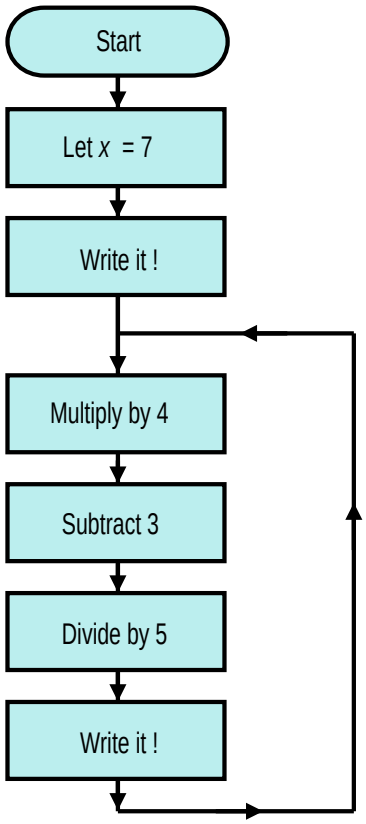
Lesson 6

Numerical Methods : Year 2

6.1 TEST

Question 1



(i) List the first ten numbers produced by the iteration described in the flow chart.

[3 marks]

(ii) This iteration is heading towards a fixed value.
What do you think the fixed value is ?

[1 mark]

(iii) What happens if you start the iteration using the fixed value ?

[1 mark]

(iv) The part **(iii)** method of verifying the fixed value only works for an iteration with a fixed value which is a rational number.
What is a rational number ?

[1 mark]

Question 2

A sequence, G, has the iteration formula;

$$G_{n+1} = \sqrt{\frac{13}{G_n}} \qquad with \qquad G_1 = 13$$

(i) Calculate the values of G_2, G_3, G_4 ..... G_{12} writing each to 3 decimal places.

[3 marks]

(ii) The fixed value is $(13)^{\frac{1}{3}}$.

Prove this by algebraically manipulating the equation

$$x = \sqrt{\frac{13}{x}}$$

into the form

$$x = \sqrt[3]{13}$$

[3 marks]

Question 3

This question is about the function

$$f(x) = e^x - \frac{1}{x}$$

(i) On one graph sketch the curves $y = e^x$ and $y = \frac{1}{x}$
Indicate where your sketch shows that $f(x) = 0$ has a root.

[3 marks]

(ii) Calculate **(a)** $f(0.5)$
(b) $f(0.6)$

[2 marks]

(iii) How do the part **(ii)** answers prove that there is a root to the equation $f(x) = 0$ in the interval $0.5 \leqslant x \leqslant 0.6$?

[1 mark]

(iv) Show that $f(x) = 0$, implies $x = e^{-x}$

[2 marks]

(v) Using the iterative formula

$$x_{n+1} = e^{-x_n} \qquad with \qquad x_0 = 0.55$$

to find x_1, x_2 and x_3

[2 marks]

(vi) Keep iterating, finally stating the root of $f(x) = 0$ to about 8 decimal places.

[1 mark]

Question 4

C3 Examination question from January 2006, Q5.

$$f(x) = 2x^3 - x - 4$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$$

[3 marks]

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)}$$

with $x_o = 1.35$, to find, to 2 decimal places, the value of x_1, x_2 and x_3.

[3 marks]

The only real root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.

[3 marks]

Question 5

C3 Examination question from June 2011, Q2.

$$f(x) = 2\sin(x^2) + x - 2. \qquad 0 \leqslant x < 2\pi$$

(a) Show that $f(x) = 0$ has a root α between $x = 0.75$ and $x = 0.85$.

[2 marks]

The equation $f(x) = 0$ can be written as $x = [\arcsin(1 - 0.5x)]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = [\arcsin(1 - 0.5x)]^{\frac{1}{2}}, \qquad x_0 = 0.8$$

to find the values of x_1, x_2 and x_3, giving your answers to 5 decimal places.

[3 marks]

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

[3 marks]

Question 6

C3 Examination question from January 2012, Q6.

$$f(x) = x^2 - 3x + 2\cos\left(\frac{1}{2}x\right). \qquad 0 \leqslant x < \pi$$

(a) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$

[2 marks]

The curve with equation $y = f(x)$ has a minimum point P.

(b) Show that the x-coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$$

[4 marks]

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}, \qquad x_0 = 2$$

find the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

[3 marks]

(d) By choosing a suitable interval, show that the x-coordinate of P is 1.9078 correct to 4 decimal places.

[3 marks]