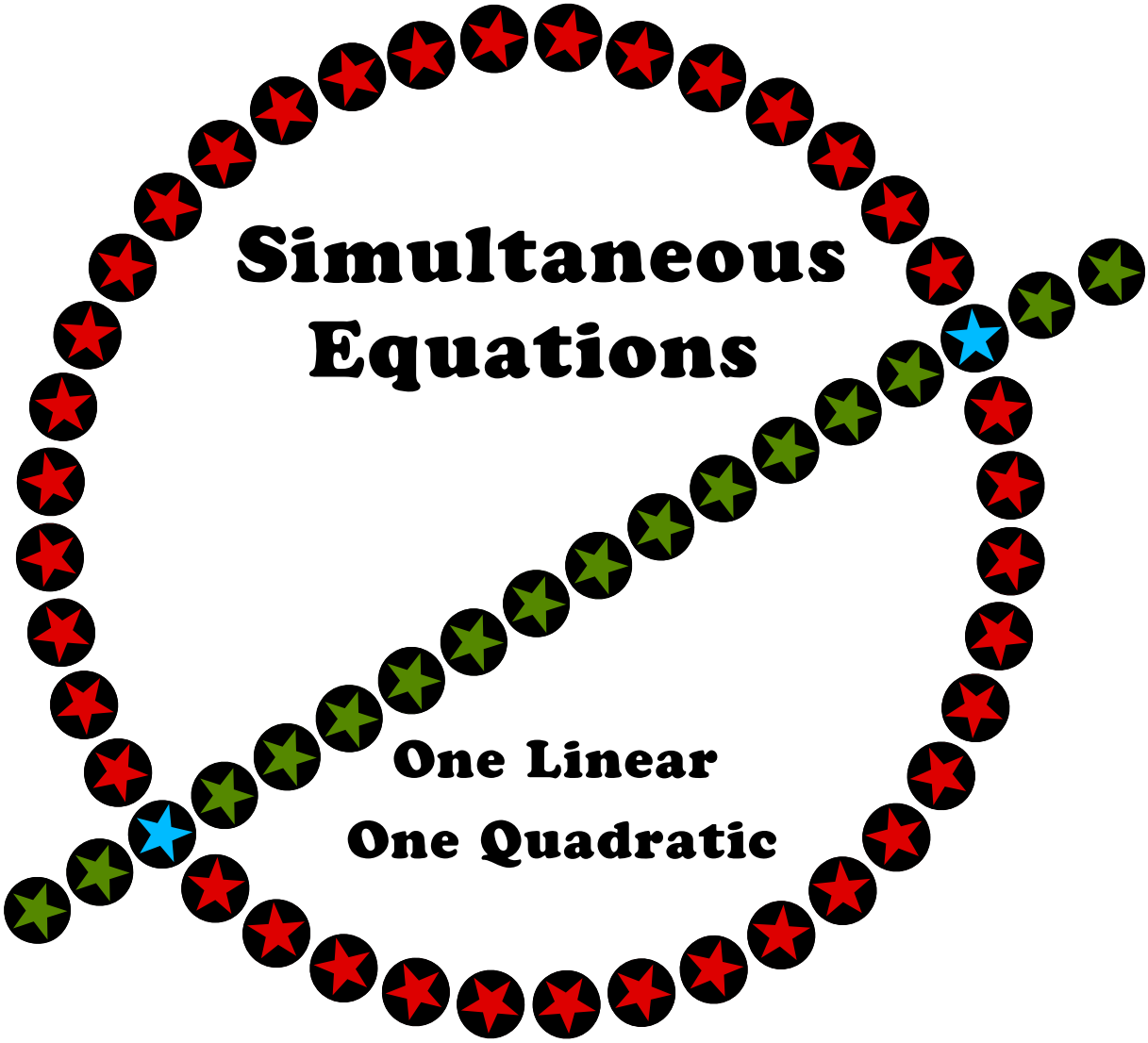

Simultaneous
Equations
One Linear
One Quadratic

Simultaneous Equations

One Linear and One Quadratic

Lesson 1

GCSE Mathematics

Simultaneous Equations III

1.1 Introduction

Previously, we have looked at solving simultaneous equations where both equations were linear. That is, they were *line like*. Geometrically, this equivalent to finding the crossing point of two straight lines.

Now we move on to the situation where one of the two equations is quadratic.

This could be a circle, for example;

$$x^2 + y^2 = 64$$

or a parabola, for example;

$$y = x^2 + 2x + 1$$

What follows next is an example of an examination question. We're not going to solve those yet. Rather we look at what algebraic techniques, studied previously, will be involved in solving such a question.

GCSE, June 2011, paper 3H, Q22

Solve the simultaneous equations,

$$y = 2x - 3 \qquad (\,line\,)$$

$$x^2 + y^2 = 2 \qquad (\,circle\,)$$

Geometrically, this is asking for the points (if any) where the line cuts the circle. Typically there will be two points but there could be one or none.

This topic is really five previous topics glued together.

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

We'll revise each of these separately, then look at how to glue the bits together.

In an examination, a *solve these simultaneous equations question* is worth about six marks. The bits glued together will also be tested separately elsewhere in the exam. So, being good at this topic is potentially worth about 6% in each of the two examination papers.

1.2 FOIL

This is a first and crucial step. If you get this wrong, the mark scheme may state that no marks are to be awarded. So an error here is 'game over': *Go direct to Jail : Do not pass Go, Do not collect £200.*

Questions

Expand the brackets;

(i) $(x+6)(x+5)$

(ii) $(x+1)(4x+7)$

(iii) $(2x+5)(3x+4)$

(iv) $(8x+5)(11x+9)$

(v) $(x+8)(x-3)$

(vi) $(x-5)(3x+4)$

(vii) $(x-7)(5x-2)$

(viii) $(x+5)^2$

(ix) $(2x+7)^2$

(x) $(5x-3)^2$

The last three questions, where the bracket is squared, are the most important.

1.3 Gathering together like terms

Example

Simplify $3x^2 - 3 + 4x + 2x + x^2 + 7$

Try this, then check with the answer at the bottom of the following page.

Questions

Simplify each of the following;

(i) $x^2 + 5x + 7x + 11$

(ii) $2x^2 + 11x - 6x + 3$

(iii) $5x^2 + 4x + 3 + 5x - 2$

(iv) $x^2 + 4x - 7 - 9x + 3$

(v) $9x^2 + 13x - 6x^2 + 3 - x$

(vi) $3x^2 + 14x + (5x - 3) - x^2$

(vii) $5x^2 - 7x - (4x + 3)$ Be careful expanding the brackets !

(viii) $x^2 + 7x + (7 - 5x) - 3$

(ix) $8x^2 + 14x - \left(4x^2 - 3x\right) - x^2$

(x) $4\left(3x^2 + 2x\right) - (7 - 5x) - 1$

1.4 Rearranging equations into the form $f(x) = 0$

Example

Rearrange the following equation into the form $f(x) = 0$

$5x^2 + 4x + 8 = 2x + 5$

Try this, then check with the answer at the bottom of the following page.

Questions

Rearrange each of the following equation into the form $f(x) = 0$

(i) $8x^2 + 9x + 5 = 4x + 2$

(ii) $2x^2 + 3x + 11 = 7x + 3$

(iii) $7x^2 - 5x + 1 = 3x - 7$

(iv) $9x^2 + 7x - 8 = 5 - 3x$

Previous page answer: $4x^2 + 6x + 4$

(v) $8x^2 + 14x - 8 = 2x^2 - 3x + 2$

(vi) $2x^2 + 15x - 4 = 2x^2 + 3x + 28$

(vii) $5x^2 + (6x - 5) = 9 - 2x + x^2$

(viii) $11x^2 - (4x + 3) = 5x^2 - 15$

(ix) $8x^2 - (7x - 6) = 5 - 3x$

(x) $7x^2 - (3 + 7x) = 2x^2 - (6x - 1)$

1.5 Mixture

Before tackling the last two items on the introduction's lists (next lesson) here are some question that mix the first three ideas.

Questions

Rearrange each of the following equation into the form $f(x) = 0$

(i) $(x + 4)(x + 6) = 5x + 3$

(ii) $(3x + 9)(2x + 3) = 8x - 13$

(iii) $(x + 5)(x - 5) = 7x + 13$

(iv) $(x + 4)(3x - 5) = 2x^2 - 5$

(v) $(x + 7)^2 = 4x + 7$

(vi) $(2x - 3)^2 = (x + 2)^2$

Previous page answer: $5x^2 + 2x + 3 = 0$

1.6 Answers

GCSE Mathematics
Simultaneous Equations III

1.6.1 Solutions (1.2 FOIL)

(i) $x^2 + 11x + 30$ **(ii)** $4x^2 + 11x + 7$

(iii) $6x^2 + 23x + 20$ **(iv)** $88x^2 + 127x + 45$

(v) $x^2 + 5x - 24$ **(vi)** $3x^2 - 11x - 20$

(vii) $5x^2 - 37x + 14$ **(viii)** $x^2 + 10x + 25$

(ix) $4x^2 + 28x + 49$ **(x)** $25x^2 - 30x + 9$

1.6.2 Solutions (1.3 Gathering together like terms)

(i) $x^2 + 12x + 11$ **(ii)** $2x^2 + 5x + 3$

(iii) $5x^2 + 9x + 1$ **(iv)** $x^2 - 5x - 4$

(v) $3x^2 + 12x + 3$ **(vi)** $2x^2 + 19x - 3$

(vii) $5x^2 - 11x - 3$ **(viii)** $x^2 + 2x + 4$

(ix) $3x^2 + 17x$ **(x)** $12x^2 + 13x - 8$

1.6.3 Solutions (1.4 Rearranging equations into the form $f(x) = 0$)

(i) $8x^2 + 5x + 3 = 0$ **(ii)** $2x^2 - 4x + 8 = 0$

(iii) $7x^2 - 8x + 8 = 0$ **(iv)** $9x^2 + 10x - 13 = 0$

(v) $6x^2 + 17x - 10 = 0$ **(vi)** $12x - 32 = 0$

(vii) $4x^2 + 8x - 14$ **(viii)** $6x^2 - 4x + 12 = 0$

(ix) $8x^2 - 4x + 1 = 0$ **(x)** $5x^2 - x - 4 = 0$

1.6.4 Solutions (1.5 Mixture)

(i) $x^2 + 5x + 21 = 0$

(ii) $6x^2 + 19x + 40 = 0$

(iii) $x^2 - 7x - 38 = 0$

(iv) $x^2 + 7x - 15 = 0$

(v) $x^2 + 10x + 42$

(vi) $3x^2 - 16x + 5 = 0$

Lesson 2

GCSE Mathematics
Simultaneous Equations III

2.1 Recap (Homework)

To solve questions like the following;

GCSE, June 2011, paper 3H, Q22
Solve the simultaneous equations

$$y = 2x - 3 \qquad \text{(line)}$$

$$x^2 + y^2 = 2 \qquad \text{(circle)}$$

We need to be competent at the following five topics;

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

2.2 Revision Exercise

Question 1

- Expanding brackets, FOIL

Expand the brackets;

(i) $(x + 6)(x + 11)$

(ii) $(3x + 5)(4x - 3)$

(iii) $(x + 4)^2$

Question 2

- Gathering together like terms

Simplify each of the following;

(i) $x^2 + 9x + 17x + 18$

(ii) $7x^2 + 17x - 3x^2 + 13 - 5x$

(iii) $15x^2 - (6x - 7) + 6x^2 - 15$

Question 3

- Rearranging equations into the form $f(x) = 0$

Rearrange each of the following equation into the form $f(x) = 0$

(i) $19x^2 + 2x + 6 = 5x + 3$

(ii) $5x^2 + 12x - 8 = 2x^2 - 7x + 8$

(iii) $13x^2 - (7x - 6) = 1 - 4x$

Question 4

This question is a mix the first three ideas.

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$

Rearrange each of the following equation into the form $f(x) = 0$

(i) $(x + 5)(x + 8) = 4x + 3$

(ii) $(x + 5)^2 = 7x + 9$

(iii) $(3x - 2)^2 = (x + 1)^2$

2.3 Answers

2.3.1 Solutions (2.2 Revision Exercise)

Answer 1

(i) $x^2 + 17x + 66$ **(ii)** $12x^2 + 11x - 15$

(iii) $x^2 + 8x + 16$

Answer 2

(i) $x^2 + 26x + 18$ **(ii)** $4x^2 + 12x + 13$

(iii) $21x^2 - 6x - 8$

Answer 3

(i) $19x^2 - 3x + 3 = 0$ **(ii)** $3x^2 + 19x - 16 = 0$

(iii) $13x^2 - 3x + 5 = 0$

Answer 4

(i) $x^2 + 9x + 37 = 0$ **(ii)** $x^2 + 3x + 16 = 0$

(iii) $8x^2 - 14x + 3 = 0$

Lesson 3

GCSE Mathematics

Simultaneous Equations III

3.1 Quadratics That Readily Factorise

In general, a quadratic equation is an equation of the form $y = ax^2 + bx + c$.

When given a specific quadratic to work on the letters a, b and c will be replaced with numbers. You might, for example be asked to ***factorise*** the quadratic equation,

$$y = x^2 + 11x + 24$$

In this example $a = 1$, $b = 11$ and $c = 24$.

The 1, 11 and 24 are constants in this example. They are consistent, meaning they don't change. The x and the y are variables; they are free to vary which means that changing one, changes the other.

The word ***factorise*** in this context means to ***make brackets***.

To keep things simple (to begin with) we will work on questions where $a = 1$. When asked to factorise such a quadratic the task is to find an equivalent expression of the form $y = (x + A)(x + B)$ where A and B are two real numbers (called the ***negative roots*** of the quadratic) that are to be found.

3.2 The Theory

We want to rewrite $y = x^2 + bx + c$

in the form $y = (x + A)(x + B)$

In the space below, expand the brackets, gather together like terms and write two conclusions in which you relate A and B to b and c.

3.3 Three Examples

Example #1

Factorise $y = x^2 + 11x + 24$

Example #2

Factorise $y = x^2 + 3x - 10$

Example #3

Factorise $y = 3x^2 - 15x + 18$

3.4 Exercise

Question 1

Factorise;

(i) $y = x^2 + 10x + 21$

(ii) $y = x^2 + 11x + 30$

(iii) $y = x^2 + 9x + 14$

(iv) $y = x^2 + 8x + 15$

(v) $y = x^2 + 14x + 33$

(vi) $y = x^2 + 6x + 9$

(vii) $y = x^2 + 10x + 9$

(viii) $y = x^2 + 14x + 13$

(ix) $y = 10\,x^2 + 140x + 480$

(x) $y = 2\,x^2 + 36x + 154$

Question 2

Factorise;

(i) $y = x^2 + 2x - 3$

(ii) $y = x^2 + 5x - 14$

(iii) $y = x^2 + 9x - 22$

(iv) $y = x^2 + 2x - 15$

(v) $y = x^2 - 2x - 15$

(vi) $y = x^2 - 4x - 21$

(vii) $y = x^2 - 8x - 20$

(viii) $y = x^2 - 8x - 33$

(ix) $y = 3x^2 - 9x - 120$

(x) $y = 5x^2 - 30x - 200$

Question 3

Factorise;

(i) $y = x^2 - 5x + 6$

(ii) $y = x^2 - 8x + 15$

(iii) $y = x^2 - 10x + 21$

(iv) $y = x^2 - 9x + 20$

(v) $y = x^2 - 10x + 25$

(vi) $y = x^2 - 7x + 6$

(vii) $y = x^2 - 10x + 16$

(viii) $y = x^2 - 8x + 12$

(ix) $y = 4x^2 - 60x + 176$

(x) $y = 10x^2 - 140x + 490$

Question 4

Factorise;

(i) $y = x^2 + 15x + 50$ **(ii)** $y = x^2 + 5x - 50$

(iii) $y = x^2 - 15x + 50$ **(iv)** $y = x^2 - 5x - 50$

Question 5

A quadratic has two real negative roots, *A* and *B*.
The sum of the negative roots is 13.
The product of the negative roots is 42.
What are the two negative roots ?

Question 6

A quadratic has two real negative roots, *A* and *B*.
The sum of the negative roots is 5.
The product of the negative roots is – 204.
What are the two negative roots ?

3.5 Answers

3.5.1 Solution to 3.2 The Theory

Knowing that quadratics factorise into $(x + A)(x + B)$ means that the process can be "reverse engineered" by expanding the brackets.

$$(x + A)(x + B) = x^2 + Ax + Bx + AB$$

$$= x^2 + (A + B)x + AB$$

Compare this with $y = x^2 + bx + c$ (where $a = 1$)

$$A + B = b \text{ and } AB = c$$

3.5.2 Solution to 3.3 Three Examples

Example #1

Factorise $y = x^2 + 11x + 24$

A	B	
1	24	
2	12	
3	8	<===
4	6	

$$y = (x + 3)(x + 8)$$

Example #2

Factorise $y = x^2 + 3x - 10$

A	B	
-1	10	
-2	5	<===
-5	2	
-10	1	

$$y = (x - 2)(x + 5)$$

Example #3

Factorise $y = 3x^2 - 15x + 18$

$$y = 3(x^2 - 5x + 6)$$

A	B	
-2	-3	<===

$$y = 3(x - 2)(x - 3)$$

3.5.2 Solutions to 3.4 Exercise

Answer 1

(i) $y = (x + 3)(x + 7)$ **(ii)** $y = (x + 5)(x + 6)$

(iii) $y = (x + 2)(x + 7)$ **(iv)** $y = (x + 3)(x + 5)$

(v) $y = (x + 3)(x + 11)$ **(vi)** $y = (x + 3)^2$

(vii) $y = (x + 1)(x + 9)$ **(viii)** $y = (x + 1)(x + 13)$

(ix) $y = 10(x + 6)(x + 8)$ **(x)** $y = 2(x + 7)(x + 11)$

Answer 2

(i) $y = (x + 3)(x - 1)$ **(ii)** $y = (x + 7)(x - 2)$

(iii) $y = (x + 11)(x - 2)$ **(iv)** $y = (x + 5)(x - 3)$

(v) $y = (x - 5)(x + 3)$ **(vi)** $y = (x - 7)(x + 3)$

(vii) $y = (x - 10)(x + 2)$ **(viii)** $y = (x - 11)(x + 3)$

(ix) $y = 3(x - 8)(x + 5)$ **(x)** $y = 5(x - 10)(x + 4)$

Answer 3

(i) $y = (x - 2)(x - 3)$ **(ii)** $y = (x - 3)(x - 5)$

(iii) $y = (x - 3)(x - 7)$ **(iv)** $y = (x - 4)(x - 5)$

(v) $y = (x - 5)^2$ **(vi)** $y = (x - 6)(x - 1)$

(vii) $y = (x - 8)(x - 2)$ **(viii)** $y = (x - 6)(x - 2)$

(ix) $y = 4(x - 4)(x - 11)$ **(x)** $y = 10(x - 7)^2$

Answer 4

(i) $y = (x + 5)(x + 10)$ **(ii)** $y = (x + 10)(x - 5)$

(iii) $y = (x - 5)(x - 10)$ **(iv)** $y = (x - 10)(x + 5)$

Answer 5

$A = 6,\ B = 7$ (or vice versa)

Answer 6

$A = -12$ or $B = 17$ (or vice versa)

Lesson 4

GCSE Mathematics

Simultaneous Equations III

4.1 Quadratic Equations With Integer Roots

A quadratic equation is an equation of the form,

$$y = ax^2 + bx + c$$

where a, b and c are constants, and x and y are variables.

Last lesson, a large number of quadratic expressions where factorised. The ability to factorise a quadratic expression is one of the key steps in how the quadratic equations in this lesson may be solved.

We are particularly interested in finding any value of x that make $y = 0$. This is because zero in mathematics is a very special number, not least because of the following theorem, The Product of Zero Theorem.

The Product of Zero Theorem

Given two numbers which have a product of zero,
either the first number or the second number must be zero.

If $p \times q = 0$

Then either $p = 0$ or $q = 0$

The word "or" means either $p = 0$ or $q = 0$ or both $p = 0$ and $q = 0$
It is called an "inclusive or".
(FYI : "Exclusive or", *xor*, excludes the possibility that both are zero)

Note that it is only when $p \times q = 0$ that you get such insightful information about p or q. To see this consider the situation where $p \times q = 24$. Solutions to this, even just in positive integers, include (1, 24), (2, 12), (3, 8), (4, 6), (6, 4), (8, 3), (12, 2), (24, 1). The constraints upon p or q when the product is zero are very strong, which is otherwise not so.

4.2 Example

The Question : Solve, $x^2 + 6x + 8 = 0$

The Answer :

$$x^2 + 6x + 8 = 0$$

$$(x + 4)(x + 2) = 0 \qquad \text{Factorise}$$

$$\text{Either } x + 4 = 0 \text{ or } x + 2 = 0 \qquad \text{The Product of Zero Theorem}$$

$$\therefore x = -4 \text{ or } x = -2 \qquad \text{The "roots" of the quadratic}$$

Notice : Each of the two answers could be substituted back into the quadratic equation to check they are correct.

4.3 Exercise

Question 1

Solve these quadratic equations. Every solution should use the word “either” and the word “or” at the appropriate step.

(i) $x^2 + 8x + 15 = 0$

(ii) $x^2 + 5x + 4 = 0$

(iii) $x^2 + 9x + 14 = 0$

(iv) $x^2 + 9x + 18 = 0$

(v) $x^2 + 15x + 44 = 0$

(vi) $x^2 + 11x + 24 = 0$

(vii) $3x^2 + 33x + 90 = 0$ **(viii)** $10x^2 + 120x + 270 = 0$

Hint : Start by dividing through by 3

(ix) $4x^2 + 60x + 200 = 0$ **(x)** $5x^2 + 60x + 160 = 0$

4.4 Example

The Question: Solve, $x^2 - 6x - 55 = 0$

The Answer :

$$x^2 - 6x - 55 = 0$$

$$(x - 11)(x + 5) = 0 \qquad \text{Factorise}$$

$$\text{Either } x - 11 = 0 \text{ or } x + 5 = 0 \qquad \text{The Product of Zero Theorem}$$

$$\therefore x = 11 \text{ or } x = -5 \qquad \text{The "roots" of the quadratic}$$

Be Careful : By far the most common mistake is to get the signs of the answers wrong; $(x - 11)$ is a factor of the quadratic, and $x = 11$ is a root.

4.5 Exercise

Question 1

Solve these quadratic equations. Every solution should use the word "either" and the word "or" at the appropriate step.

(i) $x^2 - 5x - 14 = 0$ **(ii)** $x^2 - 4x - 21 = 0$

(iii) $x^2 - 10x - 24 = 0$ **(iv)** $x^2 - x - 30 = 0$

(v) $x^2 - 6x - 40 = 0$

(vi) $x^2 + 6x - 16 = 0$

(vii) $3x^2 + 24x - 99 = 0$

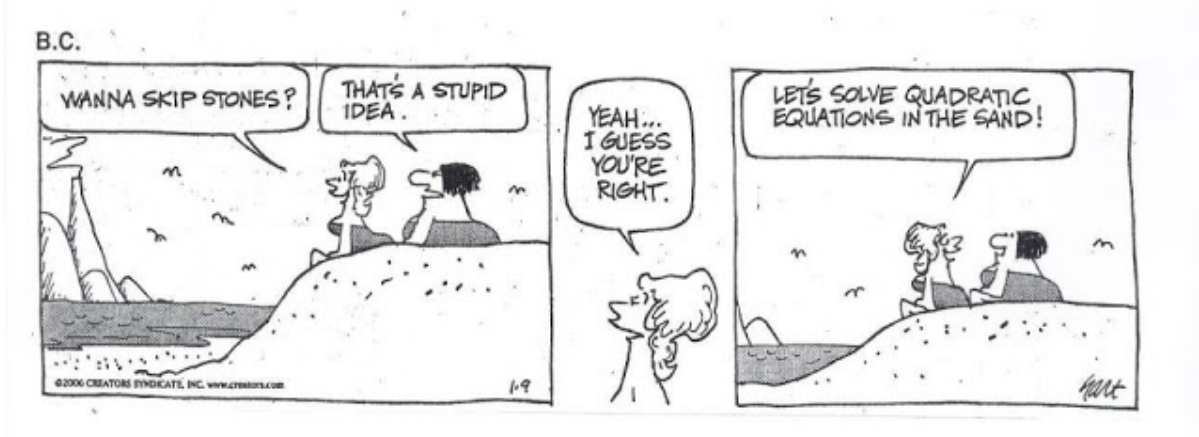
(viii) $7x^2 + 35x - 350 = 0$

(ix) $2x^2 + 32x - 34 = 0$

(x) $3x^2 + 9x - 120 = 0$

(xi) $16x^2 + 256x - 272 = 0$

(xii) $15x^2 + 45x - 600 = 0$



4.3 Answers

GCSE Mathematics
Simultaneous Equations III

4.3.1 Solutions (4.2 Example)

$$x^2 + 6x + 8 = 0$$

$$(x + 4)(x + 2) = 0$$

Either $x + 4 = 0$ or $x + 2 = 0$

$$\therefore\ x = -4 \text{ or } x = -2$$

4.3.2 Solutions (4.3 Exercise)

(i)	$-5, -3$	**(ii)**	$-4, -1$
(iii)	$-7, -2$	**(iv)**	$-6, -3$
(v)	$-11, -4$	**(vi)**	$-8, -3$
(vii)	$-6, -5$	**(viii)**	$-9, -3$
(ix)	$-10, -5$	**(x)**	$-8, -4$

4.3.3 Solutions (4.4 Example)

$$x^2 - 6x - 55 = 0$$

$$(x - 11)(x + 5) = 0$$

Either $x - 11 = 0$ or $x + 5 = 0$

$$\therefore\ x = 11 \text{ or } x = -5$$

4.3.4 Solutions (4.5 Exercise)

(i)	$-2, 7$	**(ii)**	$-3, 7$
(iii)	$-2, 12$	**(iv)**	$-5, 6$
(v)	$-4, 10$	**(vi)**	$-8, 2$
(vii)	$-11, 3$	**(viii)**	$-10, 5$
(ix)	$-17, 1$	**(x)**	$-8, 5$
(xi)	$-17, 1$	**(xii)**	$-8, 5$

Lesson 5

GCSE Mathematics

Simultaneous Equations III

5.1 Examination Style Questions

The Question: Solve the simultaneous equations

$y = x^2$ This a quadratic curve

$y = 2x + 3$ This is a straight line

The Solution: Using *the method of substitution.*

$$x^2 = 2x + 3$$

- Rearranging equations into the form $f(x) = 0$

$$x^2 - 2x - 3 = 0$$

- Factorising quadratics

$$(x + 1)(x - 3) = 0$$

- Solving quadratic equations

$$\text{Either } x + 2 = 0 \text{ or } x - 3 = 0$$

$$x = -1 \text{ or } x = 3$$

But this is not the final answer !

The final answer is the points where the straight line intersects the quadratic curve.

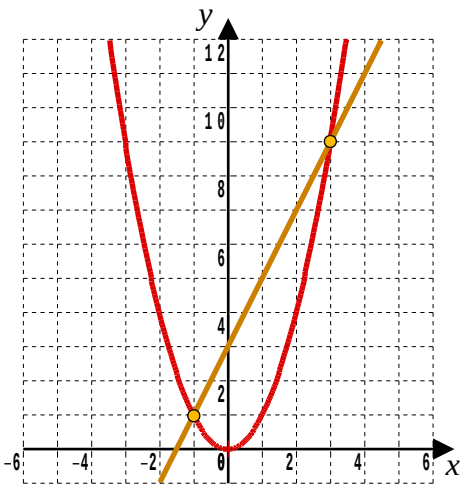
Use the equation of the line $y = 2x + 3$ with x is 2 and then with x is 3 to get,

$$\text{The Final Solutions} : (-1, 1) \text{ or } (3, 9)$$

5.2 Understanding What Has Been Achieved

On the graph below, the quadratic curve $y = x^2$ has been plotted.

So too, has the straight line, $y = 2x + 3$



The line meets the curve at $(-1, 1)$ and also at $(3, 9)$.

(Thus verifying, geometrically, the answers previously obtained algebraically)

5.3 Exercise

Question 1

Use the method of substitution to obtain a quadratic equation in the single variable, x. Solve your equation, and find the possible pairs of values for x and y.

(i) $y = x^2$

$y = 8x - 12$

(ii) $y = x^2$

$y = 11x - 28$

(iii) $y = x^2$

$y = 2x + 24$

(iv) $y = x^2 + 10$

$y = 4 - 7x$

(v) $y = x^2 - 14$

$y = 2x + 21$

(vi) $y = x^2 + 3$

$y = 30 - 6x$

(vii) $y = x^2 + 2x$

$y = 5x + 28$

(viii) $y = x^2 - 4x + 2$

$y = 7x - 8$

(ix) $y = x^2 + 3x - 10$

$y = 4x + 20$

(x) $y = x^2$

$y = 7x - 12$

5.4 Examination Question

GCSE, November 2006, paper 3H, Q18

Solve the simultaneous equations

$$y = x^2$$

$$y = 2x + 15$$

[5 marks]

5.5 Homework

GCSE Mathematics

Simultaneous Equations III

Question 1

Use the method of substitution to obtain a quadratic equation in only one variable. Solve your equation, and find the possible pairs of values for x and y.

(i) $y = x^2$

$y = 10 - 3x$

(ii) $y = x^2$

$y = 8 - 7x$

(iii) $y = x^2$

$y = 5x + 36$

(iv) $y = x^2 + 20$

$y = 17x - 50$

(v) $y = x^2 - 2x + 1$

$y = 10x - 31$

(vi) $y = x^2 + 4x$

$y = 6x + 48$

(vii) $y = x^2 - 4x + 5$

$y = 3 - x$

(viii) $y = x^2 + 5x + 17$

$y = 2 - 3x$

5.5 Answers

5.6.1 Solutions (5.3 Exercise)

(i) $x^2 - 8x + 12 = 0$
$(x - 6)(x - 2) = 0$
Points : (2, 4) or (6, 36)

(ii) $x^2 - 11x + 28 = 0$
$(x - 4)(x - 7) = 0$
Points : (4, 16) or (7, 49)

(iii) $x^2 - 2x - 24 = 0$
$(x - 6)(x + 4) = 0$
Points : (6, 36) or (− 4, 16)

(iv) $x^2 + 7x + 6 = 0$
$(x + 1)(x + 6) = 0$
Points : (− 1, 11) or (− 6, 46)

(v) $x^2 - 2x - 35 = 0$
$(x - 7)(x + 5) = 0$
Points : (7, 35) or (− 5, 11)

(vi) $x^2 + 6x - 27 = 0$
$(x + 9)(x - 3) = 0$
Points : (− 9, 84) or (3, 12)

(vii) $x^2 - 3x - 28 = 0$
$(x - 7)(x + 4) = 0$
Points : (7, 63) or (− 4, 8)

(viii) $x^2 - 11x + 10 = 0$
$(x - 10)(x - 1) = 0$
Points : (10, 62) or (1, − 1)

(ix) $x^2 - x - 30 = 0$
$(x - 6)(x + 5) = 0$
Points : (6, 44) or (− 5, 0)

(x) $x^2 - 7x + 12 = 0$
$(x - 3)(x - 4) = 0$
Points : (3, 9) or (4, 16)

5.6.2 Solutions (5.4 Examination Question)

$x^2 - 2x - 15 = 0$
$(x - 5)(x + 3) = 0$
Points : (5, 25) or (− 3, 9)

5.6.3 Solutions (5.5 Homework)

GCSE Mathematics
Simultaneous Equations III

(i) $x^2 + 3x - 10 = 0$
$(x + 5)(x - 2) = 0$
Points : $(-5, 25)$ or $(2, 4)$

(ii) $x^2 + 7x - 8 = 0$
$(x + 8)(x - 1) = 0$
Points : $(-8, 64)$ or $(1, 1)$

(iii) $x^2 - 5x - 36 = 0$
$(x - 9)(x + 4) = 0$
Points : $(9, 81)$ or $(-4, 16)$

(iv) $x^2 - 17x + 70 = 0$
$(x - 10)(x - 7) = 0$
Points : $(10, 120)$ or $(7, 69)$

(v) $x^2 - 12x + 32 = 0$
$(x - 4)(x - 8) = 0$
Points : $(4, 9)$ or $(8, 49)$

(vi) $x^2 - 2x - 48 = 0$
$(x - 8)(x + 6) = 0$
Points : $(8, 96)$ or $(-6, 12)$

(vii) $x^2 - 3x + 2 = 0$
$(x - 1)(x - 2) = 0$
Points : $(1, 2)$ or $(2, 1)$

(viii) $x^2 + 8x + 15 = 0$
$(x + 3)(x + 5) = 0$
Points : $(-3, 11)$ or $(-5, 17)$

Lesson 6

GCSE Mathematics

Simultaneous Equations III

6.1 Quadratic Equations With Integer Roots (Recap)

Faced with a quadratic equation of the form,

$$y = x^2 + bx + c \qquad \text{where } b \text{ and } c \text{ are integers}$$

we previously, in Lesson 3, developed a fatorisation theory based around the fact that, given the factorisation was of the form,

$$y = (x + A)(x + B) \qquad \text{where } A \text{ and } B \text{ are integers}$$

then,

$$A + B = b \quad \text{and} \quad AB = c$$

Proof:

Given that the quadratics factorise into $(x + A)(x + B)$ means that by expanding the brackets the original quadratic equation must be obtained;

$$(x + A)(x + B) = x^2 + Ax + Bx + AB$$

$$= x^2 + (A + B)x + AB$$

Compare this with $y = x^2 + bx + c$ (where the coefficient of x^2 is 1)

$$A + B = b \quad \text{and} \quad AB = c$$

The theory develops further to solve such equations when $y = 0$

$$x^2 + bc + c = 0$$

$$(x + A)(x + B) = 0$$

$$\text{Either } x + A = 0 \qquad \text{or } x + B = 0$$

$$x = -A \qquad\qquad x = -B$$

The solutions to such a quadratic equation are called the roots of the equation.

Example #1

Find the roots of the quadratic equation $y = x^2 - 2x - 3$

6.2 Quadratic Equations With Rational Roots

Faced with a quadratic equation of the form,

$y = ax^2 + bx + c$ where a, b and c are integers with $\left(\text{preferably with } hcf\{a, b, c\} = 1\right)$

then, if the factorisation is of the form,

$$y = (Px + Q)(Rx + S)$$

$$\text{then } PR = a \quad \text{and} \quad PS + QR = b \quad \text{and} \quad QS = c$$

Proof:

Given that the quadratics factorise into $(Px + Q)(Rx + S)$ means that by expanding the brackets the original quadratic equation must be obtained;

$$(Px + Q)(Rx + S) = PRx^2 + PSx + QRx + QS$$

$$= PRx^2 + (PS + QR)x + QS$$

Compare this with $y = ax^2 + bx + c$

$$PR = a \quad \text{and} \quad PS + QR = b \quad \text{and} \quad QS = c$$

Example #2

Factorise $y = 15x^2 + 11x + 2$

Example #3

Find the roots of the quadratic equation $y = 6x^2 - x - 2$

6.3 Exercise

Question 1

Factorise;

(i) $y = 2x^2 + 13x + 15$ **(ii)** $y = 9x^2 + 24x + 7$

(iii) $y = 7x^2 + 23x + 6$ **(iv)** $y = 6x^2 + 25x + 25$

Question 2

Factorise,

(i) $y = 10x^2 - 9x + 2$ **(ii)** $y = 3x^2 - 7x + 4$

(iii) $y = 6x^2 - 13x + 5$ **(iv)** $y = 3x^2 - 20x + 33$

Question 3

Factorise;

(i) $y = 5x^2 - 33x - 14$ **(ii)** $y = 4x^2 - 4x - 3$

(iii) $y = 6x^2 + 7x - 5$ **(iv)** $y = 14x^2 + 15x - 9$

Question 4

Factorise,

(i) $y = 18x^2 + 27x + 10$ **(ii)** $y = 12x^2 + 56x + 9$

(iii) $y = 24x^2 + 7x - 6$ **(iv)** $y = 13x^2 - 43x - 36$

Question 5

Solve these quadratic equations. Every solution should use the word “either” and the word “or” at the appropriate step.

(i) $2x^2 + 7x + 3 = 0$ **(ii)** $3x^2 + 13x - 10 = 0$

(iii) $4x^2 - 12x - 7 = 0$ **(iv)** $9x^2 - 38x + 8 = 0$

Question 6

Find the roots of the quadratic equation $y = 12x^2 + 23x + 10$

Question 7

Solve the simultaneous equations,

$$y = 3x^2$$

$$y = 2x + 16$$

Show clear algebraic working.

Question 8

The line with equation $y = \frac{1}{2}x - 3$ intersects the parabola with equation $y = x^2 + 8x - 7$ at the points A and B.

Find the coordinates of A and the coordinates of B.

Show clear algebraic working.

Question 9

Work out the coordinates of the points of intersection of $x - y = 3$ and $y^2 + xy = 5$. Show clear algebraic working.

6.4 Answers

6.4.1 Solution to Example #1

$$x^2 - 2x - 3 = 0$$

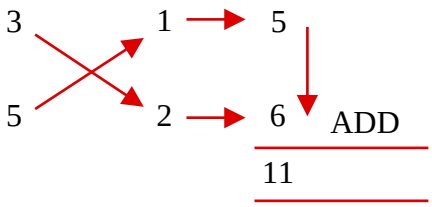
$$(x + 1)(x - 3) = 0$$

$$\text{Either} \quad x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = -1 \quad \text{or} \quad x = 3$$

6.4.2 Solution to Example #2

$$y = 15x^2 + 11x + 2$$

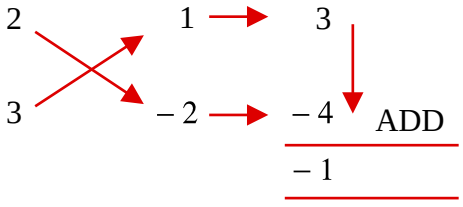


$$y = (3x + 1)(5x + 2)$$

6.4.3 Solution to Example #3

$$y = 6x^2 - x - 2$$

$$6x^2 - x - 2 = 0$$



$$(2x + 1)(3x - 2) = 0$$

$$\text{Either} \quad 2x + 1 = 0 \quad \text{or} \quad 3x - 2 = 0$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = \frac{2}{3}$$

6.4.4 Solutions to 6.3 Exercise

Answer 1

(i) $y = 2x^2 + 13x + 15$

$y = (2x + 3)(x + 5)$

(ii) $y = 9x^2 + 24x + 7$

$y = (3x + 1)(3x + 7)$

(iii) $y = 7x^2 + 23x + 6$

$y = (7x + 2)(x + 3)$

(iv) $y = 6x^2 + 25x + 25$

$y = (3x + 5)(2x + 5)$

Answer 2

(i) $y = 10x^2 - 9x + 2$

$y = (2x - 1)(5x - 2)$

(ii) $y = 3x^2 - 7x + 4$

$y = (3x - 4)(x - 1)$

(iii) $y = 6x^2 - 13x + 5$

$y = (3x - 5)(2x - 1)$

(iv) $y = 3x^2 - 20x + 33$

$y = (3x - 11)(x - 3)$

Answer 3

(i) $y = 5x^2 - 33x - 14$

$y = (5x + 2)(x - 7)$

(ii) $y = 4x^2 - 4x - 3$

$y = (2x + 1)(2x - 3)$

(iii) $y = 6x^2 + 7x - 5$

$y = (2x - 1)(3x + 5)$

(iv) $y = 14x^2 + 15x - 9$

$y = (7x - 3)(2x + 3)$

Answer 4

(i) $y = 18x^2 + 27x + 10$

$y = (3x + 2)(6x + 5)$

(ii) $y = 12x^2 + 56x + 9$

$y = (6x + 1)(2x + 9)$

(iii) $y = 24x^2 + 7x - 6$

$y = (8x - 3)(3x + 2)$

(iv) $y = 13x^2 - 43x - 36$

$y = (13x + 9)(x - 4)$

Answer 5

(i) $2x^2 + 7x + 3 = 0$

$(2x + 1)(x + 3) = 0$

Either $x = -\frac{1}{2}$ or $x = -3$

(ii) $3x^2 + 13x - 10 = 0$

$(3x - 2)(x + 5) = 0$

Either $x = \frac{2}{3}$ or $x = -5$

(iii) $4x^2 - 12x - 7 = 0$

$(2x + 1)(2x - 7) = 0$

Either $x = -\frac{1}{2}$ or $x = \frac{7}{2}$

(iv) $9x^2 - 38x + 8 = 0$

$(9x - 2)(x - 4) = 0$

Either $x = \frac{2}{9}$ or $x = 4$

Answer 6

To find the roots, set y equal to zero,

$$12x^2 + 23x + 10 = 0$$

$$(3x + 2)(4x + 5) = 0$$

$$\text{Either } x = -\frac{2}{3} \text{ or } x = -\frac{5}{4}$$

Answer 7

$$3x^2 = 2x + 16$$

$$3x^2 - 2x - 16 = 0$$

$$(x + 2)(3x - 8) = 0$$

$$\text{Either } x = -2 \text{ or } x = \frac{8}{3}$$

Answer 8

$$x^2 + 8x - 7 = \frac{1}{2}x - 3$$

$$2x^2 + 16x - 14 = x - 6$$

$$2x^2 + 15x - 8 = 0$$

$$(2x - 1)(x + 8) = 0$$

$$\text{Either } x = \frac{1}{2} \text{ or } x = -\frac{1}{8}$$

$$\text{Points are } \left(\frac{1}{2}, -\frac{11}{4}\right), \left(-\frac{1}{8}, -\frac{49}{16}\right)$$

Answer 9

$$x - y = 3 \Leftrightarrow x = y + 3$$

$$y^2 + (y + 3)y = 5$$

$$y^2 + y^2 + 3y = 5$$

$$2y^2 + 3y - 5 = 0$$

$$(y - 1)(2y + 5) = 0$$

$$\text{Either } y = 1 \quad \text{or } y = -\frac{5}{2}$$

$$\text{Points are } (4, 1) \text{ or } \left(\frac{1}{2}, -\frac{5}{2}\right)$$

Lesson 7

GCSE Mathematics

Simultaneous Equations III

7.1 Exam Bits

This is a collection of examination questions that involve the five topics we've been looking at. Often a question is testing just one of the five.

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

7.2 Exercise

Question 1

GCSE, May 2008, paper 4H, Q1

(a) Solve

$$6x + 13 = 2x + 7$$

[3 marks]

(b) Solve

$$\frac{y}{5} - 2 = 4$$

[2 marks]

Question 2

GCSE, November 2006, paper 3H, Q7

Solve the inequality

$$9x - 2 < 5x + 4$$

[3 marks]

Question 3

GCSE, June 2010, paper 4H, Q4

(a) Multiply out

$$5(n + 6)$$

[1 marks]

(b) Simplify

$$y \times y \times y \times y \times y \times y$$

[1 marks]

(c) Solve

$$4(x - 2) = 3$$

[3 marks]

Question 4

GCSE, May 2007, paper 3H, Q6

(a) Expand and simplify

$$3(4x - 5) - 4(2x + 1)$$

[2 marks]

(b) Expand and simplify

$$(y + 8)(y + 3)$$

[2 marks]

(c) Expand

$$p\left(5p^2 + 4\right)$$

[2 marks]

Question 5

GCSE, May 2007, paper 3H, Q9

(a) Solve

$$5x - 4 = 2x + 7$$

[2 marks]

(b) Solve

$$\frac{7 - 2y}{4} = 2y + 3$$

[4 marks]

Question 6

GCSE, November 2010, paper 4H, Q20

Solve the simultaneous equations

$$y = x^2$$

$$y = 7x - 10$$

[5 marks]

Question 7

GCSE, June 2010, paper 4H, Q1

Solve

$$6y - 9 = 3y + 7$$

[3 marks]

Question 8

GCSE, June 2010, paper 4H, Q13(a)

Solve

$$x^2 - 8x + 12 = 0$$

[3 marks]

Question 9

GCSE, November 2010, paper 3H, Q13(a)

Factorise

$$x^2 - 8x + 15$$

[2 marks]

Question 10

GCSE, June 2009, paper 4H, Q16

(a) Factorise

$$2x^2 - x - 3$$

[2 marks]

(b) Hence write down the solutions of

$$2x^2 - x - 3 = 0$$

[1 mark]

Question 11

GCSE, November 2009, paper 3H, Q2

Solve

$$8y - 9 = 5y + 3$$

[3 marks]

Question 12

GCSE, November 2009, paper 3H, Q9

(a) Expand and simplify fully

$$2(w - 3) + 3(w + 5)$$

[2 marks]

(b) Solve the equation

$$\frac{x + 5}{3} = 9$$

[2 marks]

(c) Solve the inequality

$$5y + 7 < 13$$

[2 marks]

Question 13

GCSE, November 2009, paper 4H, Q12(a)

Expand and simplify

$$(p + 7)(p - 4)$$

[2 marks]

Question 14

GCSE, November 2009, paper 4H, Q2

(a) Factorise

$$n^2 - 4n$$

[2 marks]

(b) Solve

$$8 - 5x = 2$$

[3 marks]

Question 15

GCSE, May 2009, paper 3H, Q5

(a) Factorise

$$p^2 + 7p$$

[2 marks]

(b) Solve

$$4 - 5x = 2$$

[3 marks]

(c) Simplify

$$t^3 \times t^6$$

[1 mark]

(d) Expand and simplify

$$3(4y + 5) - 5(2y + 3)$$

[2 marks]

Question 16

GCSE, November 2008, paper 4H, Q17(a)

Factorise

$$2x^2 + 5x + 3$$

[2 marks]

Question 17

GCSE, November 2008, paper 4H, Q6

(a) Multiply out

$$5(x - 2)$$

[2 marks]

(b) Solve the equation

$$\frac{x}{4} + 3 = 10$$

[2 marks]

(c) Solve the inequality

$$5x - 6 > 2$$

[2 marks]

Question 18

GCSE, November 2008, paper 4H, Q6

Solve

$$5(x - 4) = 35$$

[3 marks]

Question 19

GCSE, November 2007, paper 4H, Q2

(a) Factorise

$$5x - 20$$

[1 mark]

(b) Factorise

$$y^2 + 6y$$

[2 marks]

Question 20

GCSE, May 2006, paper 3H, Q13(a)

Expand and simplify

$$(3x - 5)(4x + 7)$$

[2 marks]

Question 21

GCSE, May 2006, paper 4H, Q2

(a) Factorise

$$3x^2 - 2x$$

[1 mark]

(b) Expand

$$y^3(y - 4)$$

[2 marks]

(c) Here is a formula used in physics

$$v = u + at$$

Find the value of t when $v = 30$, $u = 5$ and $a = 10$

[2 marks]

Question 22

GCSE, May 2006, paper 4H, Q8

(a) Solve

$$3\,(x + 4) = 27$$

[3 marks]

(b) Solve

$$y^2 - 2y - 120 = 0$$

[3 marks]

Question 23

GCSE, May 2006, paper 4H, Q12(a)

Factorise

$$3x^2 - 13x + 4$$

[2 marks]

Question 24

GCSE, May 2006, paper 4H, Q17

Solve the simultaneous equations;

$$y = 2x + 1$$
$$x^2 + y^2 = 13$$

[6 marks }

7.3 Answers

GCSE Mathematics
Simultaneous Equations III

7.3.1 Solutions (7.2 Exercise)

Answer 1
(a) $x = -1.5$
(b) $y = 30$

Answer 2
$x < 1.5$

Answer 3
(a) $5n + 30$
(b) y^6
(c) 2.75

Answer 4
(a) $4x - 19$
(b) $y^2 + 11y + 24$
(c) $5p^3 + 4p$

Answer 5
(a) $3\frac{2}{3}$
(b) -0.5

Answer 6
$(2,\ 4),\ (5,\ 25)$

Answer 7
$5\frac{1}{3}$

Answer 8
$x = 2,\ x = 6$

Answer 9
$(x-3)(x-5)$

Answer 10
(a) $(2x-3)(x+1)$
(b) $x = 1,\ x = \frac{3}{2}$

Answer 11
$y = 4$

Answer 12
(a) $5w + 9$
(b) $x = 22$
(c) $y < 1.2$

Answer 13
$p^2 + 3p - 28$

Answer 14
(a) $n(n-4)$
(b) $x = 1.2$

Answer 15
(a) $p(p+7)$
(b) $x = 0.4$
(c) t^9
(d) $2y$

Answer 16
$(2x+3)(x+1)$

Answer 17
(a) $5x - 10$
(b) $x = 28$
(c) $x > 1.6$

Answer 18
$x = 11$

Answer 19
(a) $5(x-4)$
(b) $y(y+6)$

Answer 20
$12x^2 + x - 35$

Answer 21
(a) $x(3x-2)$
(b) $y^4 - 4y^3$
(c) $t = 2.5$

Answer 22
(a) $x = 5$
(b) $y = -10,\ y = 12$

Answer 23
$(3x-1)(x-4)$

Answer 24

$$y = 2x + 1$$

$$x^2 + y^2 = 13$$

Substitute the equation of the line into the equation of the circle to get,

$$x^2 + (2x + 1)^2 = 13$$

$$x^2 + 4x^2 + 4x + 1 = 13$$

$$5x^2 + 4x - 12 = 0$$

$$x^2 + \frac{4}{5}x - \frac{12}{5} = 0$$

A	B
$-\frac{6}{5}$	$2 = \frac{10}{5}$

<===

$$\left(x - \frac{6}{5}\right)(x + 2) = 0$$

$$\text{Either } x = \frac{6}{5} \text{ or } x = -2$$

Now use $y = 2x + 1$ to get $(-2, -3)$ $(1.2, 3.4)$

Lesson 8

GCSE Mathematics

Simultaneous Equations III

8.1 T E S T

- Expanding brackets, FOIL
- Gathering together like terms
- Rearranging equations into the form $f(x) = 0$
- Factorising quadratics
- Solving quadratic equations

You may use a calculator

95 marks in total are available

Question 1

Find all solutions, if any, to the following equations,

(i) $5x + 13 = -27$

[2 marks]

(ii) $\frac{x}{4} = 0.625$

[2 marks]

(iii) $\frac{3x}{7} + 8 = 14$

[2 marks]

(iv) $4(x + 7) = 44$

[2 marks]

(v) $x^2 = 25$

[2 marks]

(vi) $\sqrt{x} - 1 = 2$

[2 marks]

Question 2

GCSE, June 2011, paper 3H, Q3

(a) Factorise

$$w^2 - 9w$$

[2 marks]

(b) Solve

$$5x - 1 = 2x - 7$$

[3 marks]

(c) Expand and simplify

$$(y - 7)(y + 3)$$

[2 marks]

Question 3

GCSE, June 2011, paper 4H, Q5

(a) Simplify

(i) $a \times a \times a \times a$

[1 mark]

(ii) $5a \times 6b$

[1 mark]

(iii) $q^8 \times q^2$

[1 mark }

(b) Solve

$$5 - 2y = 12$$

[2 marks]

(c) Given that, $v = w^2 - 2w$

Work out the value of v when $w = 6$

[2 marks]

Question 4

GCSE, May 2008, paper 3H, Q9

(a) Solve

$$7(x - 1) = 5 - 2x$$

[3 marks]

(b) **(i)** Solve the inequality

$$4x + 5 \leqslant 21$$

[2 marks]

(ii) n is a positive integer.
Write down all the values of n which satisfy

$$4n + 5 \leqslant 21$$

[2 marks]

Question 5

GCSE, November 2007, paper 3H, Q9

(a) Expand

$$4(v + 3)$$

[1 mark]

(b) Simplify

$$\frac{w^3 \times w^7}{w^2}$$

[2 marks]

(c) Solve the equation

$$\frac{17 - x}{7} = 3$$

[3 marks]

(d) Solve the inequality

$$4y - 5 < 6$$

[2 marks]

Question 6

GCSE, May 2008, paper 3H, Q14

(a) Factorise

$$10y - 15$$

[1 mark]

(b) Factorise completely

$$9p^2q + 12pq^2$$

[2 marks]

(c) **(i)** Factorise

$$x^2 + 6x - 16$$

[2 marks]

(ii) Solve

$$x^2 + 6x - 16 = 0$$

[1 mark]

Question 7

GCSE, June 2011, paper 4H, Q12

(i) Solve the inequality

$$2x + 13 \geqslant 6$$

[2 marks]

(ii) n is a **negative** integer.

Write down all the values of n which satisfy

$$2n + 13 \geqslant 6$$

[2 marks]

Question 8

Expand the brackets and simplify

$$(x - 5)(x + 7)$$

[2 marks]

Question 9

Expand the brackets and simplify

$$(3x + 5) (2x + 1)$$

[2 marks]

Question 10

GCSE, November 2007, paper 3H, Q13 (b) (c)

Factorise

(a)

$$x^2 - x - 12$$

[2 marks]

(b)

$$3x^2 + 7x + 2$$

[2 marks }

Question 11

Solve the simultaneous equations

$$y = x^2$$

$$y = 5x + 36$$

Show clear algebraic working.

[5 marks]

Question 12

GCSE, June 2011, paper 4H, Q15 (a)

Simplify, $\dfrac{8\,(x-3)^2}{4\,(x-3)}$

[2 marks]

Question 13

GCSE, November 2007, paper 3H, Q16 (a)

Simplify, $\dfrac{x^2-3x}{2x-6}$

[3 marks]

Question 14

Factorise,

(i) $y = 5x^2 + 12x + 4$ **(ii)** $y = 21x^2 - 10x + 1$

[4 marks]

Question 15

By first factorising, solve the following quadratic equations,

(i) $6x^2 + 11x - 10 = 0$ **(ii)** $12x^2 - 16x - 3 = 0$

[8 marks]

Question 16

The line $2y = x + 9$ intersect the curve $3x^2 + xy = 5$ at the points A and B.
Find the coordinates of A and the coordinates of B.
Show clear algebraic working.

[6 marks]

Question 17

IGCSE Examination Question from January 2021, Paper 1HR, Question 17 (Edexcel)
Solve the simultaneous equations,

$$x - 6y = 5$$

$$xy - 2y^2 = 6$$

Show clear algebraic working.

[5 marks]

Question 18

GCSE, June 2011, paper 3H, Q22

Solve the simultaneous equations

$$y = 2x - 3$$
$$x^2 + y^2 = 2$$

[6 marks]

8.2 Answers

8.2.1 Solutions (8.1 Test)

Answer 1

(i) $x = -8$
(ii) $x = 2.5$
(iii) $x = 14$
(iv) $x = 4$
(v) $x = -5, 5$
(vi) $x = 9$

Answer 2

(a) $w(w - 9)$
(b) $x = -2$
(c) $y^2 - 4y - 21$

Answer 3

(a) **(i)** a^4
(ii) $30ab$
(iii) q^{10}
(b) $y = -3.5$
(c) $v = 24$

Answer 4

(a) $x = \frac{4}{3}$

(b) **(i)** $x \leqslant 4$
(ii) 1, 2, 3, 4

Answer 5

(a) $4v + 12$

(b) w^8
(c) $x = -4$
(d) $y < 2.75$

Answer 6

(a) $5(2y - 3)$
(b) $3pq(3p + 4q)$
(c) **(i)** $(x + 8)(x - 2)$
(ii) $x = -8, 2$

Answer 7

(i) $x \geqslant -3.5$
(ii) $-3, -2, -1$

Answer 8

$x^2 + 2x - 35$

Answer 9

$6x^2 + 13x + 5$

Answer 10

(a) $(x - 4)(x + 3)$
(b) $(3x + 1)(x + 2)$

Answer 11

$x^2 - 5x - 36 = 0$
gives $(9, 81)$ or $(-4, 16)$

Answer 12

$2(x - 3)$

Answer 13

$\frac{x}{2}$

Answer 14

(i) $y = 5x^2 + 12x + 4$
$y = (5x + 2)(x + 2)$

(ii) $y = 21x^2 - 10x + 1$
$y = (3x - 1)(7x - 1)$

Answer 15

(i) $6x^2 + 11x - 10 = 0$

$(2x + 5)(3x - 2) = 0$

Either $x = -\frac{5}{2}$ or $x = \frac{2}{3}$

(ii) $12x^2 - 16x - 3 = 0$

$(2x - 3)(6x + 1) = 0$

Either $x = \frac{3}{2}$ or $x = -\frac{1}{6}$

Answer 16

$$3x^2 + xy = 5 \Leftrightarrow 6x^2 + 2xy = 10$$

$$6x^2 + x(x + 9) = 10$$

$$6x^2 + x^2 + 9x = 10$$

$$7x^2 + 9x - 10 = 0$$

$$(x + 2)(7x - 5) = 0$$

Either $x = -2$ or $x = \frac{5}{7}$

Points are $\left(-2, \frac{7}{2}\right), \left(\frac{5}{7}, \frac{34}{7}\right)$

Answer 17

$$x - 6y = 5 \Leftrightarrow x = 6y + 5$$

$$(6y + 5)y - 2y^2 = 6$$

$$6y^2 + 5y - 2y^2 = 6$$

$$4y^2 + 5y - 6 = 0$$

$$(y + 2)(4y - 3) = 0$$

Either $y = -2$ or $y = \frac{3}{4}$

Points are $(-7, -2), \left(\frac{19}{2}, \frac{3}{4}\right)$

Answer 18

$5x^2 - 12x + 7 = 0$

gives $(1.4, -0.2)$ or $(1, -1)$