriers in commission.
In a warship, speed is useful,
It's classified information as to how fast this ship can actually go but the rumour is 37 knots.
- (iii) MT Bornes
This Oil Tanker was for sale on eBay.
For just \$17 500 000, it could be yours.
Oil tankers are slow, but take miles and miles to stop or turn.
- (iv) The Bizmarck
An infamous World War II Nazi Germany warship.
There is a film about how the Royal Navy fought this ship in the Atlantic.
It sank HMS Hood with massive loss of life before itself being sunk.

YOUR JOB NOW...

Look through the specs, and note the speed and LWL for these ships.
On the introductory graph you plotted the curve;

$$Max\ Theoretical\ Speed = 1.34 \sqrt{LWL}$$

Plot with an $\times$ the four points, for the four ships given in the data sheets.

How well do these ships move compared with the theoretical maximum ?

Note : LOA is Length Over All which is not the same as LWL .

7.4 Answers

7.4.1 Solutions to the Introductory Example

(i)

$$V_{max} \propto \sqrt{LWL} \Leftrightarrow V_{max} = k \sqrt{LWL}$$

(ii) $k = 1.34$

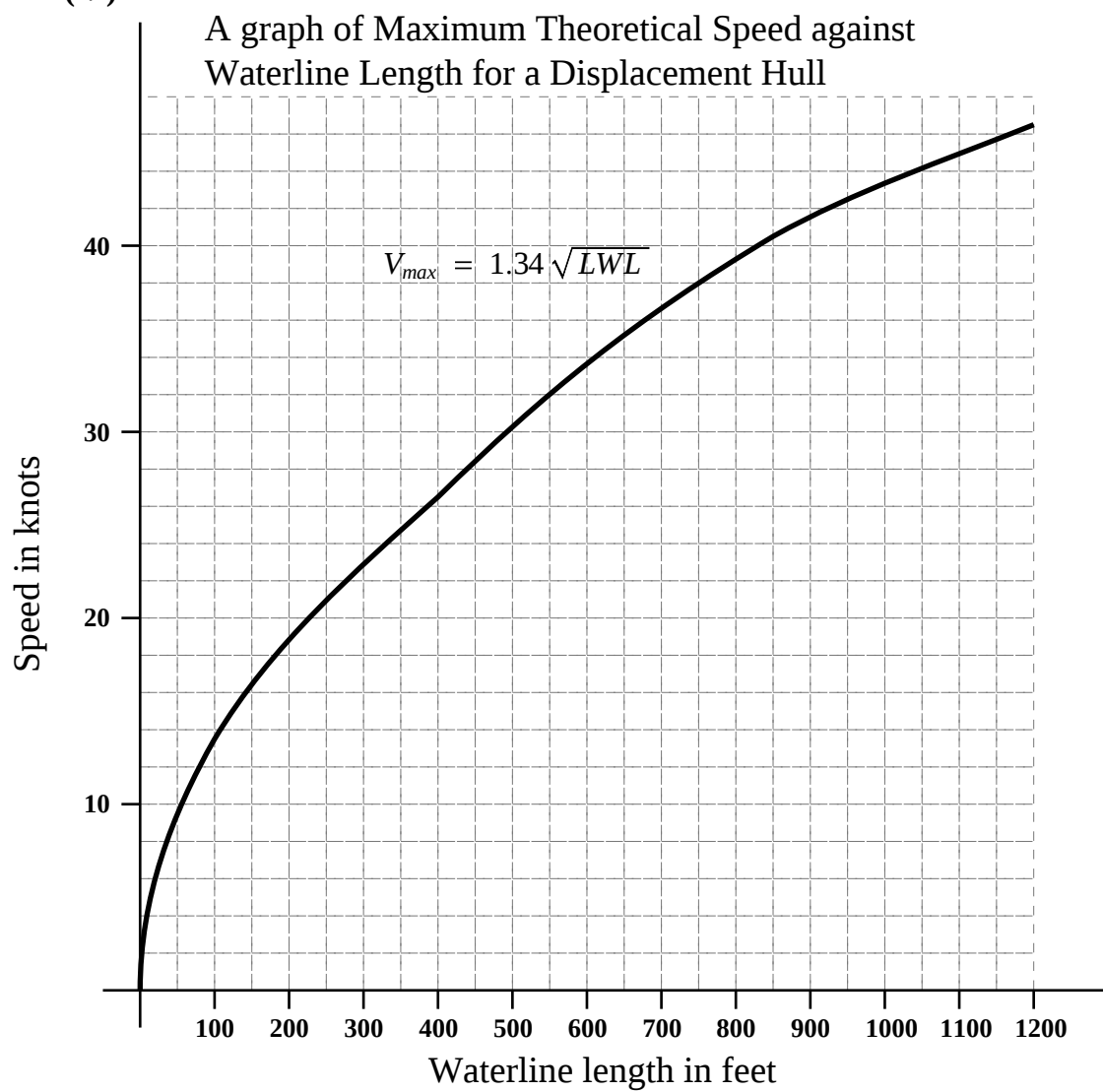
(iii)

$$V_{max} = 1.34 \sqrt{LWL}$$

(iv)

LWL	0	100	400	900	1200
V_{max}	0	13.4	26.8	40.2	46.4

(v)



7.4.2 Solutions to 7.2 Exercise

Answer 1

(i)

$$C \propto N \Leftrightarrow C = kN$$

(ii) $k = 0.2$

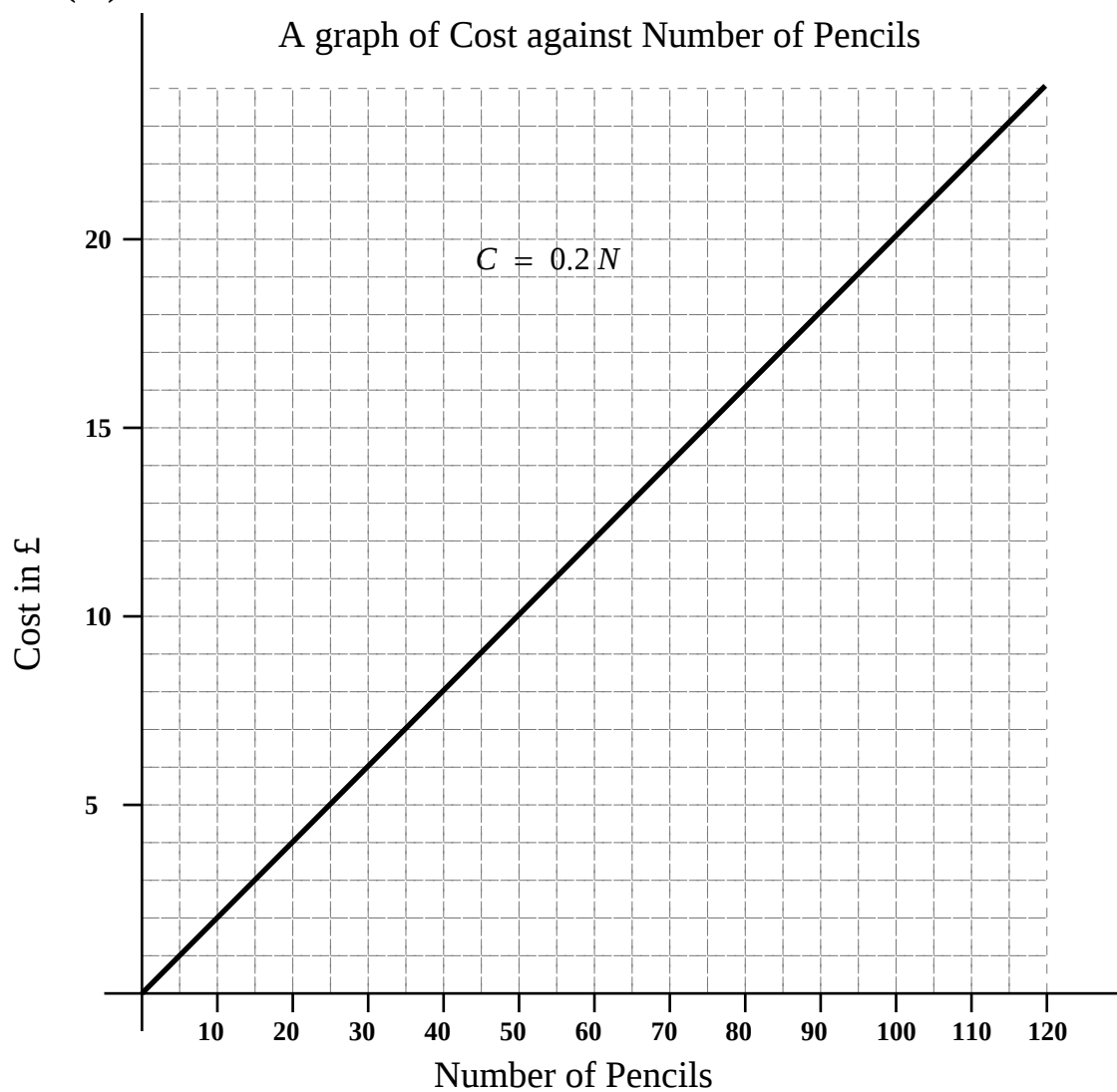
(iii)

$$C = 0.2N$$

(iv)

N	0	10	40	90	120
C	0	2	8	18	24

(v)



Answer 2**(i)**

$$p \propto r^2 \Leftrightarrow p = k r^2$$

(ii)

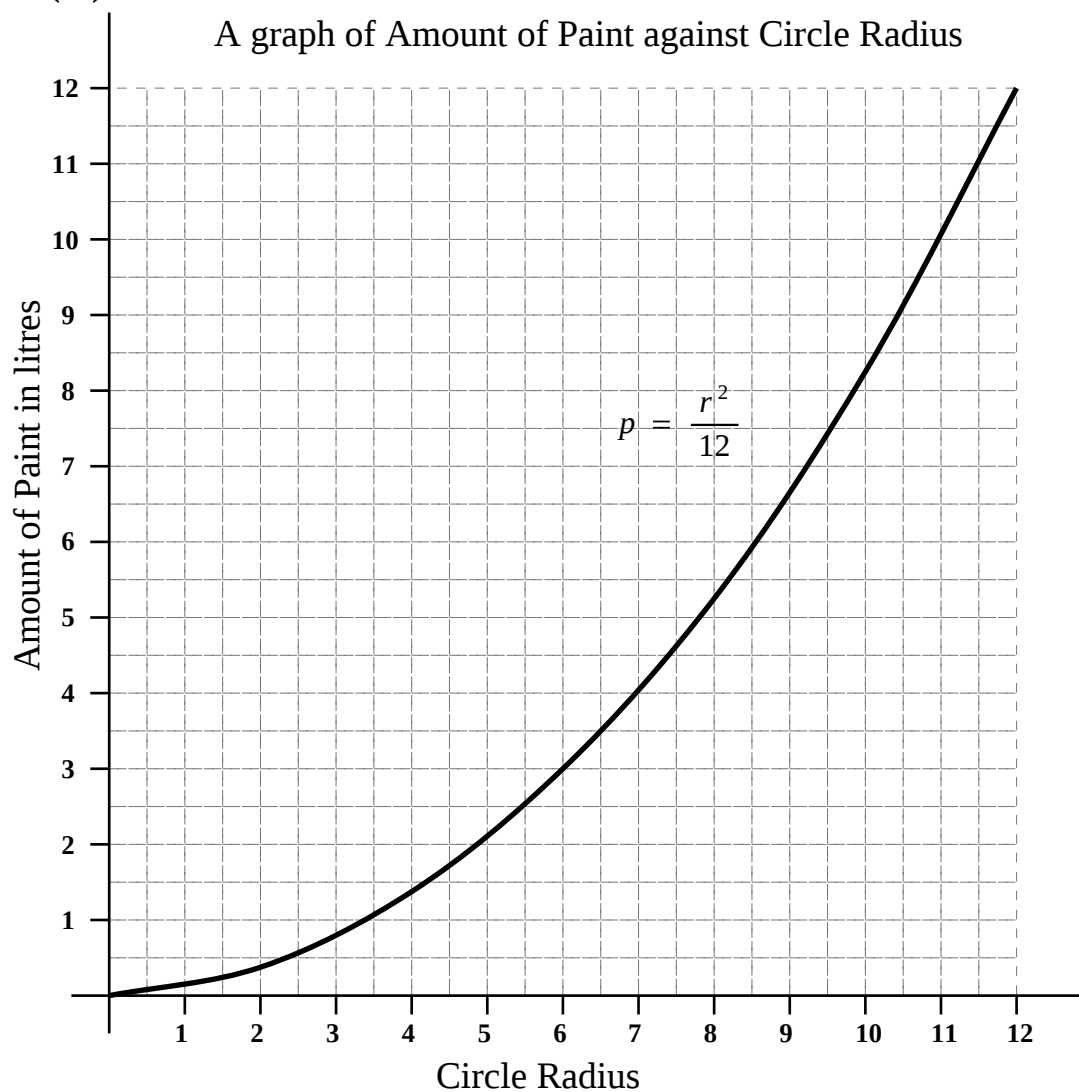
$$k = \frac{1}{12}$$

(iii)

$$p = \frac{r^2}{12} \quad \text{or} \quad p = 0.8333... r^2$$

(iv)

r	0	2	4	6	8	10	12
p	0	$\frac{1}{3}$	$1\frac{1}{3}$	3	$5\frac{1}{3}$	$8\frac{1}{3}$	12

(v)

Answer 3**(i)**

$$P \propto \frac{1}{D^2} \Leftrightarrow P = \frac{k}{D^2}$$

(ii)

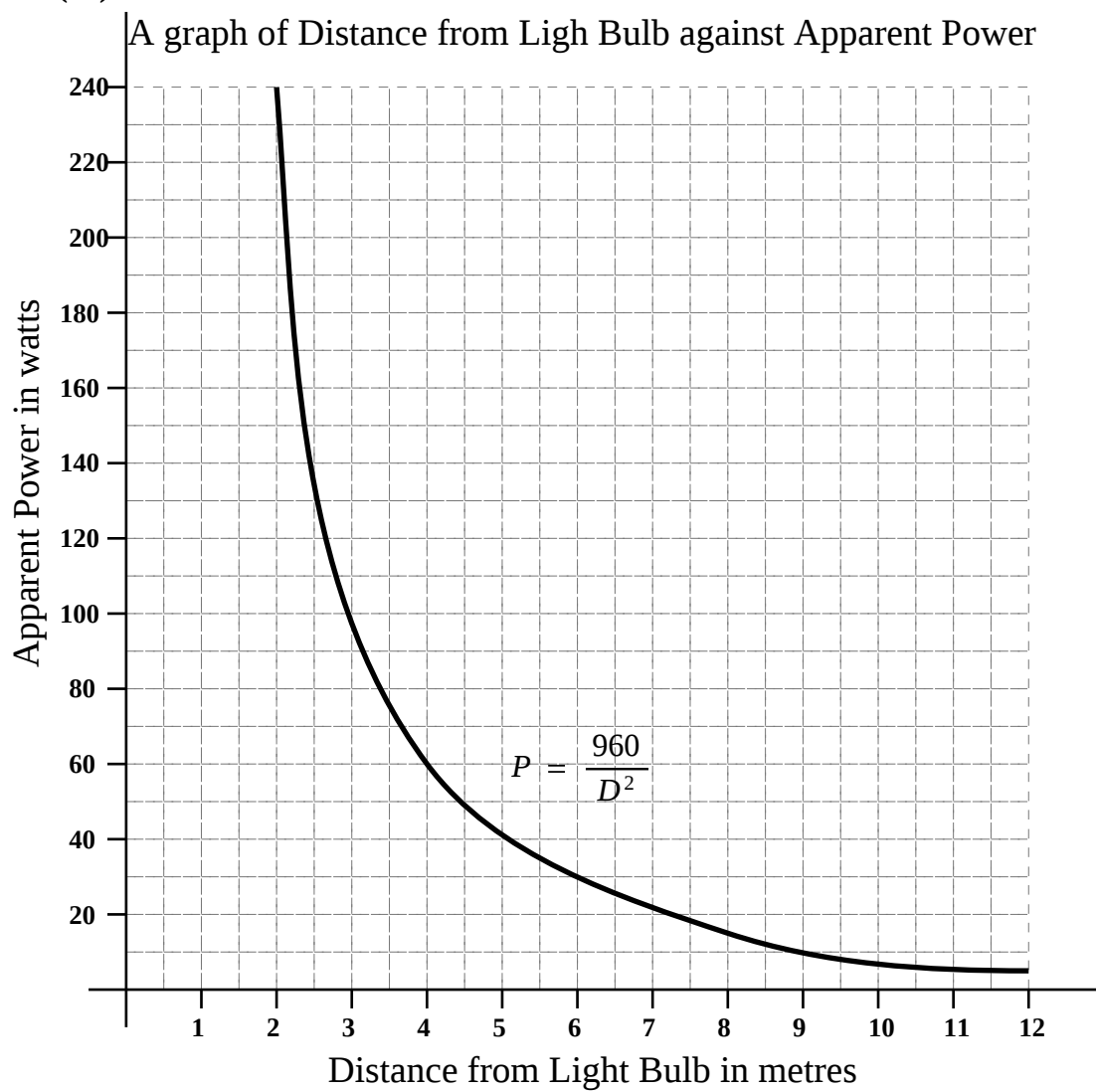
$$k = 960$$

(iii)

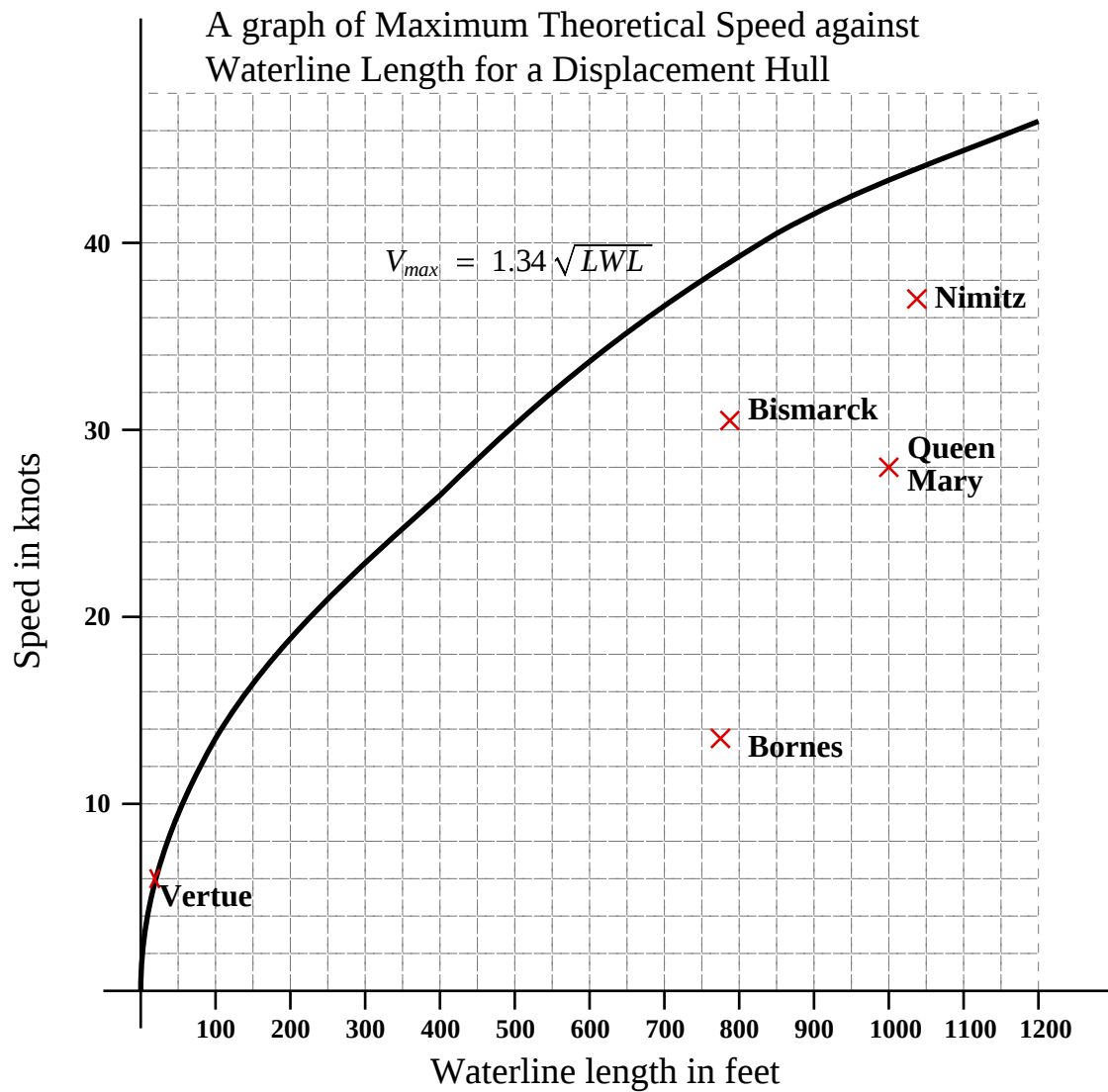
$$P = \frac{960}{D^2}$$

(iv)

D	2	4	6	8	10	12
P	240	60	$26\frac{2}{3}$	15	9.6	$6\frac{2}{3}$

(v)

7.4.3 Solutions to 7.3 Homework



Lesson 8

Proportionality : GCSE

8.1 Test

Calculator needed !

Question 1

$$y = \frac{36}{x^2}$$

- (i) Tick which applies ☐ y is directly proportional to the square of x
 ☐ y is inversely proportional to the square of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 3$, what is y ?
- (iv) When $x = 6$, what is y ?
- (v) If the value of x was increased, would the value of y increase or decrease ?

[10 marks]

Question 2

Write down the equation described by,

y is directly proportional to the square root of x

The constant of the proportionality is 7

[4 marks]

Question 3

" m is inversely proportional to the square of h "

Which one of the following is a correct translation into maths of the above words ?

$$m \propto \sqrt{h}$$

$$m \propto h^2$$

$$m \propto \frac{1}{\sqrt{h}}$$

$$m \propto \frac{1}{h^2}$$

[3 marks]

Question 4

"p is directly proportional to the square root of g"

Which one of the following is a correct translation into maths of the above words ?

$$p \propto g^2$$

$$p \propto \sqrt{g}$$

$$p \propto \frac{1}{\sqrt{g}}$$

$$p \propto \frac{1}{g^2}$$

[3 marks]

Question 5

$$y = 20x^3$$

- (i) Tick which applies
- ☐ y is directly proportional to the cube of x
- ☐ y is inversely proportional to the cube of x
- (ii) What is the constant of the proportionality ?
- (iii) When $x = 1$, what is y ?
- (iv) When $x = 3$, what is y ?
- (v) When $x = 10$, what is y ?
- (vi) If the value of x was increased, would the value of y increase or decrease ?

[12 marks]

Question 6

- (i) Write down the equation described by,
y is inversely proportional to x
The constant of the proportionality is 81

- (ii) When $x = 9$, what is y ?
- (iii) When $x = 27$, what is y ?
- (iv) Is inverse proportion best described by ☐ "MORE is MORE"
or ☐ "MORE is LESS"

[8 marks]

Question 7

The formula for the area of a triangle is $A = \frac{1}{2} b h$.

- (i)** Tick which applies
- ☐ A is directly proportional to b
- ☐ A is inversely proportional to b
- ☐ A is directly proportional to h
- ☐ A is inversely proportional to h
- (ii)** What is the constant of the proportionality?

[6 marks]

Question 8

- (i) Write down the equation described by,
 y is directly proportional to the square of x
 The constant of the proportionality is k
- (ii) If $y = 18$ when x is 3, what is the value of k ?
- (iii) Rewrite your part (i) answer with k replaced with its constant fixed value.
- (iv) When x is 15, what is y ?

[8 marks]

Question 9

The power loss, P , in an electrical wire is directly proportional to the square of the current, I , flowing through it.

- (i) Express " P is **directly** proportional to the square of I " as a proportionality.
- (ii) If, in a given wire, $P = 400$ when $I = 5$ find the constant of the proportionality.
- (iii) Write down a formula relating P and I .
- (iv) Use your formula to determine the value of P when $I = 10$.

[8 marks]

Question 10

Old examination question

(i) Express " T is **inversely** proportional to N " as a proportionality.

_____ [2 marks]

(ii) It is estimated that it will take 4 hours for 6 men to dig a hole.
By letting $T = 4$ when $N = 6$ find the constant of the proportionality.

_____ [2 marks]

(iii) Write down a formula relating T and N .

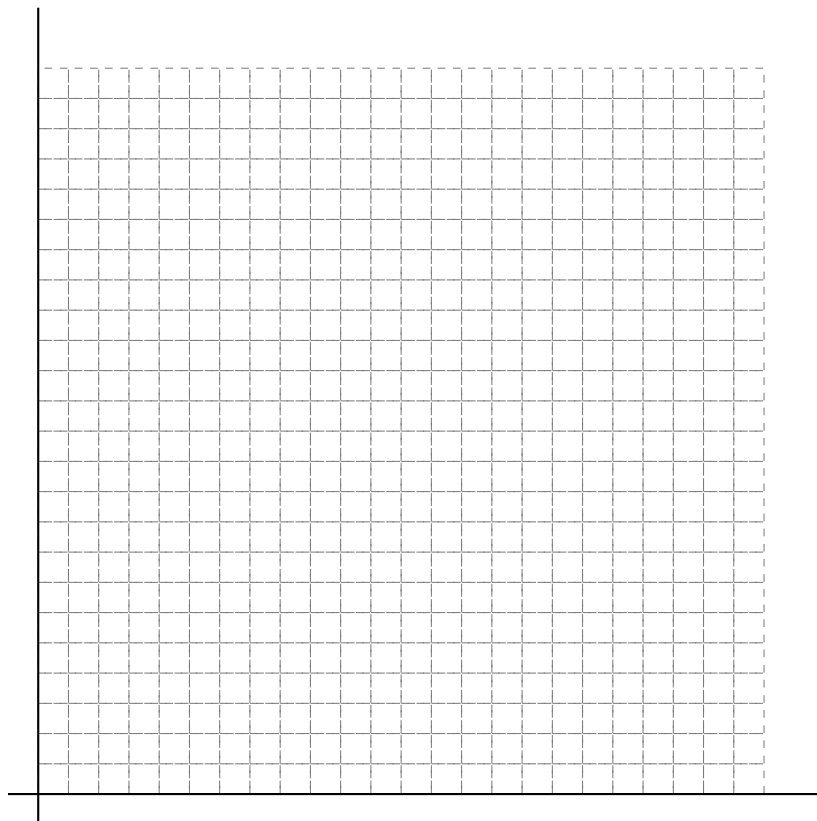
_____ [2 marks]

(iv) Use your formula to complete the following table:

N	1	2	3	4	6	8	12	24
T								

[2 marks]

(v) Plot a graph of inverse proportion for the "men digging hole".



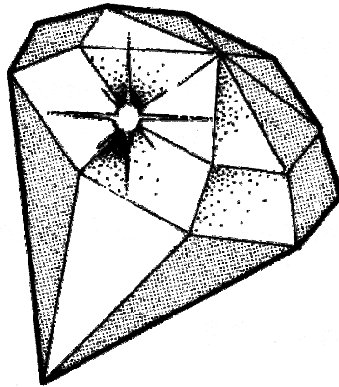
[2 marks]

Question 11

Like all great classes, Mrs Davies' decide to buy her a Christmas present.
They have discovered that Mrs Davies likes diamonds.

The value, V , of a diamond is **directly** proportional to the **square** of its weight W .
In other words,

$$V \propto W^2$$



- a) Use the fact that a diamond weighing 10 grams is worth £200 to help you write down a formula that relates the value of a diamond to its cost.

$$V = \underline{\hspace{2cm}} \quad [2 \text{ marks }]$$

- b) Show how you would use your part (a) formula to determine the value of a diamond weighing 30 grams.

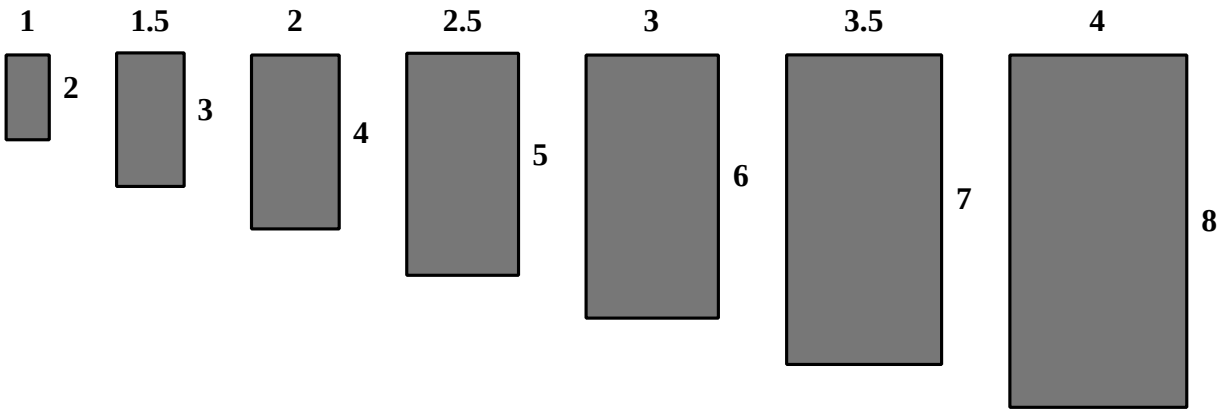
$$\underline{\hspace{2cm}} \quad [2 \text{ marks }]$$

- c) The class decide to spend £5000.
What is the weight of the diamond that Mrs Davies can look forward to receiving on Christmas day ?

$$\underline{\hspace{2cm}} \quad [2 \text{ marks }]$$

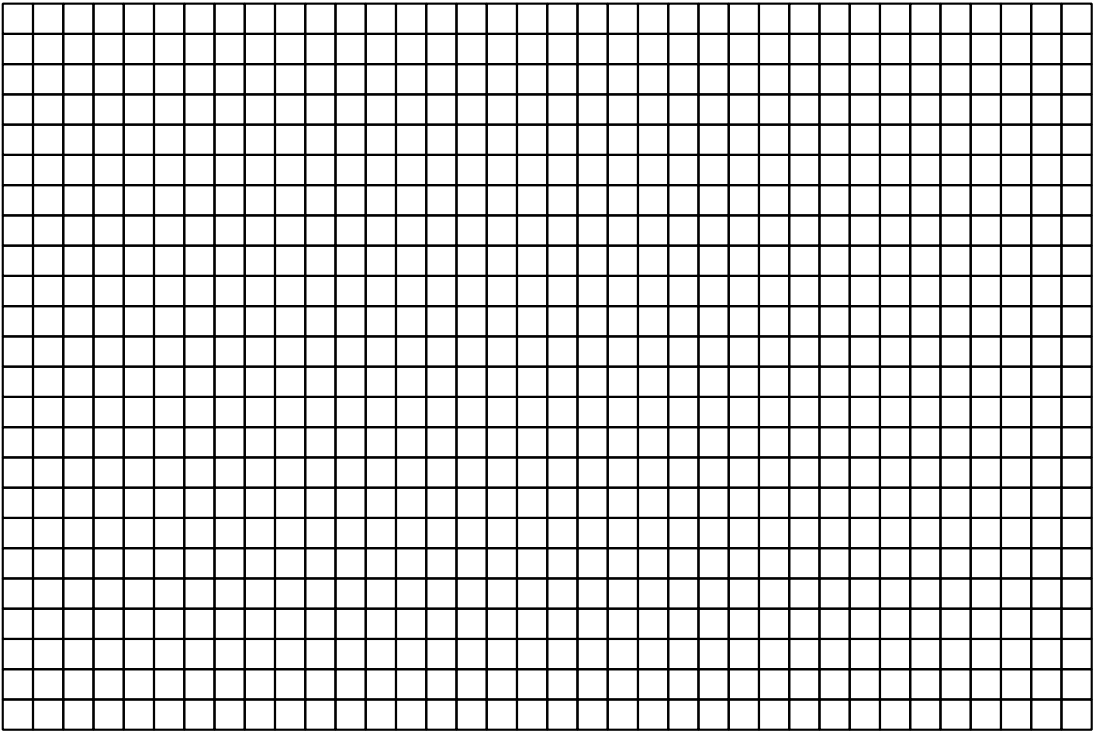
Question 12

Consider rectangles in which the height is double the width of side length.
For each of these rectangles, work out the perimeter and the area, and use the table below to summarise your results.



Measurements	1 by 2	1.5 by 3	2 by 4	2.5 by 5	3 by 6	3.5 by 7	4 by 8
Area							
Perimeter							

Plot a graph of your results.
Place Area on the x-axis, and Perimeter on the y-axis.



The Prof writes that :

$$P = k \sqrt{A}$$

[8 marks]

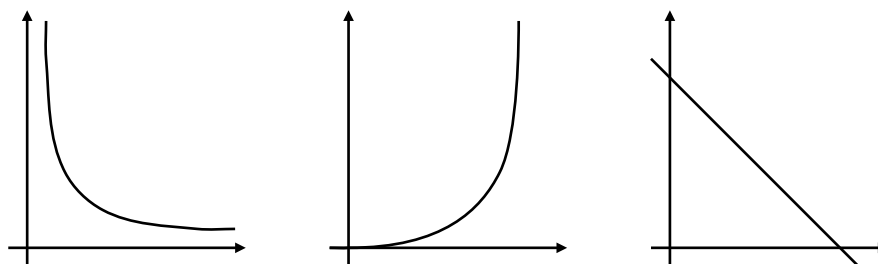
The formula for the area of a circle is $A = \pi r^2$

[8 marks]

Question 14

Indicate which one of the following graphs could represent the relation :

"m is directly proportional to the square of h"

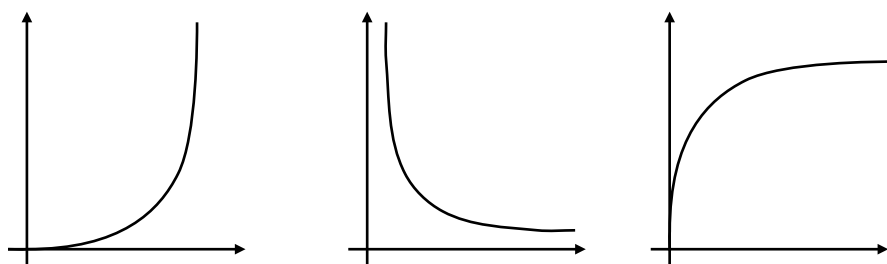


[3 marks]

Question 15

Indicate which one of the following graphs could represent the relation :

"m is inversely proportional to h"



[3 marks]

8.2 Answers

Answer 1

- (i) TICK on y is inversely proportional to the square of x .
- (ii) 36
- (iii) 4
- (iv) 1
- (v) decrease

Answer 2

$$y = 7\sqrt{x}$$

Answer 3

$$m \propto \frac{1}{h^2}$$

Answer 4

$$p \propto \sqrt{g}$$

Answer 5

- (i) TICK on y is directly proportional to the cube of x
- (ii) 20
- (iii) 20
- (iv) 540
- (v) 20 000
- (vi) increase

Answer 6

- (i)

$$y = \frac{81}{x}$$

- (ii) 9
- (iii) 3
- (iv) TICK on MORE is LESS

Answer 7

- (i) TICK on A is directly proportional to b
 TICK on A is directly proportional to h
- (ii) $\frac{1}{2}$

Answer 8

- (i) $y = kx^2$
- (ii) 2
- (iii) $y = 2x^2$
- (iv) 450

Answer 9

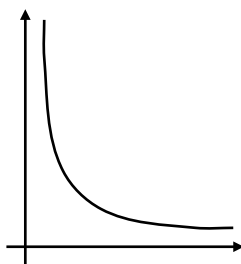
- (i) $P \propto I^2$
- (ii) 16
- (iii) $P = 16I^2$
- (iv) 1600

Answer 10

- (i) $T \propto \frac{1}{N}$
- (ii) 24
- (iii) $T = \frac{24}{N}$
- (iv)

N	1	2	3	4	6	8	12	24
T	24	12	8	6	4	3	2	1

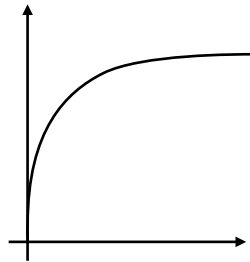
- (v)

**Answer 11**

- (a) $V = 2W^2$
- (b) £1800
- (c) 50 grammes

Answer 12

Measurements	1 by 2	1.5 by 3	2 by 4	2.5 by 5	3 by 6	3.5 by 7	4 by 8
Area	2	4.5	8	12.5	18	24.5	32
Perimeter	6	9	12	15	18	21	24

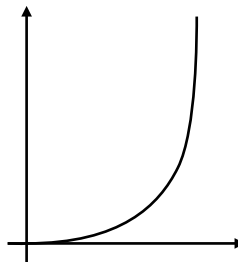


$$p = 3\sqrt{2} \sqrt{A} \quad \text{or} \quad P = 4.242 \sqrt{A}$$

127.3

Answer 13

- (i) TICK on A is directly proportional to r^2
- (ii) π
- (iii) 55.4 cm^2
- (iv) 5.64 cm

Answer 14**Answer 15**