

3.7 Answers

3.7.1 Solutions (3.3 Example)

$$\begin{aligned}f(x) &= \frac{x^3 + 4x^2 + 3}{x} \\&= \frac{x^3}{x} + \frac{4x^2}{x} + \frac{3}{x} \\&= x^2 + 4x + 3x^{-1} \\\therefore f'(x) &= 2x + 4 - 3x^{-2} \\&= 2x + 4 - \frac{3}{x^2}\end{aligned}$$

3.7.2 Solutions (3.6 Exercise)

Answer 1

$$\begin{aligned}\frac{x^2 - 7}{[(x - 2)(x - 3)]} &= A + \frac{B}{(x - 2)} + \frac{C}{(x - 3)} \\x^2 - 7 &= A(x - 2)(x - 3) + B(x - 3) + C(x - 2) \\ \text{Let } x = 2 : \quad -3 &= -B & \therefore B = 3 \\ \text{Let } x = 3 : \quad 2 &= C & \therefore C = 2 \\ \text{Let } x = 0 : \quad -7 &= 6A - 3B - 2C \\ -7 &= 6A - 9 - 4 & \therefore A = 1\end{aligned}$$

Thus

$$\frac{x^2 - 7}{(x - 2)(x - 3)} = 1 + \frac{3}{(x - 2)} + \frac{2}{(x - 3)}$$

Answer 2

$$\frac{2x^3 + 5}{[(x + 1)(x + 2)]} = Ax + B + \frac{C}{(x + 1)} + \frac{D}{(x + 2)}$$

$$2x^3 + 5 = (Ax + B)(x + 1)(x + 2) + C(x + 2) + D(x + 1)$$

$$\text{Let } x = -2 \quad -11 = -D \quad \therefore D = 11$$

$$\text{Let } x = -1 \quad 3 = C \quad \therefore C = 3$$

$$\text{Let } x = 0 \quad 5 = 2B + 2C + D$$

$$5 = 2B + 6 + 11 \quad \therefore B = -6$$

$$\text{Let } x = 1 \quad 7 = (A + B)6 + 3C + 2D$$

$$7 = 6A - 36 + 9 + 22$$

$$12 = 6A \quad \therefore A = 2$$

Thus

$$\frac{2x^3 + 5}{(x + 1)(x + 2)} = 2x - 6 + \frac{3}{(x + 1)} + \frac{11}{(x + 2)}$$

Answer 3

$$\frac{x^3 + 12x^2 + 3}{(x - 1)(x + 3)} = x + 10 + \frac{4}{(x - 1)} - \frac{21}{(x + 3)}$$

Answer 4

$$\frac{x^2 + 8x + 4}{(x + 5)(2x - 1)} = \frac{1}{2} + \frac{1}{(x + 5)} + \frac{3}{2(2x - 1)}$$