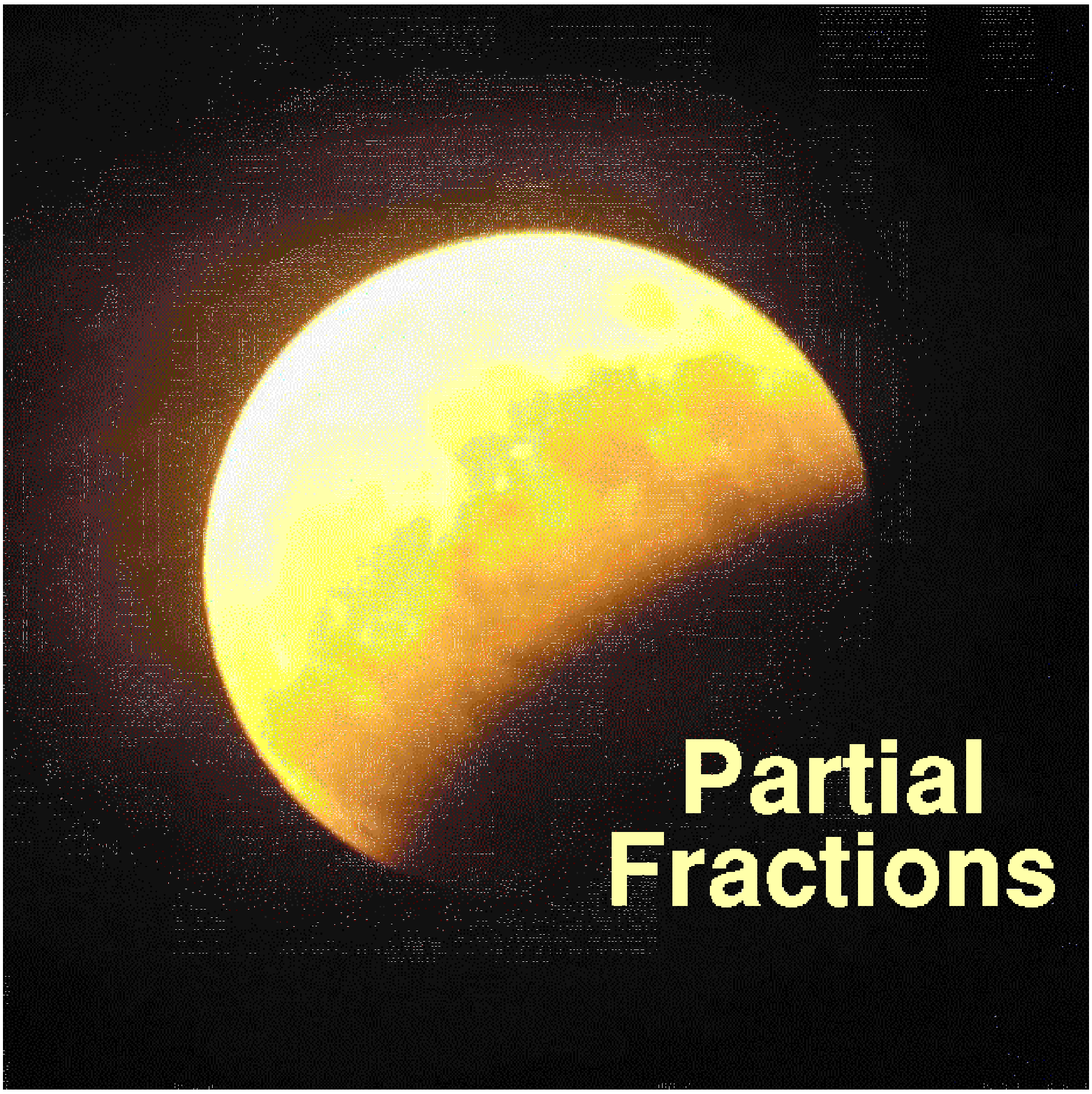
A-Level Pure Mathematics
Year 2

P A R T I A L

F R A C T I O N S



P A R T I A L F R A C T I O N S
Algebra Skills

Lesson 1

Partial Fractions : Pure Year 2

1.1 Multiplying By 1

Adding vulgar fractions is a skill mastered long ago.
However, let's just look at exactly why we do what we do.

The key idea is that multiplying by 1 is a 'no change' operation.
The number 1 can come in many forms, such as;

$$1 = \frac{5}{5} \qquad \textit{or} \qquad 1 = \frac{11}{11}$$

With that in mind, here is the full detail of how to add a couple of fractions.
The driving strategy is to get a common denominator.

$$\begin{aligned}
\frac{4}{11} + \frac{2}{5} &= \frac{4}{11} \times 1 + \frac{2}{5} \times 1 \\
&= \frac{4}{11} \times \frac{5}{5} + \frac{2}{5} \times \frac{11}{11} \\
&= \frac{4 \times 5}{55} + \frac{2 \times 11}{55} \\
&= \frac{4 \times 5 + 2 \times 11}{55} \\
&= \frac{20 + 22}{55} \\
&= \frac{42}{55}
\end{aligned}$$

The topic of Partial Fractions is about reversing this operation.

So, in the following, the task would be to find the value of the integers A and B.

$$\frac{42}{55} = \frac{A}{11} + \frac{B}{5}$$

You would know to split the denominator of 55 up into 11 and 5 because, when written as a product of primes, $55 = 11 \times 5$.
After multiplying by 55 you'd have the fact that

$$42 = 5A + 11B$$

Solving this in integers would require some 'trial and improvement' but the solution, that $A = 4$ and $B = 2$ is soon found.

1.2 Algebraic Fractions

For GCSE, adding algebraic fractions was tackled as an A/A* grade topic.
A typical question would be to write the following as a single fraction;

$$\frac{3}{(x-4)} - \frac{2}{(x-1)}$$

Again, the key idea is that multiplying by 1 is a 'no change' operation with the number 1 this time being in the forms;

$$1 = \frac{(x-1)}{(x-1)} \qquad \textit{and} \qquad 1 = \frac{(x-4)}{(x-4)}$$

Here is the detail;

$$\begin{aligned}
\frac{3}{(x-4)} - \frac{2}{(x-1)} &= \frac{3}{(x-4)} \times 1 - \frac{2}{(x-1)} \times 1 \\
&= \frac{3}{(x-4)} \times \frac{(x-1)}{(x-1)} - \frac{2}{(x-1)} \times \frac{(x-4)}{(x-4)} \\
&= \frac{3(x-1)}{(x-4)(x-1)} - \frac{2(x-4)}{(x-1)(x-4)} \\
&= \frac{3(x-1) - 2(x-4)}{(x-4)(x-1)} \\
&= \frac{3x - 3 - 2x + 8}{(x-4)(x-1)} \\
&= \frac{x+5}{(x-4)(x-1)}
\end{aligned}$$

1.3 Partial Fractions Exam Style Question

Question : Write the following as partial fractions.

$$\frac{x+5}{(x-4)(x-1)}$$

From section 1.2 we know what the answer is going to be.

That is;

$$\frac{x+5}{(x-4)(x-1)} = \frac{3}{(x-4)} - \frac{2}{(x-1)}$$

But we need to develop a method of obtaining this without such prior knowledge.

1.4 The Square Bracket Method*

We begin by making an assumption about the format of the answer.
Thus;

$$\frac{x+5}{(x-4)(x-1)} = \frac{A}{(x-4)} + \frac{B}{(x-1)}$$

Put square brackets around the question's denominator.

$$\frac{x+5}{[(x-4)(x-1)]} = \frac{A}{(x-4)} + \frac{B}{(x-1)}$$

We're going to multiply through by the square bracket.
However, to lessen the clutter, we're not going to actually write the contents of the square brackets in each time.

$$\frac{(x+5)\,[\,\ldots\ldots\,]}{[(x-4)(x-1)]} = \frac{A\,[\,\ldots\ldots\,]}{(x-4)} + \frac{B\,[\,\ldots\ldots\,]}{(x-1)}$$

With 'virtual cancelling' this becomes;

$$x+5 = A(x-1) + B(x-4)$$

And this must be true for all values of *x*.

Let $x = 1$: $6 = -3B$ $\therefore B = -2$

Let $x = 4$: $9 = 3A$ $\therefore A = 3$

And so;

$$\frac{x+5}{(x-4)(x-1)} = \frac{3}{(x-4)} - \frac{2}{(x-1)}$$

1.5 Exercise

Question 1

Write the following as partial fractions.

$$\frac{x+7}{(x-5)(x+1)}$$

Question 2

Write the following as partial fractions.

$$\frac{3x+11}{x^2+6x+5}$$

Question 3
Write the following as partial fractions.

$$\frac{3(3x + 1)}{x^2 - 9}$$

HINT : Difference of two squares.

* This is a method of my own devising.

I find it preferable to 'the cover up rule' as students continue to have an understanding of why it works which is not typically the case with the rival method

1.6 Exercise

Question 1

Write as a single fraction;

$$\frac{3}{(x-2)} + \frac{4}{(x+5)}$$

Question 2

Write as a single fraction;

$$\frac{8}{(x-5)} - \frac{3}{(x+2)}$$

Question 3

Write the following as partial fractions;

$$\frac{3x^2 - 19x + 24}{(x-1)(x-2)(x-3)}$$

Question 4
Write the following as partial fractions;

$$\frac{10}{3x^3 - x^2 - 2x}$$

Question 5

Write the following as partial fractions;

$$\frac{3x}{(x-1)(x+2)(2x-1)}$$

Puzzles
(not too hard)

Question 6

$$\frac{16}{21} = \frac{A}{3} + \frac{B}{7}$$

Find the value of the integers A and B.

Question 7

$$\frac{7}{15} = \frac{A}{3} + \frac{B}{5}$$

Find the value of the integers A and B.

1.7 Answers

1.7.1 Solutions (1.5 Exercise)

Answer 1

$$\frac{x+7}{(x-5)(x+1)} = \frac{2}{(x-5)} - \frac{1}{(x+1)}$$

Answer 2

$$\frac{3x+11}{x^2+6x+5} = \frac{1}{(x+5)} + \frac{2}{(x+1)}$$

Answer 3

$$\frac{3(3x+1)}{x^2-9} = \frac{5}{(x-3)} + \frac{4}{(x+3)}$$

1.7.2 Solutions (1.6 Exercise)

Answer 1

$$\frac{3}{(x-2)} + \frac{4}{(x+5)} = \frac{7(x+1)}{(x-2)(x+5)}$$

Answer 2

$$\frac{8}{(x-5)} - \frac{3}{(x+2)} = \frac{5x+31}{(x-5)(x+2)}$$

Answer 3

$$\frac{3x^2-19x+24}{(x-1)(x-2)(x-3)} = \frac{4}{(x-1)} + \frac{2}{(x-2)} - \frac{3}{(x-3)}$$

Answer 4

$$\frac{10}{3x^3-x^2-2x} = -\frac{5}{x} + \frac{9}{(3x+2)} + \frac{2}{(x-1)}$$

Answer 5

$$\frac{3x}{(x-1)(x+2)(2x-1)} = \frac{1}{(x-1)} - \frac{2}{5(x+2)} - \frac{6}{5(2x-1)}$$

Answer 6

$$\frac{16}{21} = \frac{1}{3} + \frac{3}{7}$$

Answer 7

$$\frac{7}{15} = \frac{2}{3} - \frac{1}{5}$$

Lesson 2

2.1 The Format Of The Answer

Previously, it was shown that

$$\frac{x+5}{(x-4)(x-1)} = \frac{3}{(x-4)} - \frac{2}{(x-1)}$$

In obtaining the Right Hand Side it was necessary to have some idea as to the form of the answer.
The form of the answer is not always as obvious as you might at first think.

2.2 A Repeated Factor

Spot the pattern ....

$$\frac{x+1}{(x+3)^4} = \frac{A}{(x+3)} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3} + \frac{D}{(x+3)^4}$$

$$\frac{x+1}{(x+3)^3} = \frac{A}{(x+3)} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3}$$

$$\frac{x+1}{(x+3)^2} =$$

2.3 Exercise

Question 1

Write the following as partial fractions;

$$\frac{x+1}{(x+3)^2}$$

Question 2

Write the following as partial fractions;

$$\frac{2x^2 - 5x + 7}{(x-2)(x-1)^2}$$

Question 3

Write the following as partial fractions;

$$\frac{x^2 - 20x - 11}{(x+1)^2 (x-4)}$$

Question 4

C4 Examination question from June 2011, Q1.

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$$

Find the values of the constants A, B and C.

[4 marks]

Question 5

Write the following as partial fractions;

$$\frac{2x^2 + 9x + 2}{(x + 5)^3}$$

Question 6

Write as a single fraction;

$$\frac{8}{(x+1)} + \frac{3}{(x+1)^2} + \frac{5}{(x+1)^3}$$

Question 7

Write the following as partial fractions;

$$\frac{8(9x-10)}{x^2(x+4)^2}$$

Puzzle (Not too hard !)

Question 8

$$\frac{17}{18} = \frac{A}{2} + \frac{B}{3} + \frac{C}{9}$$

Find the positive integer value of A, B and C.

2.4 Answers

2.4.1 Solutions (2.3 Exercise)

Answer 1

$$\frac{x+1}{(x+3)^2} = \frac{1}{(x+3)} - \frac{2}{(x+3)^2}$$

Answer 2

$$\frac{2x^2 - 5x + 7}{(x-2)(x-1)^2} = \frac{5}{x-2} - \frac{3}{x-1} - \frac{4}{(x-1)^2}$$

Answer 3

$$\frac{x^2 - 20x - 11}{(x+1)^2(x-4)} = \frac{4}{x+1} - \frac{2}{(x+1)^2} - \frac{3}{x-4}$$

Answer 4

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{4}{(x-1)} + \frac{3}{(x-1)^2} + \frac{1}{(2x+1)}$$

Answer 5

$$\frac{2x^2 + 9x + 2}{(x+5)^3} = \frac{2}{x+5} - \frac{11}{(x+5)^2} + \frac{7}{(x+5)^3}$$

Answer 6

$$\frac{8}{(x+1)} + \frac{3}{(x+1)^2} + \frac{5}{(x+1)^3} = \frac{8x^2 + 19x + 16}{(x+1)^3}$$

Answer 7

$$\frac{8(9x - 10)}{x^2(x+4)^2} = \frac{7}{x} - \frac{5}{x^2} - \frac{7}{x+4} - \frac{23}{(x+4)^2}$$

Answer 8

$$\frac{17}{18} = \frac{1}{2} + \frac{1}{3} + \frac{1}{9}$$

Lesson 3

Partial Fractions : Pure Year 2

3.1 Top Heavy Fractions

In numberwork, a "Top Heavy" fraction is one in which the numerator is of greater magnitude that the denominator.

For example,

$$\frac{14}{3}$$

This can handled in a variety of ways, such as,

$$\frac{14}{3} = 4\,\frac{2}{3} = 4.\dot{6}$$

As far as partial fractions are concerned, in algebra, we'd effectively do this;

$$\frac{14}{3} = 4 + \frac{2}{3}$$

3.2 Recognizing a Top Heavy Algebraic Fraction

A "Top Heavy" algebraic fraction is one in which the degree of the numerator is equal to, or greater than, the degree of the denominator.

Consider,

$$f(x) = \frac{x^3 + 4x^2 + 3}{x}$$

The degree of the numerator is 3.
The degree of the denominator is 1.
$\therefore$ It's "Top Heavy".

3.3 Example of Partial Fractions with a Top Heavy Fraction.

Differentiate $f(x)$.

3.4 Analysis of the Example

The crucial step in the example was the 'wedging' of the fraction.
That is,

$$\frac{x^3 + 4x^2 + 3}{x} = \frac{x^3}{x} + \frac{4x^2}{x} + \frac{3}{x}$$
$$= x^2 + 4x + \frac{3}{x}$$

So, the form of the Partial Fraction was,

$$\frac{x^3 + 4x^2 + 3}{[x]} = Ax^2 + Bx + C + \frac{D}{x}$$

and the Partial Fraction approach would be: Begin by assuming the solution was of that form and continued by multiplying through by $[x]$.
The partial fraction end point is finding that $A = 1$, $B = 4$, $C = 0$ and $D = 3$.
Of course, such a simple question would not be tackled using Partial Fractions !

3.5 The form of Top Heavy Partial Fractions

Spot the pattern ....

$$\frac{x^4 - 7}{(x-2)(x-3)} = Ax^2 + Bx + C + \frac{D}{(x-2)} + \frac{E}{(x-3)}$$

$$\frac{x^3 - 7}{(x-2)(x-3)} = Ax + B + \frac{C}{(x-2)} + \frac{D}{(x-3)}$$

$$\frac{x^2 - 7}{(x-2)(x-3)} =$$

3.6 Exercise

Question 1

Write as partial fractions,

$$\frac{x^2 - 7}{(x-2)(x-3)}$$

Question 2
Write as partial fractions,

$$\frac{2x^3 + 5}{(x + 1)(x + 2)}$$

Question 3

Write as partial fractions,

$$\frac{x^3 + 12x^2 + 3}{(x - 1)(x + 3)}$$

Question 4

Write as partial fractions,

$$\frac{x^2 + 8x + 4}{(x + 5)(2x - 1)}$$

(Expect some answers to be fractions)

3.7 Answers

3.7.1 Solutions (3.3 Example)

$$
\begin{aligned}
f(x) &= \frac{x^3 + 4x^2 + 3}{x} \\
&= \frac{x^3}{x} + \frac{4x^2}{x} + \frac{3}{x} \\
&= x^2 + 4x + 3x^{-1} \\
\therefore f'(x) &= 2x + 4 - 3x^{-2} \\
&= 2x + 4 - \frac{3}{x^2}
\end{aligned}
$$

3.7.2 Solutions (3.6 Exercise)

Answer 1

$$\frac{x^2 - 7}{[(x-2)(x-3)]} = A + \frac{B}{(x-2)} + \frac{C}{(x-3)}$$

$$x^2 - 7 = A(x-2)(x-3) + B(x-3) + C(x-2)$$

Let $x = 2$: $-3 = -B$ $\therefore B = 3$

Let $x = 3$: $2 = C$ $\therefore C = 2$

Let $x = 0$: $-7 = 6A - 3B - 2C$

$-7 = 6A - 9 - 4$ $\therefore A = 1$

Thus

$$\frac{x^2 - 7}{(x-2)(x-3)} = 1 + \frac{3}{(x-2)} + \frac{2}{(x-3)}$$

Answer 2

$$\frac{2x^3+5}{[(x+1)(x+2)]} = Ax + B + \frac{C}{(x+1)} + \frac{D}{(x+2)}$$

$$2x^3 + 5 = (Ax + B)(x+1)(x+2) + C(x+2) + D(x+1)$$

Let $x = -2$	$-11 = -D$	$\therefore\ D = 11$
Let $x = -1$	$3 = C$	$\therefore\ C = 3$
Let $x = 0$	$5 = 2B + 2C + D$	
	$5 = 2B + 6 + 11$	$\therefore\ B = -6$
Let $x = 1$	$7 = (A + B)6 + 3C + 2D$	
	$7 = 6A - 36 + 9 + 22$	
	$12 = 6A$	$\therefore\ A = 2$

Thus

$$\frac{2x^3+5}{(x+1)(x+2)} = 2x - 6 + \frac{3}{(x+1)} + \frac{11}{(x+2)}$$

Answer 3

$$\frac{x^3+12x^2+3}{(x-1)(x+3)} = x + 10 + \frac{4}{(x-1)} - \frac{21}{(x+3)}$$

Answer 4

$$\frac{x^2+8x+4}{(x+5)(2x-1)} = \frac{1}{2} + \frac{1}{(x+5)} + \frac{3}{2(2x-1)}$$

Lesson 4

Partial Fractions : Pure Year 2

4.1 Non-Linear Factors in the Denominator

$$f(x) = \frac{3x + 11}{x^2 + 6x + 5}$$

In order to write $f(x)$ in partial fractions, first factorise the denominator.
Thus;

$$\frac{3x + 11}{x^2 + 6x + 5} = \frac{3x + 11}{(x + 5)(x + 1)}$$

This particular question was tackled in Exercise 1.5, Question 2;

$$\frac{3x + 11}{x^2 + 6x + 5} = \frac{1}{(x + 5)} + \frac{2}{(x + 1)}$$

But what if the quadratic factor could not be factorised ?
For example, $x^2 + x + 2$ cannot be factorised as it has a negative discriminant.

4.2 Example

Write in partial fractions,

$$\frac{8x + 7}{(x^2 + x + 1)(x + 2)}$$

The form of the answer will be;

$$\frac{8x + 7}{(x^2 + x + 1)(x + 2)} = \frac{Ax + B}{(x^2 + x + 1)} + \frac{C}{(x + 2)}$$

(In general the numerator is of degree one less than the denominator)

Now try to determine A, B and C.

4.3 Exercise

Question 1

Write in partial fractions;

$$\frac{3(x+2)}{(2x-1)(x^2+1)}$$

Question 2

Write in partial fractions;

$$\frac{6}{(x^2+2)(1-x)}$$

Question 3

(a) Write in partial fractions in the form given;

$$\frac{x^2 + 6x + 4}{(x^3 - 2x)} = \frac{A}{x} + \frac{Bx + C}{x^2 - 2}$$

(b) Write in partial fractions in the form given;

$$\frac{x^2 + 6x + 4}{(x^3 - 2x)} = \frac{A}{x} + \frac{B}{x - \sqrt{2}} + \frac{C}{x + \sqrt{2}}$$

4.4 Answers

4.4.1 Solutions (4.2 Example)

$$\frac{8x+7}{[(x^2+x+1)(x+2)]} = \frac{Ax+B}{(x^2+x+1)} + \frac{C}{(x+2)}$$

$$8x+7 = (Ax+B)(x+2) + C(x^2+x+1)$$

$Let\ x = -2 \qquad -9 = 3C \qquad \therefore\ C = -3$

$Let\ x = \ 0 \qquad 7 = 2B + C$

$7 = 2B - 3 \qquad \therefore\ B = 5$

$Let\ x = \ 1 \qquad 15 = (A+5)3 + (-3)(3)$

$24 = 3A + 15 \qquad \therefore\ A = 3$

Thus,

$$\frac{8x+7}{(x^2+x+1)(x+2)} = \frac{3x+5}{(x^2+x+1)} - \frac{3}{(x+2)}$$

4.4.2 Solutions (4.3 Exercise)

Answer 1

$$\frac{3(x+2)}{[(2x-1)(x^2+1)]} = \frac{A}{(2x-1)} + \frac{Bx+C}{(x^2+1)}$$

$$3x+6 = A(x^2+1) + (Bx+C)(2x-1)$$

$Let\ x = \frac{1}{2} \qquad \frac{3}{2} + 6 = A(\frac{1}{4} + 1)$

$\frac{15}{2} = A\frac{5}{4} \qquad \therefore\ A = 6$

$Let\ x = 0 \qquad 6 = A - C \qquad \therefore\ C = 0$

$Let\ x = 1 \qquad 9 = 2A + B \qquad \therefore\ B = -3$

Thus,

$$\frac{3(x+2)}{(2x-1)(x^2+1)} = \frac{6}{(2x-1)} - \frac{3x}{(x^2+1)}$$

Answer 2

$$\frac{6}{[(x^2+2)(1-x)]} = \frac{Ax+B}{(x^2+2)} + \frac{C}{(1-x)}$$

$$6 = (Ax+B)(1-x) + C(x^2+2)$$

$Let\ x = 1 \qquad 6 = 3C \qquad \therefore\ C = 2$

$Let\ x = 0 \qquad 6 = B + 2C \qquad \therefore\ B = 2$

$Let\ x = -1 \qquad 6 = (-A+2)2 + 6 \qquad \therefore\ A = 2$

$Thus,$

$$\frac{6}{(x^2+2)(1-x)} = \frac{2x+2}{(x^2+2)} + \frac{2}{(1-x)}$$

Answer 3

$$\frac{x^2+6x+4}{[x(x^2-2)]} = \frac{A}{x} + \frac{Bx+C}{x^2-2}$$

$$x^2+6x+4 = A(x^2-2) + (Bx+C)x$$

$Let\ x = 0 \qquad 4 = -2A \qquad \therefore A = -2$

$Let\ x = 1 \qquad 11 = 2 + B + C \qquad \therefore\ B + C = 9$

$Let\ x = -1 \qquad (-1) = 2 + B - C \qquad \therefore\ B - C = -3$

$2B = 6 \qquad \therefore\ B = 3$

$\therefore\ C = 6$

$Thus,$

$$\frac{x^2+6x+4}{x(x^2-2)} = -\frac{2}{x} + \frac{3x+6}{x^2-2}$$

Answer 4

$$\frac{x^2+6x+4}{x(x-\sqrt{2})(x+\sqrt{2})} = -\frac{2}{x} + \frac{3(1+\sqrt{2})}{2(x-\sqrt{2})} + \frac{3(1-\sqrt{2})}{2(x+\sqrt{2})}$$

Lesson 5

Partial Fractions : Pure Year 2

5.1 Past Paper Partial Fractions

Question 1

C4 examination question from June 2005, Q3 (a)

Express in partial fractions;

$$\frac{5x + 3}{(2x - 3)(x + 2)}$$

[3 marks]

Question 2

C4 examination question from June 2006, Q2 (a)

$$f(x) = \frac{3x - 1}{(1 - 2x)^2} \qquad |x| < \frac{1}{2}$$

Given that, for $x \neq \frac{1}{2}$

$$\frac{3x - 1}{(1 - 2x)^2} = \frac{A}{(1 - 2x)} + \frac{B}{(1 - 2x)^2}$$

where A and B are constants, find the values of A and B

[3 marks]

Question 3

C4 examination question from June 2010, Q5 (a)

$$\frac{2x^2 + 5x - 10}{(x-1)(x+2)} = A + \frac{B}{x-1} + \frac{C}{x+2}$$

Find the values of the constants *A*, *B* and *C*

[4 marks]

Question 4

C4 examination question from January 2013, Q3

Express in partial fractions;

$$\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)}$$

[4 marks]

Question 5

C4 examination question from June 2008, Q7 (a)

Express in partial fractions;

$$\frac{2}{(4 - y^2)}$$

[3 marks]

5.2 Answers

5.2.1 Solutions (5.1 Exercise)

Answer 1

$$\frac{5x + 3}{(2x - 3)(x + 2)} = \frac{3}{(2x - 3)} + \frac{1}{(x + 2)}$$

$$i.e. \quad A = 3. \; B = 1$$

Answer 2

$$\frac{3x - 1}{(1 - 2x)^2} = -\frac{3}{2(1 - 2x)} + \frac{1}{2(1 - 2x)^2}$$

$$i.e. \quad A = -\frac{3}{2}, \quad B = \frac{1}{2}$$

Answer 3

$$\frac{2x^2 + 5x - 10}{(x - 1)(x + 2)} = 2 - \frac{1}{x - 1} + \frac{4}{x + 2}$$

$$i.e. \quad A = 2, \quad B = -1, \quad C = 4$$

Answer 4

$$\frac{9x^2 + 20x - 10}{(x + 2)(3x - 1)} = 3 + \frac{2}{x + 2} - \frac{1}{3x - 1}$$

$$i.e. \quad A = 3, \quad B = 2, \quad C = -1$$

Answer 5

$$\frac{2}{(4 - y^2)} = \frac{2}{(2 + y)(2 - y)} = \frac{1}{2(2 + y)} + \frac{1}{2(2 - y)}$$

$$i.e. \quad Both\ are\ \frac{1}{2}\ however\ \ A\ \ and\ \ B\ \ are\ assigned$$

Lesson 6

Partial Fractions : Pure Year 2

6.1 More Past Paper Partial Fractions (Homework)

Question 1

C4 examination question from June 2009, Q3 (a)

$$f(x) = \frac{4 - 2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

Find the values of the constants A, B and C

[4 marks]

Question 2

C4 examination question from June 2011, Q1

$$\frac{9x^2}{(x-1)^2(2x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(2x+1)}$$

Find the values of the constants A, B and C

[4 marks]

Question 3

C4 examination question from June 2007, Q4 (a)

$$\frac{2(4x^2+1)}{(2x+1)(2x-1)} = A + \frac{B}{(2x+1)} + \frac{C}{(2x-1)}$$

Find the values of the constants A, B and C

[4 marks]

6.2 Answers

6.2.1 Solutions (6.1 Exercise)

Answer 1

$$\frac{4 - 2x}{(2x + 1)(x + 1)(x + 3)} = \frac{4}{2x + 1} - \frac{3}{x + 1} + \frac{1}{x + 3}$$

$$i.e. \quad A = 4. \quad B = -3, \quad C = 1$$

Answer 2

$$\frac{9x^2}{(x - 1)^2 (2x + 1)} = \frac{4}{(x - 1)} + \frac{3}{(x - 1)^2} + \frac{1}{(2x + 1)}$$

$$i.e. \quad A = 4, \quad B = 3, \quad C = 1$$

Answer 3

$$\frac{2(4x^2 + 1)}{(2x + 1)(2x - 1)} = 2 - \frac{2}{(2x + 1)} + \frac{2}{(2x - 1)}$$

$$i.e. \quad A = 2, \quad B = -2, \quad C = 2$$