

4.4 Answers

4.4.1 Solutions (4.2 Example)

$$\frac{8x + 7}{[(x^2 + x + 1)(x + 2)]} = \frac{Ax + B}{(x^2 + x + 1)} + \frac{C}{(x + 2)}$$
$$8x + 7 = (Ax + B)(x + 2) + C(x^2 + x + 1)$$

$$\text{Let } x = -2 \quad -9 = 3C \quad \therefore C = -3$$

$$\text{Let } x = 0 \quad 7 = 2B + C$$
$$7 = 2B - 3 \quad \therefore B = 5$$

$$\text{Let } x = 1 \quad 15 = (A + 5)3 + (-3)(3)$$
$$24 = 3A + 15 \quad \therefore A = 3$$

Thus,

$$\frac{8x + 7}{(x^2 + x + 1)(x + 2)} = \frac{3x + 5}{(x^2 + x + 1)} - \frac{3}{(x + 2)}$$

4.4.2 Solutions (4.3 Exercise)

Answer 1

$$\frac{3(x + 2)}{[(2x - 1)(x^2 + 1)]} = \frac{A}{(2x - 1)} + \frac{Bx + C}{(x^2 + 1)}$$
$$3x + 6 = A(x^2 + 1) + (Bx + C)(2x - 1)$$

$$\text{Let } x = \frac{1}{2} \quad \frac{3}{2} + 6 = A\left(\frac{1}{4} + 1\right)$$
$$\frac{15}{2} = A\frac{5}{4} \quad \therefore A = 6$$

$$\text{Let } x = 0 \quad 6 = A - C \quad \therefore C = 0$$

$$\text{Let } x = 1 \quad 9 = 2A + B \quad \therefore B = -3$$

Thus,

$$\frac{3(x + 2)}{(2x - 1)(x^2 + 1)} = \frac{6}{(2x - 1)} - \frac{3x}{(x^2 + 1)}$$

Answer 2

$$\frac{6}{[(x^2 + 2)(1 - x)]} = \frac{Ax + B}{(x^2 + 2)} + \frac{C}{(1 - x)}$$

$$6 = (Ax + B)(1 - x) + C(x^2 + 2)$$

$$\text{Let } x = 1 \quad 6 = 3C \quad \therefore C = 2$$

$$\text{Let } x = 0 \quad 6 = B + 2C \quad \therefore B = 2$$

$$\text{Let } x = -1 \quad 6 = (-A + 2)2 + 6 \quad \therefore A = 2$$

Thus,

$$\frac{6}{(x^2 + 2)(1 - x)} = \frac{2x + 2}{(x^2 + 2)} + \frac{2}{(1 - x)}$$

Answer 3

$$\frac{x^2 + 6x + 4}{[x(x^2 - 2)]} = \frac{A}{x} + \frac{Bx + C}{x^2 - 2}$$

$$x^2 + 6x + 4 = A(x^2 - 2) + (Bx + C)x$$

$$\text{Let } x = 0 \quad 4 = -2A \quad \therefore A = -2$$

$$\text{Let } x = 1 \quad 11 = 2 + B + C \quad \therefore B + C = 9$$

$$\text{Let } x = -1 \quad (-1) = 2 + B - C \quad \therefore B - C = -3$$

$$2B = 6 \quad \therefore B = 3$$

$$\therefore C = 6$$

Thus,

$$\frac{x^2 + 6x + 4}{x(x^2 - 2)} = -\frac{2}{x} + \frac{3x + 6}{x^2 - 2}$$

Answer 4

$$\frac{x^2 + 6x + 4}{x(x - \sqrt{2})(x + \sqrt{2})} = -\frac{2}{x} + \frac{3(1 + \sqrt{2})}{2(x - \sqrt{2})} + \frac{3(1 - \sqrt{2})}{2(x + \sqrt{2})}$$