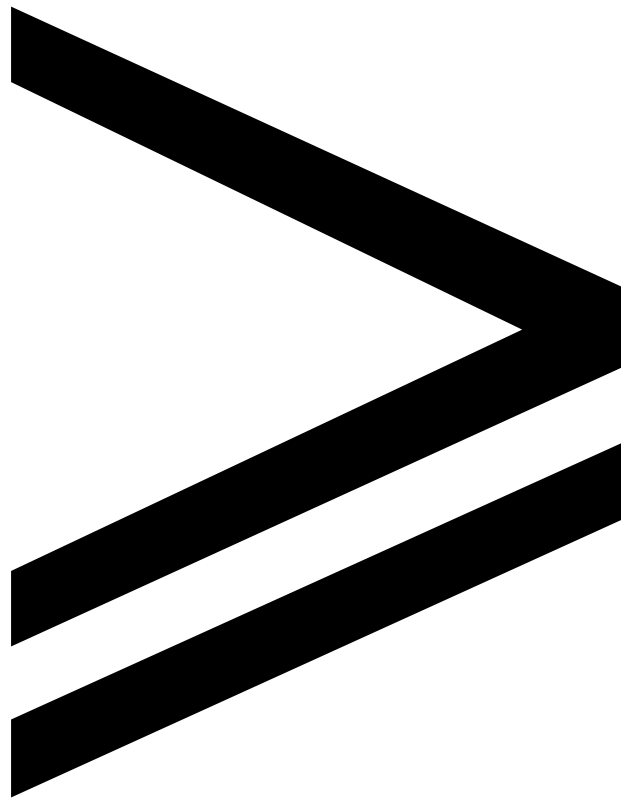


GCSE Mathematics

INEQUALITIES



Lesson 1

GCSE : Algebra of Inequalities

1.1 Solving Simple Equations & Inequalities

Do NOT use a calculator

Question 1

Solve the following equations;

(i) $x + 11 = 57$

(ii) $x - 13 = 14$

(iii) $5x = 115$

(iv) $5x + 8 = 33$

(v) $4x - 7 = 33$

(vi) $8x + 3 = 7$

(vii) $2(x + 4) = 18$

(viii) $13x = 52$

(ix) $19 - x = 8$

(x) $x + 7 = -15$

(xi) $x + 26 = 15$

(xii) $14 - x = 16$

Question 2

Study the following table;

SYMBOL	MEANING
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to

For each of the following, state if the inequality written down is TRUE or FALSE.

(i) $\pounds 5 > 50\text{p}$

(ii) $\pounds 0.40 < 50\text{p}$

(iii) $1\text{kg} > 100\text{g}$

(iv) $40\text{km/h} > 50\text{mph}$

(v) $3\text{km} < 300\text{m}$

(vi) $16\text{cm} > 160\text{mm}$

Question 3

I am about to roll two standard six sided dice and add together the spots showing.
Which of the following is ***always true*** about the number of spots showing ?

spots ≥ 25 spots < 2 spots > 8 spots ≤ 12

Question 4

I was born in June but it was so long ago that I have forgotten what date in June.
Which of the following is ***definitely true*** about that date.

date = 1st date $\geq 1^{\text{st}}$ date $< 1^{\text{st}}$ date $> 1^{\text{st}}$

Question 5

You can solving the following simple inequalities as if they are equations with an equals sign in the place of the inequality symbol.

Be sure to have an inequality sign in your answer.

(i) $4x \geq 24$

(ii) $x + 5 < 27$

(iii) $x - 8 \leq 17$

(iv) $5x + 11 > 26$

(v) $6x + 8 \geq 32$

(vi) $4x - 14 < 18$

(vii) $20 + 3x > 53$

(viii) $3 (x + 6) > 21$

(ix) $2 (3x + 5) \geq 22$

(x) $15x \leq 105$

(xi) $25x + 10 \leq 85$

(xii) $12x > 144$

1.2 Solutions (1.1 Exercise)

Answer 1

(i)	46	(ii)	27
(iii)	23	(iv)	5
(v)	10	(vi)	0.5
(vii)	5	(viii)	4
(ix)	11	(x)	- 22
(xi)	-11	(xii)	- 2

Answer 2

(i)	TRUE	(ii)	TRUE
(iii)	TRUE	(iv)	FALSE
(v)	FALSE	(vi)	FALSE

Answer 3

spots $\leq$ 12

Answer 4

date $\geq$ 1st

Answer 5

The inequality sign MUST be present in the answer

(i)	$x \geq 6$	(ii)	$x < 22$
(iii)	$x \leq 25$	(iv)	$x > 3$
(v)	$x \geq 4$	(vi)	$x < 8$
(vii)	$x > 11$	(viii)	$x > 1$
(ix)	$x \geq 2$	(x)	$x \leq 7$
(xi)	$x \leq 3$	(xii)	$x > 12$

Lesson 2

GCSE : Algebra of Inequalities

2.1 Equations & Inequalities Practice

Do NOT use a calculator

Question 1

Solve the following equations;

(i) $x + 0.5 = 5.7$

(ii) $x - 2.4 = 4.1$

(iii) $0.2x = 20$

(iv) $3x + 0.8 = 6.8$

(v) $40x - 7 = 393$

(vi) $8x + 3 = 67$

(vii) $20 (x + 4) = 100$

(viii) $12x = 36$

(ix) $- 5x = 65$

(x) $x + 2.6 = - 3.5$

(xi) $2x + 2.6 = 5.6$

(xii) $2.4 + 3x = 8.7$

Question 2

Study the following example;

EXAMPLE	MEANING
$7 < x < 11$	x is more than 7 and less than 11

List all of the following which **could be** the value of x in the above example;

-2 0 3 7 9 10 11 12

Question 3

For each of the following state if the given inequality is TRUE or FALSE.

- (i) $10\text{p} < \text{£}1 < 150\text{p}$ (ii) $\text{£}0.40 < 50\text{p} < \text{£}0.30$
- (iii) $1\text{mm} < 1\text{cm} < 1\text{m}$ (iv) $0.04\text{m} < 40\text{m} < 40\text{cm}$
- (v) $3\text{km} < 300\text{m} < 3\text{cm}$ (vi) $\frac{1}{4} < \frac{1}{3} < \frac{1}{2}$

Question 4

I am about to roll two standard six sided dice and add together the spots showing.
Which of the following is **always true** about the number of spots showing ?

- $4 < \text{spots} \leq 7$ $2 \leq \text{spots} \leq 12$
- $2 \leq \text{spots} < 12$ $0 \leq \text{spots} \leq 5$

Question 5

In my wallet I have exactly 4 coins.
Which of the following is **definitely true** about the amount of money in my wallet ?

- $5\text{p} < \text{money} < \text{£}1$ $4\text{p} < \text{money} \leq \text{£}2$
- $4\text{p} \leq \text{money} \leq \text{£}8$ $0\text{p} \leq \text{money} < \text{£}3$

Question 6

You can solving the following simple inequalities as if they are equations with an equals sign in the place of the inequality symbol.

Be sure to have an inequality sign in your answer.

(i) $4x < 28$

(ii) $x + 0.5 \geq 7.2$

(iii) $2x - 0.9 \leq 7.1$

(iv) $0.5x + 150 > 160$

(v) $6x + 38 \leq 32$

(vi) $1.3x + 1.3 < 3.9$

(vii) $6 (x + 2) < 42$

(viii) $3 (x + 2) + 4 (x + 3) \leq 53$

(ix) $4 (2x + 1) \leq 24$

(x) $2 (5x + 1) + 3 (x + 8) < 65$

2.2 Solutions (2.1 Exercise)

Answer 1

(i)	5.2	(ii)	6.5
(iii)	100	(iv)	2
(v)	10	(vi)	8
(vii)	1	(viii)	3
(ix)	- 13	(x)	- 6.1
(xi)	1.5	(xii)	2.1

Answer 2

9, 10

Answer 3

(i)	TRUE	(ii)	FALSE
(iii)	TRUE	(iv)	FALSE
(v)	FALSE	(vi)	TRUE

Answer 4

$2 \leq \text{spots} \leq 12$

Answer 5

$4\text{p} \leq \text{money} \leq \text{£}8$

Answer 6

The inequality sign MUST be present in the answer

(i)	$x < 7$	(ii)	$x \geq 6.7$
(iii)	$x \leq 4$	(iv)	$x > 20$
(v)	$x \leq -1$	(vi)	$x < 2$
(vii)	$x < 5$	(viii)	$x \leq 5$
(ix)	$x \leq 2.5$	(x)	$x < 3$

Lesson 3

GCSE : Algebra of Inequalities

3.1 Homework on Equations & Inequalities

Do NOT use a calculator

Question 1

Solve the following equations;

(i) $\frac{x}{4} + 5 = 8$

(ii) $\frac{x+5}{4} = 8$

(iii) $\frac{x}{6} + 3 = 11$

(iv) $\frac{x+3}{6} = 8$

(v) $\frac{x}{3} + 12 = 19$

(vi) $\frac{x+12}{3} = 19$

(vii) $\frac{2x}{3} + 5 = 13$

(viii) $\frac{2x+5}{3} = 7$

(ix) $\frac{5x+2}{4} = 8$

(x) $\frac{5x}{4} + 2 = 22$

(xi) $\frac{2x+6}{5} = 6$

(xii) $\frac{2x}{5} + 6 = 18$

Question 2

Study the following example;

EXAMPLE	MEANING
$-2 < x \leq 1$	x is more than -2 and less than or equal to 1

List all of the following which ***could be*** the value of x in the above example;

- 4 - 3 - 2 - 1 0 1 2 3 4

Question 3

For each of the following state if the given inequality is TRUE or FALSE.

(i) $5p < £0.06 < 7p$

(ii) $£0.40 < 40p < £0.50$

(iii) $1m < 10cm < 100mm$

(iv) $0.04m < 40cm < 4m$

(v) $3km < 30\,000m < 30\,000\,000cm$

(vi) $\frac{1}{2} < \frac{1}{4} < \frac{1}{3}$

Question 4

I am about to roll two standard six sided dice and add together the spots showing.
Which of the following is ***always true*** about the number of spots showing ?

$$-1 < \text{spots} < 26$$

$$26 < \text{spots} < -1$$

$$3 < \text{spots} < 12$$

$$-3 < \text{spots} < -1$$

Question 5

You can solving the following simple inequalities as if they are equations with an equals sign in the place of the inequality symbol.

Be sure to have an inequality sign in your answer.

(i) $\frac{x}{7} + 3 \geq 5$

(ii) $\frac{x+3}{7} \geq 5$

(iii) $\frac{x}{11} + 3 < 9$

(iv) $\frac{x+3}{11} > 9$

(v) $\frac{x}{2} - 8 \leq 3$

(vi) $\frac{x-8}{2} < 3$

(vii) $\frac{5x}{3} - 7 \geq 43$

(viii) $\frac{5x-8}{4} < 13$

$$\text{(ix) } \quad \frac{x}{6} - \frac{1}{3} \leq \frac{1}{6}$$

$$\text{(x) } \quad \frac{x}{4} - \frac{4}{3} < \frac{5}{12}$$

$$\text{(xi) } \quad \frac{x}{2} + \frac{x}{3} \geq 15$$

$$\text{(xii) } \quad \frac{2x}{3} + \frac{x}{2} > 14$$

3.2 Solutions (3.1 Homework)

Answer 1

(i) $x = 12$

(iii) $x = 48$

(v) $x = 21$

(vii) $x = 12$

(ix) $x = 6$

(xi) $x = 12$

(ii) $x = 27$

(iv) $x = 45$

(vi) $x = 45$

(viii) $x = 8$

(x) $x = 16$

(xii) $x = 30$

Answer 2

-1, 0, 1

Answer 3

(i) TRUE

(iii) FALSE

(v) TRUE

(ii) FALSE

(iv) TRUE

(vi) FALSE

Answer 4

- 1 < spots < 26

Answer 5

(i) $x \geq 14$

(iii) $x < 66$

(v) $x \leq 22$

(vii) $x \geq 30$

(ix) $x \leq 3$

(xi) $x \geq 18$

(ii) $x \geq 32$

(iv) $x > 96$

(vi) $x < 14$

(viii) $x < 12$

(x) $x < 7$

(xii) $x > 12$

Lesson 4

GCSE : Algebra of Inequalities

4.1 Multiplication By A Negative Number

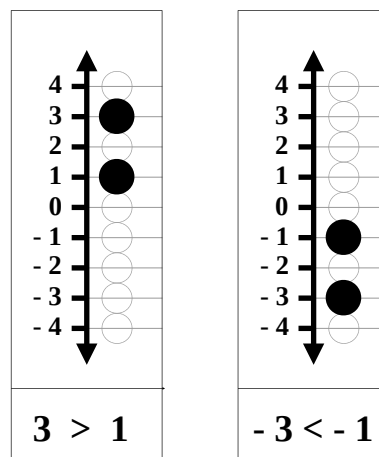
The inequalities looked at so far have behaved in the same manner as equations.

For Example;

Finding the x from the inequality $8x + 9 < 33$ is done in a very similar manner to finding the x from the equation $8x + 9 = 33$.

$$\begin{array}{rcl} 8x + 9 < 33 & | & \\ 8x < 24 & | & -9 \\ x < 3 & | & \div 8 \end{array} \qquad \begin{array}{rcl} 8x + 9 = 33 & | & \\ 8x = 24 & | & -9 \\ x = 3 & | & \div 8 \end{array}$$

There is a complication that arises when solving some inequalities.
To understand the complication think about this diagram;



IMPORTANT RESULT

If we multiply (or divide) both sides of an inequality by a negative number then we must reverse the direction of the inequality

IMPLICATION

You cannot multiply (or divide) both sides of an inequality by x when the sign of x is not known because you would not know if to reverse the inequality or not.

4.2 Three Questions

Solve the following inequalities;

(i) $21 - x < 17$

(ii) $43 + 4x \geq 23$

(iii) $31 - 3x > 88$

4.3 Answers to The Three Questions

(i) $x > 4$

(ii) $x \geq -5$

(iii) $x < -19$

4.4 Exercise : Inequalities That Sometimes Need Reversing

Do NOT use a calculator

Question 1

Solve the following inequalities;

(i) $4x < 12$

(ii) $-5x \geq 35$

(iii) $-3x > -15$

(iv) $7x > -28$

(v) $-6x \leq 3$

(vi) $10x > -4$

(vii) $-x \geq 1$

(viii) $4 - x \geq 11$

(ix) $4 > x$

(x) $-4 > x$

(xi) $7x \geq -3.5$

(xii) $-2x + 7 > 5$

Question 2

Solve the following inequalities;

(i) $5x + 7 < 22$

(ii) $8 + 6x > -10$

(iii) $-7 + 2x \geq 13$

(iv) $14 - 3x \leq 5$

(v) $3(x - 5) > 9$

(vi) $0.4x + 2.4 < 4.8$

Question 3

In a field are a lot of sheep.

I'm not sure exactly how many.

Let's say that there are x sheep in the field.

224 sheep are taken out of the field.

The number of sheep now in the field is less than 100.

Write down, and solve, an inequality for x .

Question 4

Let A be the number of prime numbers less than 25.

Let B be the number of prime numbers less than 101.

Which one of the following statements is true ?

(a) $A > B$

(b) $A < B$

(c) $A = B$

Question 5

If x is an *integer*, list all of the values of x for which it is true that;

$$3 < x \leq 7$$

Question 6

If x is an *integer*, list all of the values of x for which it is true that;

$$-2 \leq x < 6$$

Question 7

If x is a *positive integer*, list all of the values of x for which it is true that;

$$x \leq 4$$

Question 8

If Mr X is a teenager, which one of the following inequalities most accurately describes his possible ages.

$$12 \leq X < 19$$

$$13 < X \leq 19$$

$$12 < X \leq 19$$

$$13 \leq X < 19$$

Question 9

Two sides of a mathematician's triangle are 8 cm and 13 cm exactly.

Write down an inequality in the form

$$a \leq x \leq b$$

where x is the length of the third side, and a & b are numbers the values of which are to be stated.

Question 10

Solve the following inequalities;

(i) $\frac{4x}{3} + 5 < 13$

(ii) $23 - 3x > -1$

(iii) $7x + 9 > 3x + 37$

(iv) $38 - 3x > 65$

Question 11

Calculate;

$$(4)^2 \quad (3)^2 \quad (-1)^2 \quad \text{and} \quad (-2)^2$$

If x is an integer, list **ALL** values of x for which $x^2 \leq 9$.

Question 12

In this question x is NOT an integer.

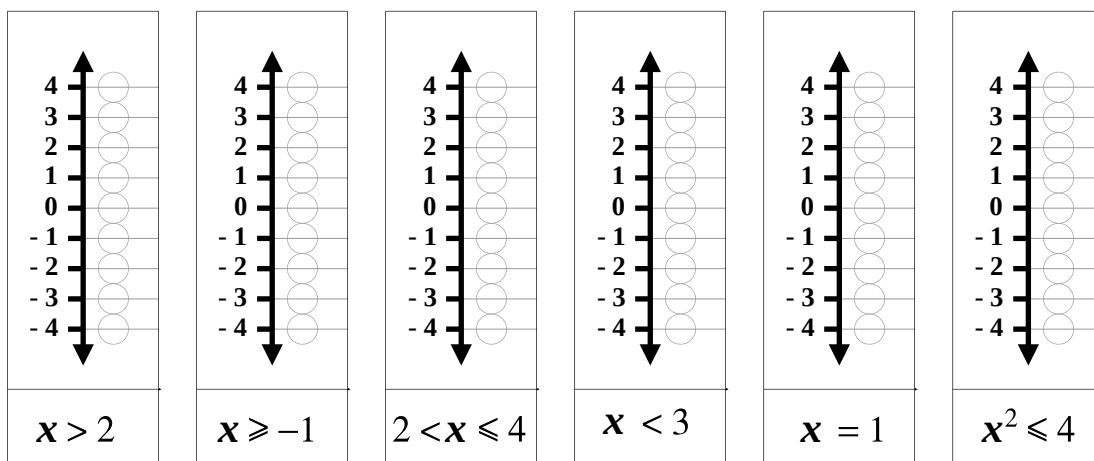
Also you are told that $x^2 < 16$.

Solve this inequality and write your answer in the form

$a < x < b$, where a and b are numbers you must find.

Question 13

On these number ladders draw the inequalities indicated;



Question 14

To the nearest cm, a plank of wood is 123 cm.

Write down an inequality of the form;

$$a \leq x < b$$

where x is the exact length of the plank, and a & b are numbers, the values of which you must state.

Out Of Interest :

a is referred to as the lower bound of the plank's length, and b the upper bound.

4.5 Solutions (4.4 Exercise)

Answer 1

(i) $x < 3$

(iii) $x < 5$

(v) $x \geq -0.5$

(vii) $x \leq -1$

(ix) $x < 4$

(xi) $x \geq -0.5$

(ii) $x \leq -7$

(iv) $x > -4$

(vi) $x > -0.4$

(viii) $x \leq -7$

(x) $x < -4$

(xii) $x < 1$

Answer 2

(i) $x < 3$

(iii) $x \geq 10$

(v) $x > 8$

(ii) $x > -3$

(iv) $x \geq 3$

(vi) $x < 6$

Answer 3

$x - 224 < 100$ or $x < 324$

Answer 4

Option (b) $A < B$

Answer 5

4, 5, 6, 7

Answer 6

-2, -1, 0, 1, 2, 3, 4, 5

Answer 7

1, 2, 3, 4

Answer 8

$12 < X \leq 19$

Answer 9

$5 \text{ cm} < x < 21 \text{ cm}$

Scope for discussion about use of $\leq$ or $<$

Answer 10

(i) $x < 6$

(iii) $x > 7$

(ii) $x < 8$

(iv) $x < - 9$

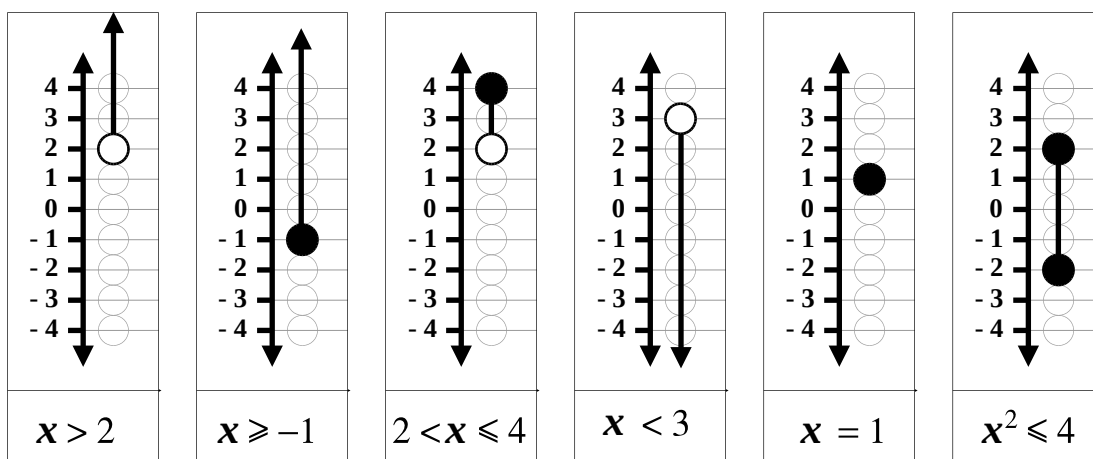
Answer 11

$16 \qquad 9 \qquad 1 \qquad 4$

$-3, -2, -1, 0, 1, 2, 3$

Answer 12

$- 4 < x < 4$

Answer 13**Answer 14**

$122.5 \text{ cm} \leq x < 123.5 \text{ cm}$

Lesson 5

GCSE : Algebra of Inequalities

5.1 Sandwich Inequalities

$$6 \text{ cm} < x < 11 \text{ cm}$$

The above inequality states that the unknown, x , has a value *between* 6 cm and 11 cm.
We say that x is sandwiched between the values of 6 and 11 cm.

Sandwich inequalities are always written using $<$ or $\leq$
Not $>$ or $\geq$

5.2 Examples

$$\begin{array}{l} - 10 \leq 7x + 4 < 39 \\ - 14 \leq 7x < 35 \\ - 2 \leq x < 5 \end{array} \quad \left| \begin{array}{l} \\ - 4 \\ \div 7 \end{array} \right.$$

$$\begin{array}{l} 10 \leq 43 - 3x < 37 \\ - 33 \leq -3x < -6 \\ - 11 \leq -x < -2 \\ 11 \geq x > 2 \\ 2 < x \leq 11 \end{array} \quad \left| \begin{array}{l} \\ - 43 \\ \div 3 \\ \times (-1) \\ \text{rewrite} \end{array} \right.$$

5.3 Exercise

Do NOT use a calculator

Question 1

Solve the following inequalities

(i) $9 < 2x + 3 < 15$

(ii) $8 < 3x - 4 < 17$

(iii) $- 8 < 5x + 2 \leq 47$

(iv) $- 5 \leq 4x - 9 \leq 15$

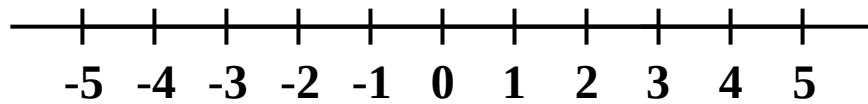
(v) $8 \leq \frac{x}{2} + 3 < 21$

(vi) $7 \leq \frac{x}{5} + 13 < 63$

Question 2

GCSE exam question, November 2007, 4H.

(a) On the number line, show the inequality $- 2 < x \leq 3$



[2 marks]

(b) n is an integer.

Write down all the possible values of n which satisfy the inequality

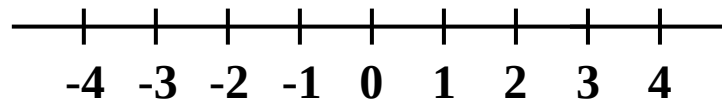
$$-1 \leq n < 4$$

[2 marks]

Question 3

(i) Solve the inequality $-10 \leq 3x - 7 < 2$

(ii) On the number line, represent the solution to part (i).

**Question 4**

If x is an *integer*, list all of the values of x for which it is true that;

$$-3.5 < x \leq 2$$

Question 5

GCSE exam question, November 2008, 4H.

Solve the inequality $5x - 6 > 2$

You must show sufficient working.

[2 marks]

Question 6

GCSE exam question from May 2008, 3H.

(i) Solve the inequality $4x + 5 \leq 21$

(ii) n is a positive integer.

Write down all the values of n which satisfy $4n + 5 \leq 21$

[4 marks]

Question 7

GCSE exam question from January 2012, 4H.

- (i) Solve the inequalities

$$-6 < 4x < 8$$

- (ii) n is an integer.

Write down all the values of n which satisfy

$$-6 < 4n \leq 8$$

[4 marks]

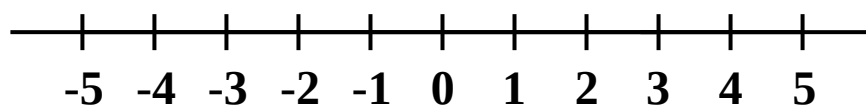
Question 8

GCSE exam question from January 2013, 3H

- (i) Solve the inequalities

$$-2 < x + 2 \leq 5$$

- (ii) On the number line, represent the solution to part (i).



[4 marks]

Question 9

GCSE exam question from November 2006, 3H.

Solve the inequality

$$9x - 2 < 5x + 4$$

[3 marks]

Question 10

Solve the following inequalities

(i) $31 \leq 3x + 25 < 37$

(ii) $-50 < 7x + 27 \leq 48$

(iii) $-8 < 8 - x \leq 28$

(iv) $-36 \leq 9x - 99 \leq 99$

(v) $9 \leq 3 - 2x < 23$

(vi) $9 \leq 13 - 4x < 65$

Question 11

(i) In this question x is an integer.

Also $x^2 < 25$.

List all possible values of x .

(ii) If y is not restricted to being an integer, solve $y^2 < 25$.

Your answer should be in the form

$$a < y < b$$

for some values of a and b which you must find.

Question 12

GCSE exam question from May 2008, 4H.

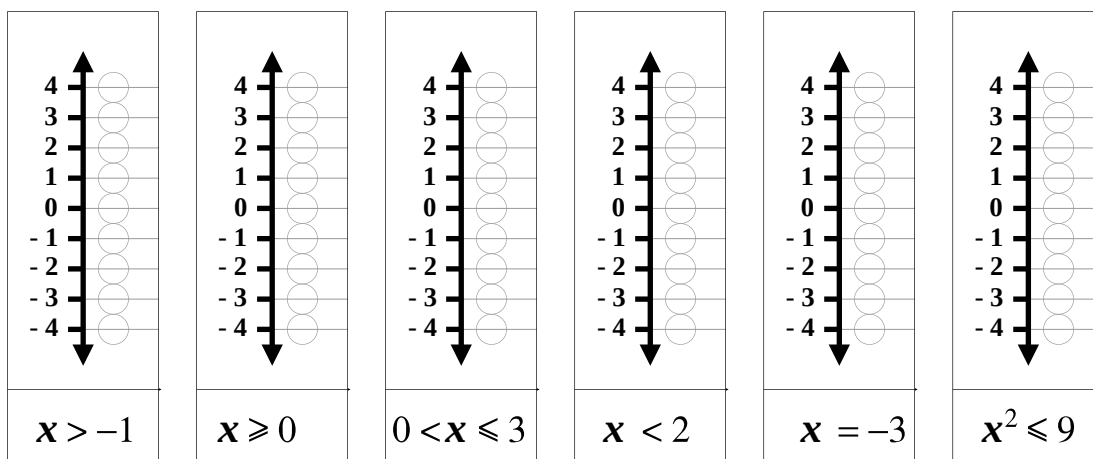
Solve the inequality

$$y^2 < 9$$

[2 marks]

Question 13

On these number ladders draw on the inequalities indicated



Question 14

GCSE exam question from June 2011, 4H.

- (i) Solve the inequality

$$2x + 13 \geq 6$$

- (ii) n is a **negative** integer.

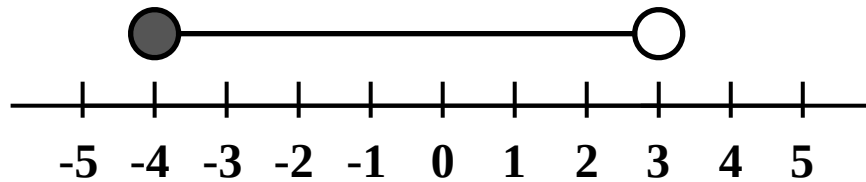
Write down all the values of n which satisfy $2n + 13 \geq 6$

[4 marks]

Question 15

GCSE exam question from June 2010, 3H.

- (a)



An inequality is shown on the number line.

Write down this inequality

- (b) (i) Solve the inequality $2x + 9 > 1$

- (ii) n is a **negative** integer.

Write down all the values of n which satisfy $2n + 9 > 1$

[6 marks]

Question 16

Write down all values of x that satisfy the following.

(i) $3x + 5 \leq 17$, where x is a positive, even integer.

(ii) $5x - 13 < 52$, where x is a positive, square number.

(iii) $7x + 17 < 141$, where x is a positive, prime number.

5.4 Answers (5.3 Exercise)

Answer 1

(i) $3 < x < 6$

(ii) $4 < x < 7$

(iii) $-2 < x \leq 9$

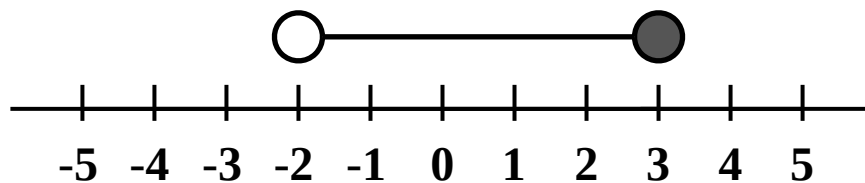
(iv) $1 \leq x \leq 6$

(v) $10 \leq x < 36$

(vi) $-30 \leq x < 250$

Answer 2

(a)

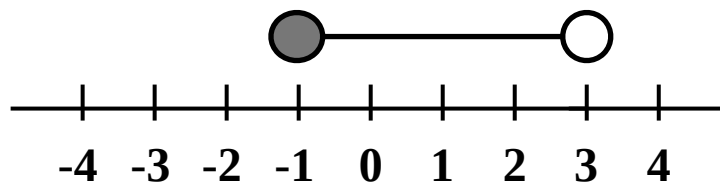


(b) $n \in \{ -1, 0, 1, 2, 3 \}$

Answer 3

(i) $-1 \leq x < 3$

(ii)



Answer 4

$x \in \{ -3, -2, -1, 0, 1, 2 \}$

Answer 5

$x > 1.6$

Answer 6

(i) $x \leq 4$

(ii) $1, 2, 3, 4$

Answer 7

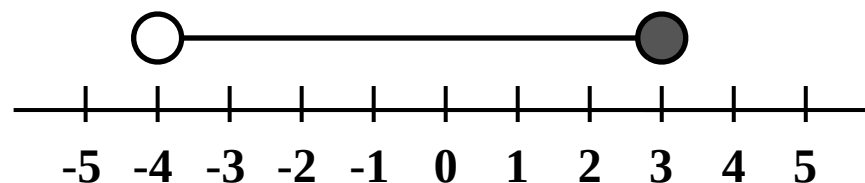
(i) $-1.5 < x < 2$

(ii) $-1, 0, 1$

Answer 8

(i) $-4 < x \leq 3$

(ii)



Answer 9

$$x < 1.5$$

Answer 10

(i) $2 \leq x < 4$

(ii) $-11 < x \leq 3$

(iii) $-20 \leq x < 16$

(iv) $7 \leq x \leq 22$

(v) $-10 < x \leq -3$

(vi) $-13 < x \leq 1$

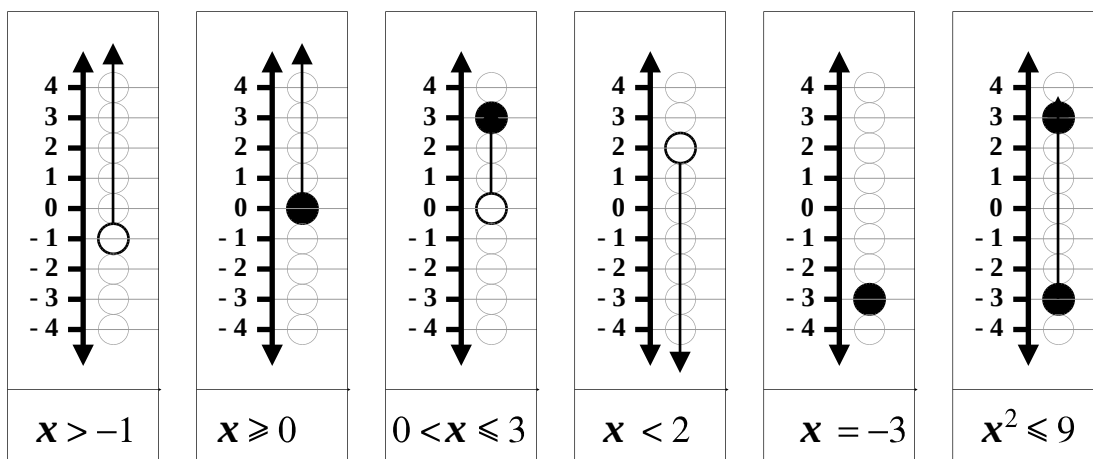
Answer 11

(i) $x \in \{ -4, -3, -2, -1, 0, 1, 2, 3, 4 \}$

(ii) $-5 < y < 5$

Answer 12

$$-3 < y < 3$$

Answer 13**Answer 14**

(i) $x \geq -3.5$

(ii) $x \in \{ -3, -2, -1 \}$

Answer 15

(a) $-4 \leq x < 3$

(b) (i) $x > -4$

(ii) $n \in \{ -3, -2, -1 \}$

Answer 16

(i) $x \in \{ 2, 4 \}$

(ii) $x \in \{ 1, 4, 9 \}$

(iii) $x \in \{ 2, 3, 5, 7, 11, 13, 17 \}$

Lesson 6

GCSE : Algebra of Inequalities

6.1 Graphs Of Inequalities

In this Lesson we:

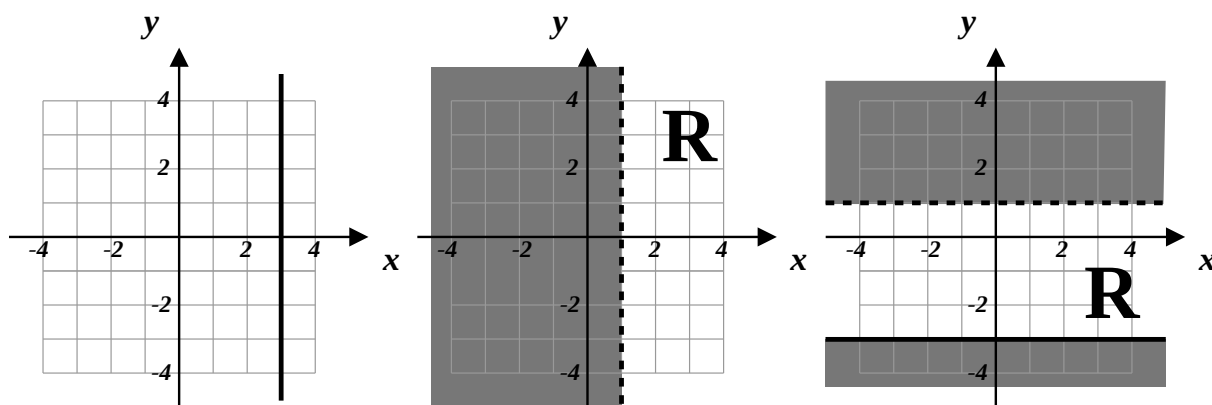
- Look at the graph of an inequality and write down its algebraic description.
- Start with the algebraic description of an inequality and draw its graph.

6.2 Example

The Question

Look at the one equality and two inequality graphs, given below.

- (i) Give the equation of the line on the first graph.
- (ii) Give the inequality that describes the region marked with an **R** on each of the second and third graphs.



The Answer

- (i) The first graph is of the equality $x = 3$.

- (ii) The second graph is of the inequality $x > 1$.

The region **R** is *wanted*.

R in this question is the unshaded part of the graph.

In words; "More than 1 across"

The boundary line is broken, so the boundary itself is NOT wanted.

That is why the inequality $>$ is used rather than $\geq$.

The third graph of the inequality $-3 \leq y < 1$.

The region **R** is *wanted*.

R in this question is the unshaded part of the graph.

In words; "Heights between -3 and 1, -3s included, 1s not".

The boundary line $y = -3$ is solid, so that boundary is wanted. Thus, $\leq$

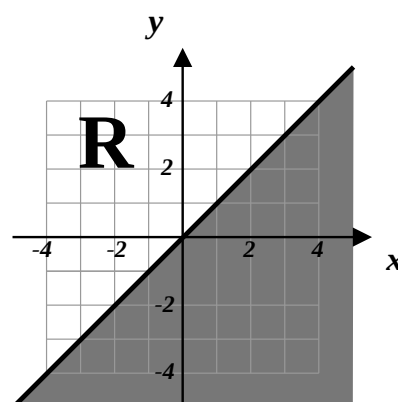
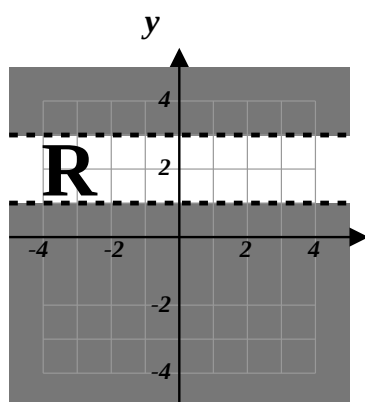
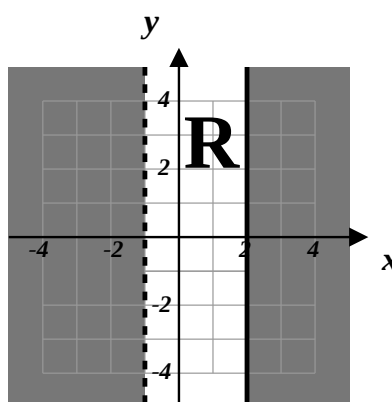
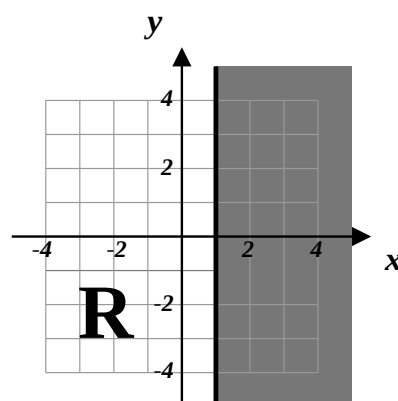
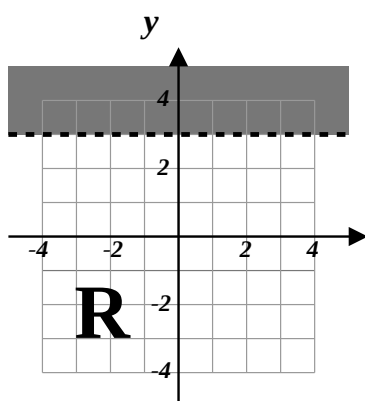
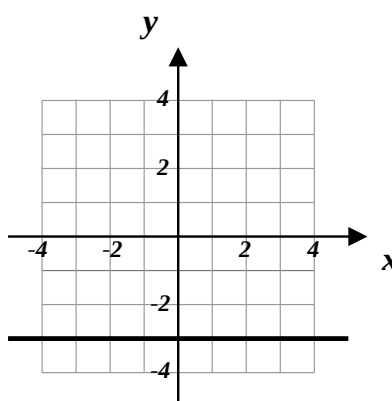
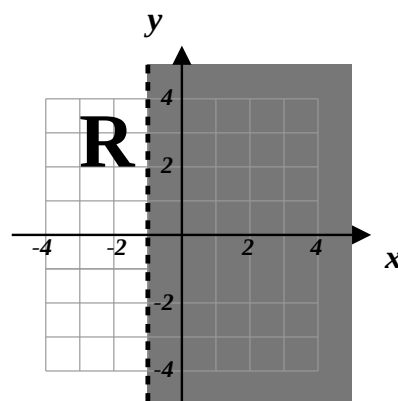
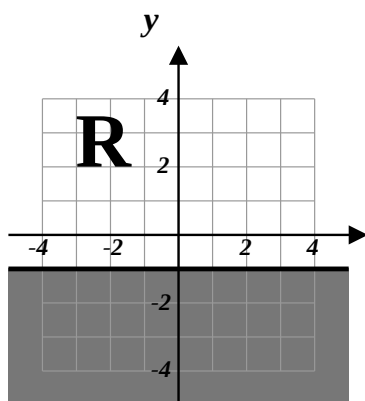
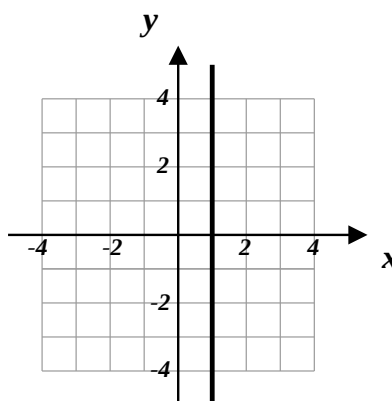
The boundary line $y = 1$ is broken, so that boundary is NOT wanted. Thus, $<$

6.3 Exercise

Question 1

For each graph, write down the algebraic equation or inequality shown.

Regions unshaded are marked **R** and contain the points *wanted*.

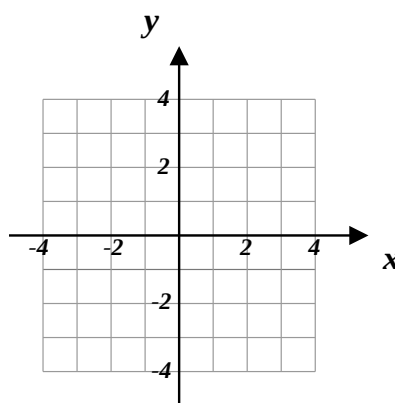


Question 2

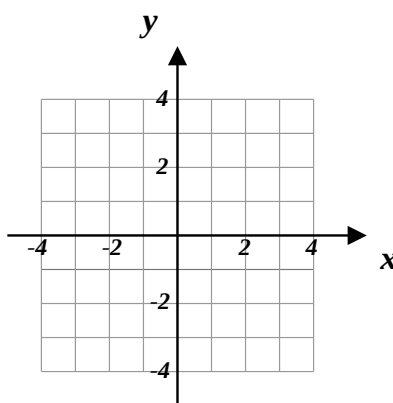
Draw the graph of the equation or inequality specified.

Shade the region **unwanted**.

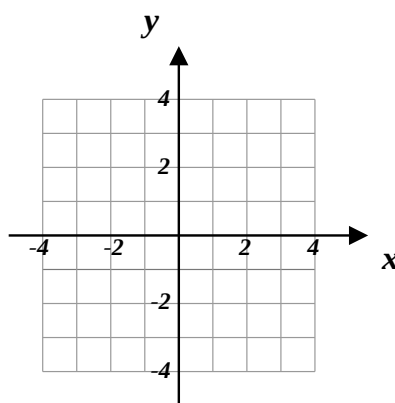
Mark the region specified by the inequality with an **R**.



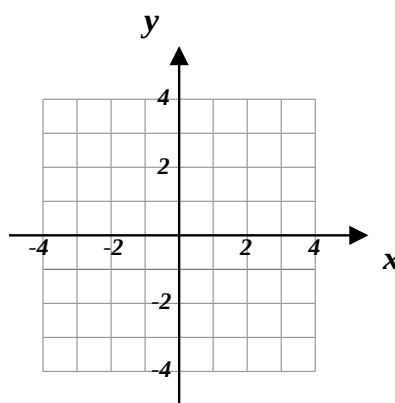
$$y = 3$$



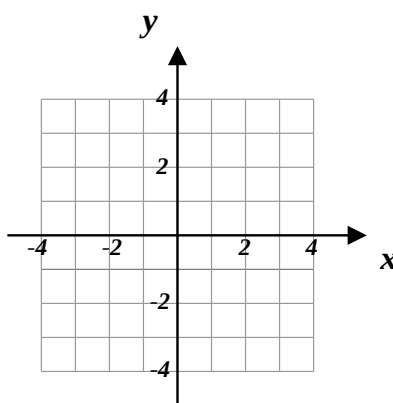
$$y \leq -1$$



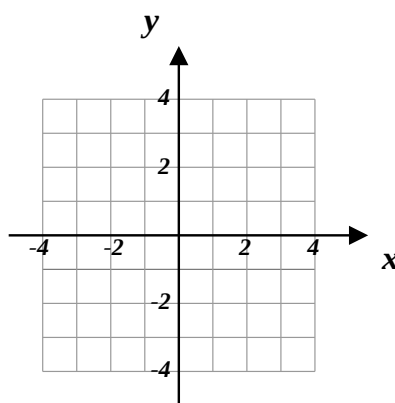
$$x < 3$$



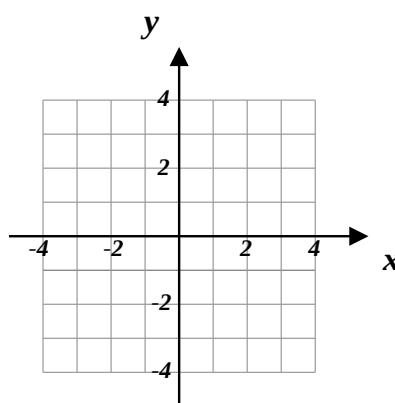
$$x > 2$$



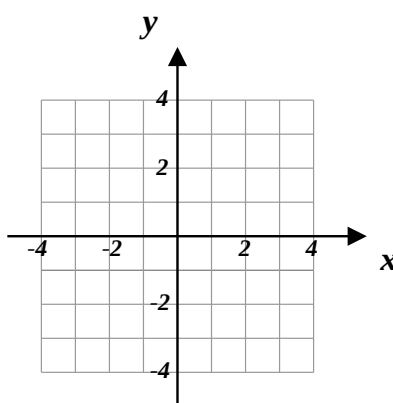
$$x = -2$$



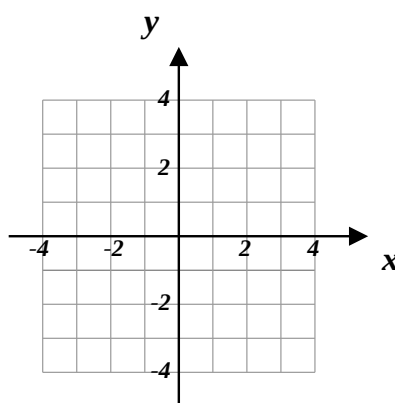
$$y \leq 0$$



$$-3 < x \leq -1$$



$$-2 \leq y < 1$$



$$y < x$$

6.4 Answers

Answer 1

$$x = 1$$

$$y = -3$$

$$-1 < x \leq 2$$

$$y \geq -1$$

$$y < 3$$

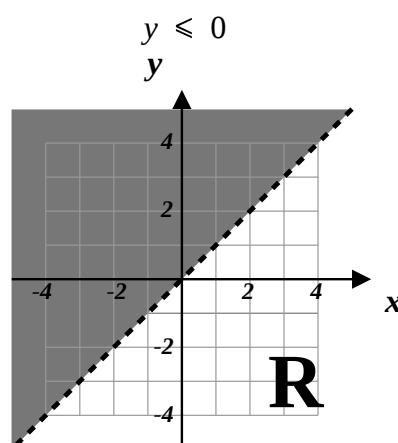
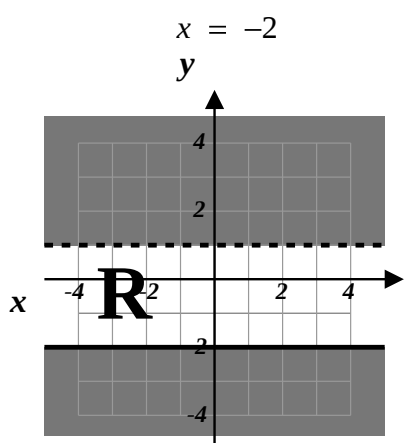
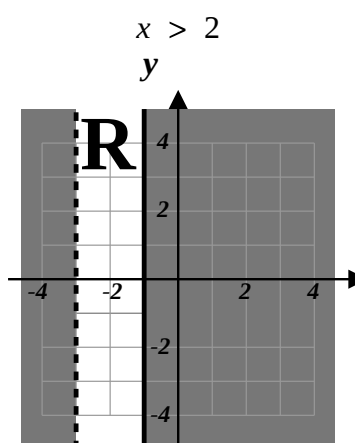
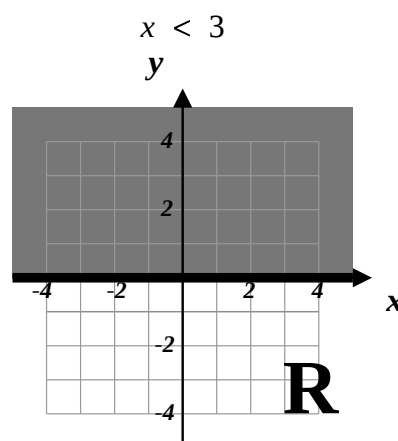
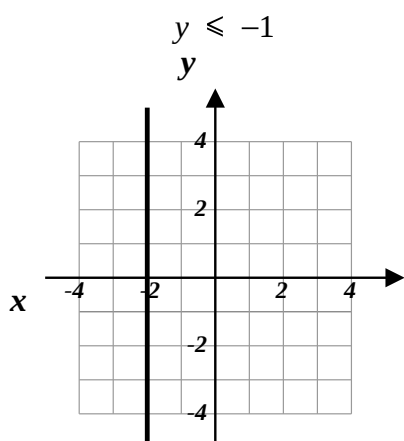
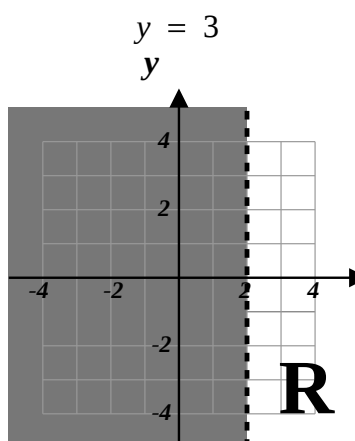
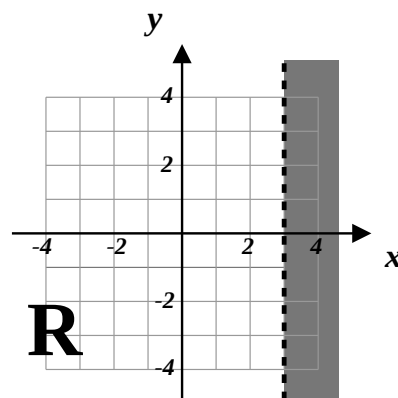
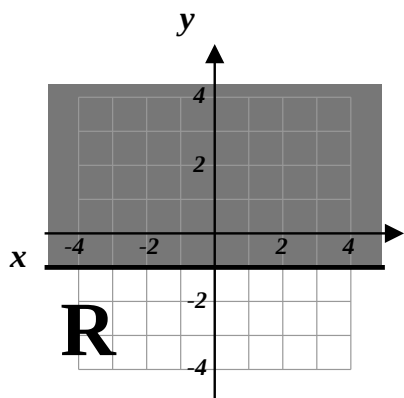
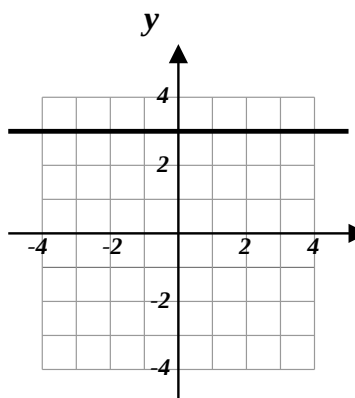
$$1 < y < 3$$

$$x < -1$$

$$x \leq 1$$

$$y \geq x$$

Answer 2



$$-3 < x \leq -1$$

$$-2 \leq y < 1$$

$$y < x$$

Lesson 7

GCSE : Algebra of Inequalities

7.1 Multi-Graph Inequalities

GCSE questions often ask for an inequality multi-graph.

- This is two or three inequalities on one graph.

7.2 Example

The Question

Show, by shading on the grid, the region which satisfies these inequalities

$$y \leq -1 \quad \text{and} \quad y > -3 \quad \text{and} \quad y \leq x$$

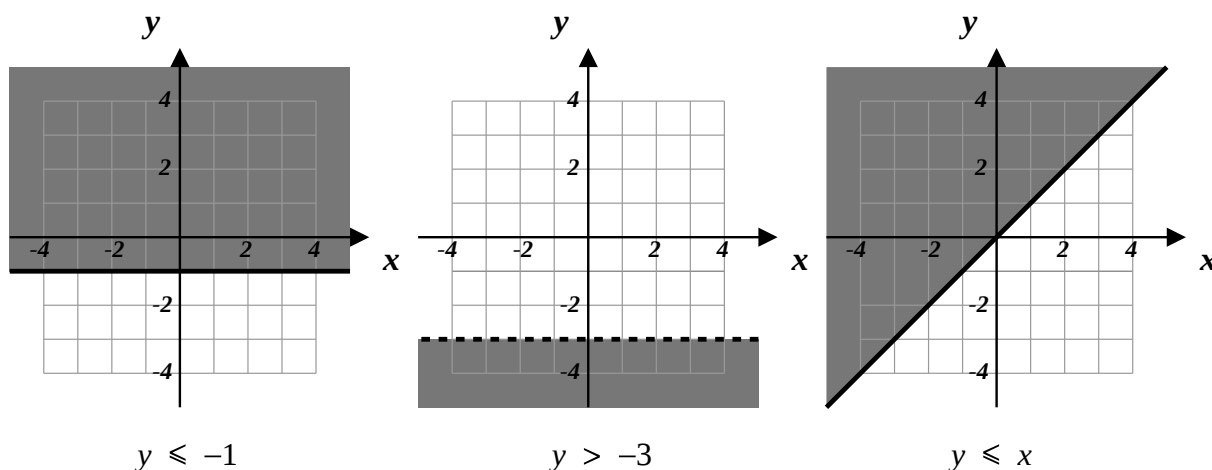
Label your region **R**.

The Answer

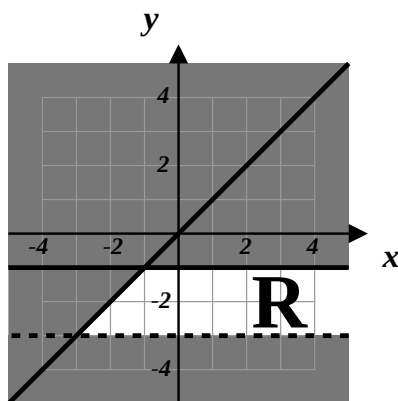
The question requests the three inequalities on one graph, but here they are first drawn separately to help you see what is going on.

Notice that the region **NOT** wanted is shaded.

Think of it as shading the bit to be thrown away.



Putting the three together gives the proper answer. The region wanted is the white bit. It's crucial you do as the question ask and mark the region wanted with an **R**.



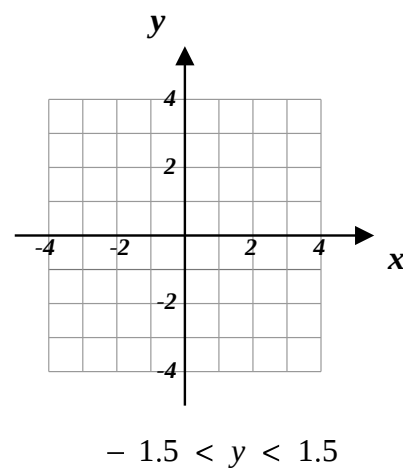
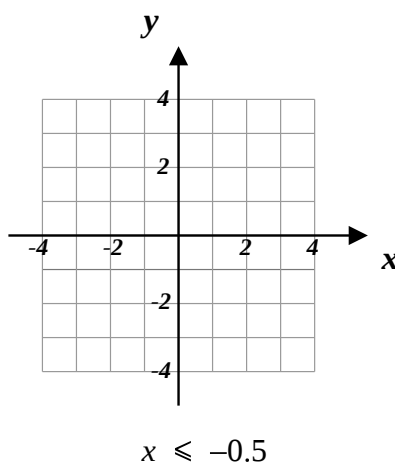
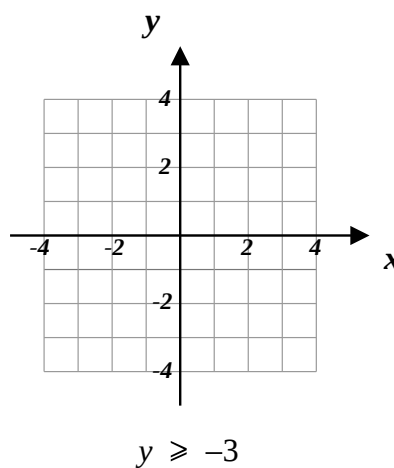
7.3 Exercise

Question 1

Show, by shading on the grid, the region which satisfies these inequalities.

Label your region **R**.

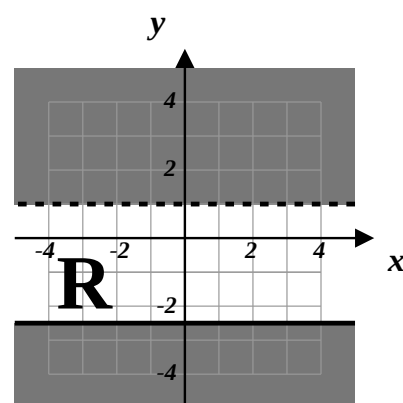
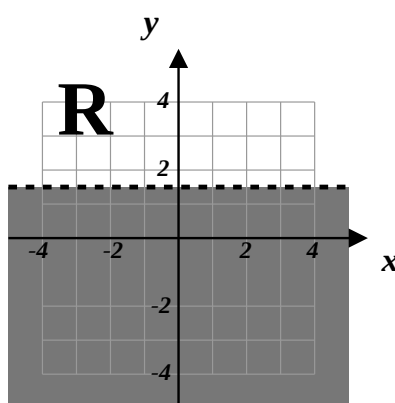
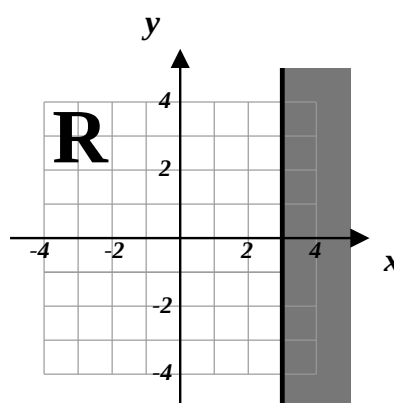
(Try doing this by shading the part to be *thrown away*)



Question 2

Describe the region that has NOT been shaded.

In each case, it is marked **R**.



Advertisement

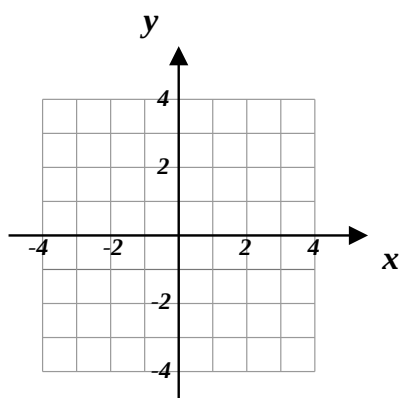
BUY MORE THAN CAR INSURANCE

Question 3

Show, by shading on the grid, the region which satisfies these two inequalities.

$$y \leq 2 \quad \text{and} \quad -1 < x \leq 3$$

Label your region **R**.

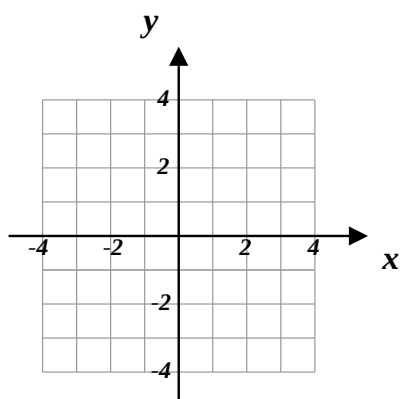


Question 4

Show, by shading on the grid, the region which satisfies these two inequalities.

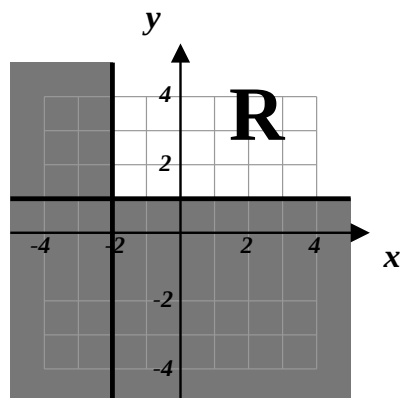
$$y \leq x \quad \text{and} \quad -1 < y \leq 3$$

Label your region **R**.



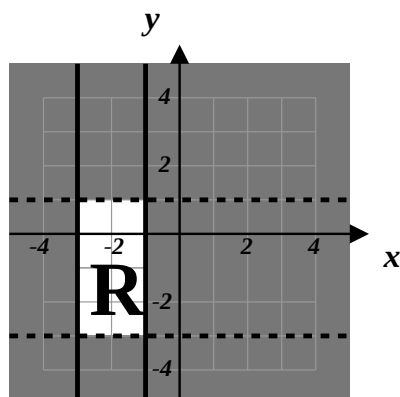
Question 5

Describe the region marked **R**.



Question 6

Describe the region marked **R**.



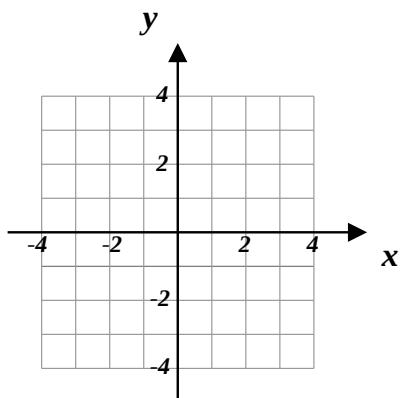
Question 7

GCSE Examination Question from May 2006, Paper 3H, Q9.

Show, by shading on the grid, the region which satisfies these two inequalities.

$$1 \leq x \leq 3 \quad \text{and} \quad -4 < y \leq -2$$

Label your region **R**.



[3 marks]

My Cat says "More is Less"

> = <

7.4 Answers

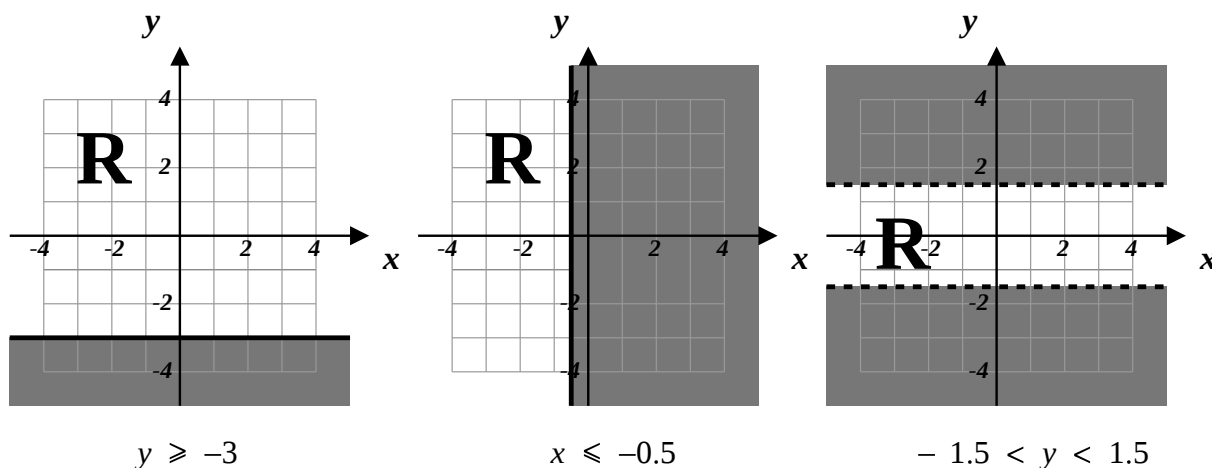
Answer 1

I've shaded the region **NOT** wanted and to be *thrown away*.

The shading is unimportant.

It's in which region you place the letter **R** that is crucial.

Also look carefully at each boundary : broken or a solid line ?



Answer 2

The region R is;

$$x \leq 3$$

$$y > 1.5$$

$$-2.5 \leq y < 1$$

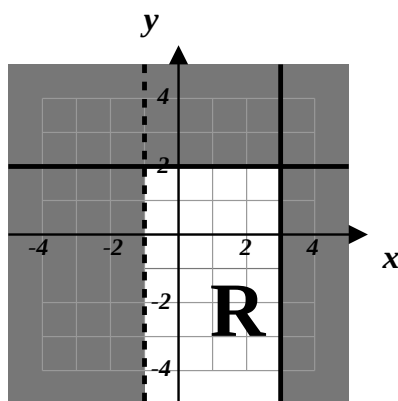
Answer 3

I've shaded the region **NOT** wanted and to be *thrown away*.

The shading is unimportant.

It's in which region you place the letter **R** that is crucial.

Also look carefully at each boundary : broken or a solid line ?



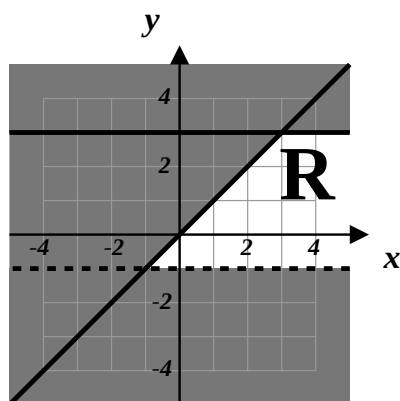
Answer 4

I've shaded the region **NOT** wanted and to be *thrown away*.

The shading is unimportant.

It's in which region you place the letter **R** that is crucial.

Also look carefully at each boundary : broken or a solid line ?



Answer 5

$$x \geq -2 \quad \text{and} \quad y \geq 1$$

Answer 6

$$-3 \leq x \leq -1 \quad \text{and} \quad -3 < y < 1$$

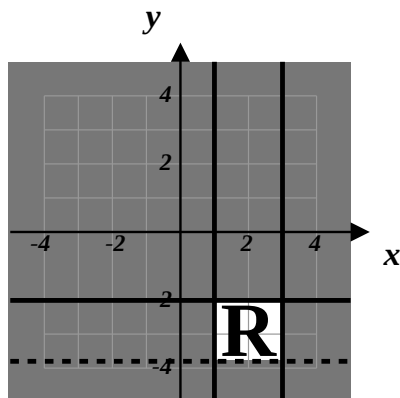
Answer 7

I've shaded the region **NOT** wanted and to be *thrown away*.

The shading is unimportant.

It's in which region you place the letter **R** that is crucial.

Also look carefully at each boundary : broken or a solid line ?





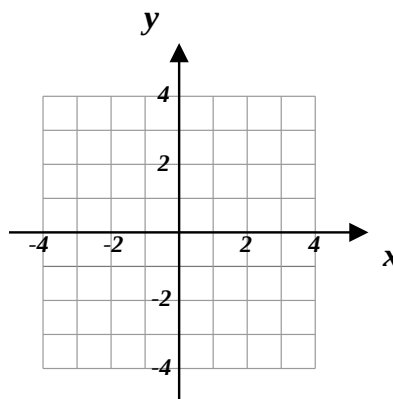
GCSE : Algebra of Inequalities

- Revise equations of sloping lines.
- Tackle inequality question involving sloping lines.

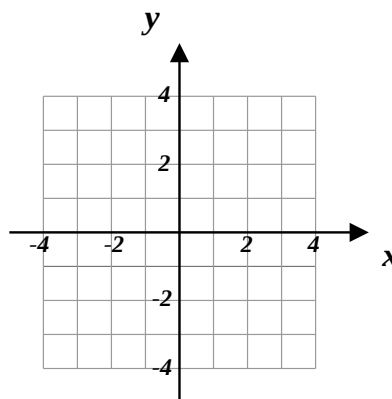
Sloping lines almost all have equations of the form $y = mx + c$, where c is the y -axis intercept and m is the gradient of the line;

$$m = \frac{\Delta y}{\Delta x}$$

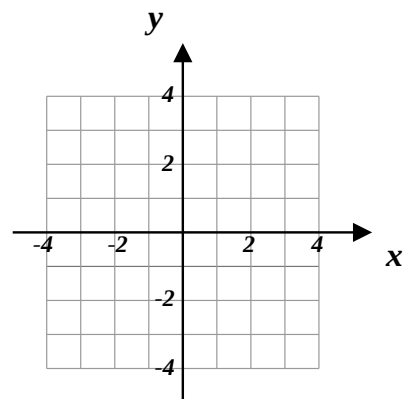
Plot the given lines. Then check your answers with mine, over the page.



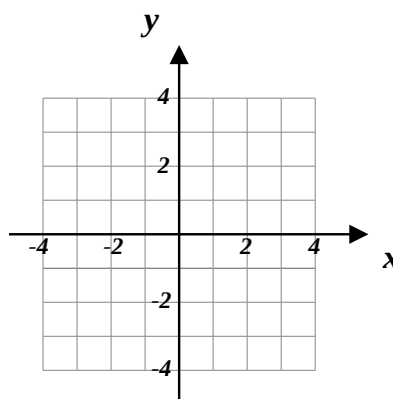
$$y = 2x - 1$$



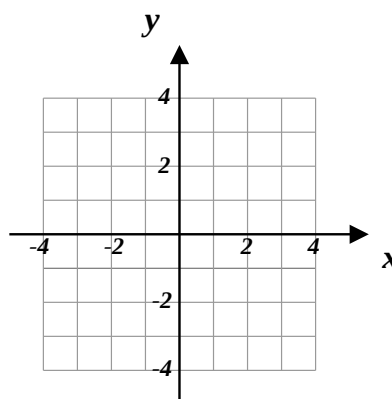
$$y = 3x - 2$$



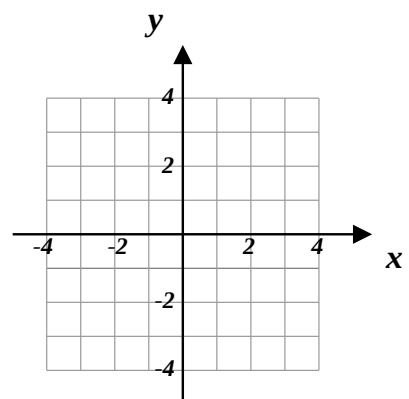
$$y = x$$



$$y = -2x + 3$$

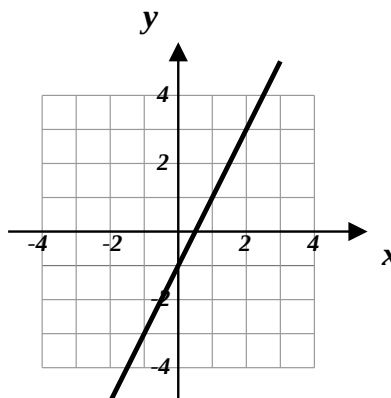


$$y = x + 2$$

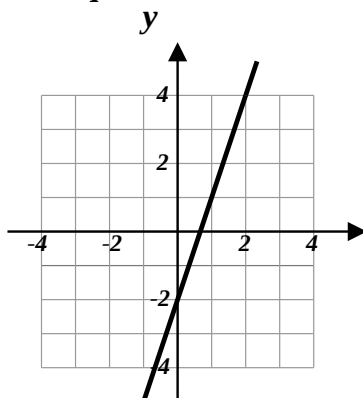


$$y = -x + 2$$

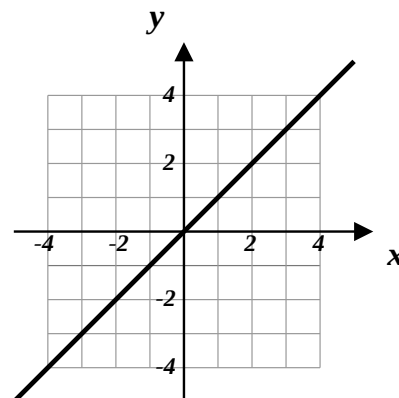
8.4 Answers to the revision questions



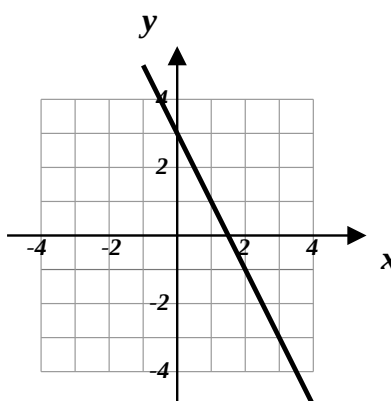
$$y = 2x - 1$$



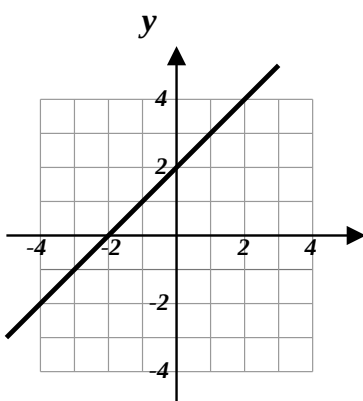
$$y = 3x - 2$$



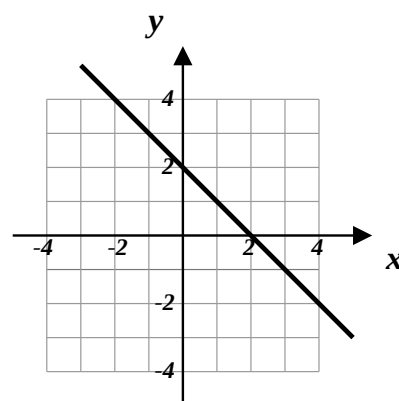
$$y = x$$



$$y = -2x + 3$$



$$y = x + 2$$



$$y = -x + 2$$

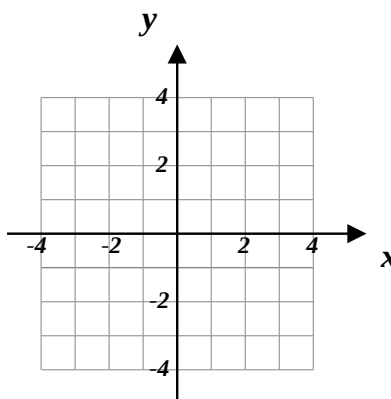
8.5 Tri to try

Show, by shading on the grid, the region which satisfies these inequalities.

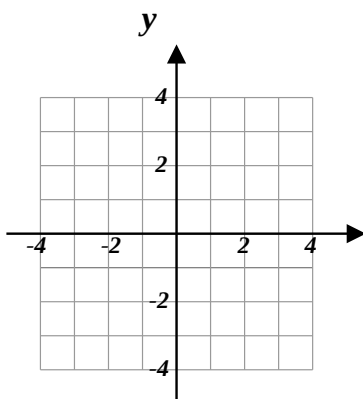
Keep in mind that $<$ and $>$ require broken lines, $\leq$ and $\geq$ solid.

Also, y is *height* so ask yourself if you want *heights* above or below the line.

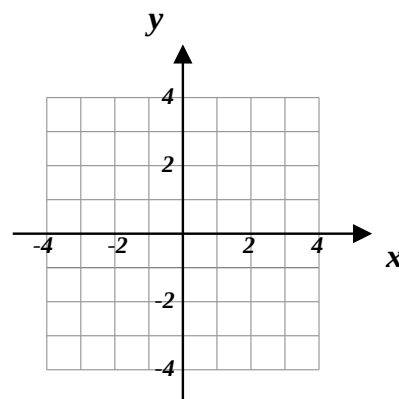
Label the region specified by the inequality with an **R**.



$$y < 2x - 3$$



$$y \geq x - 1$$

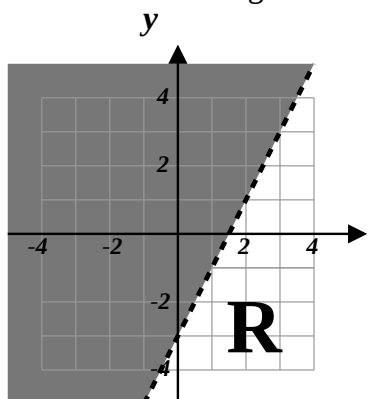


$$y \leq 0.5x + 1$$

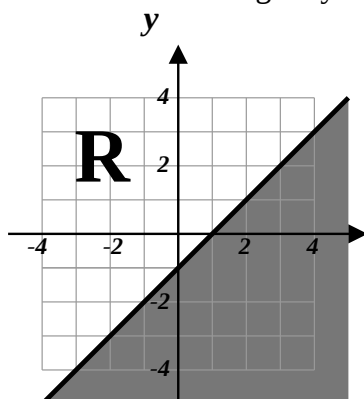
8.6 Answers to "Tri to try"

I've shaded the region **NOT** wanted and to be *thrown away*.

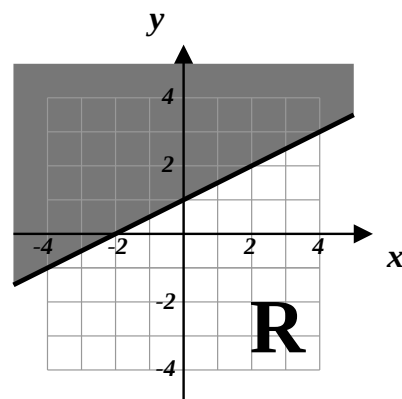
The shading is unimportant. It's in which region you place the letter **R** that is crucial.



$$y < 2x - 3$$



$$y \geq x - 1$$

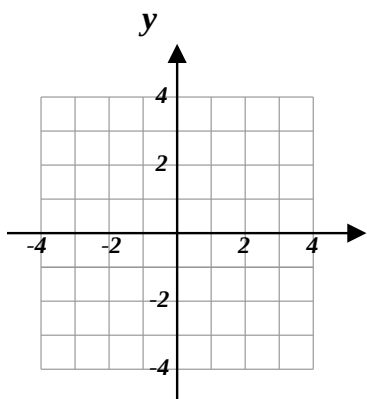


$$y \leq 0.5x + 1$$

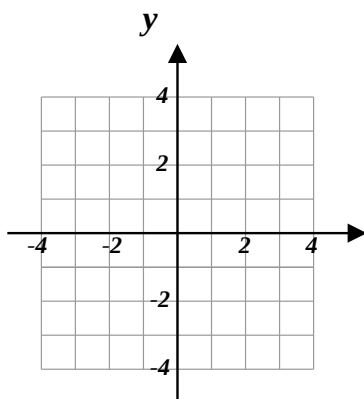
8.7 Exercise

Question 1

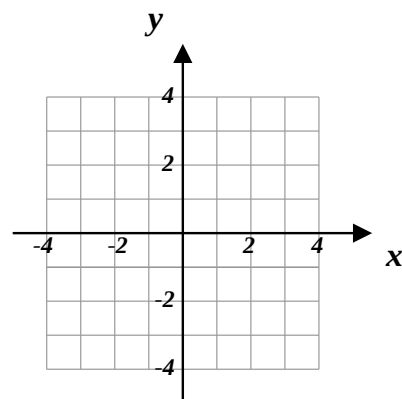
Plot the given lines.



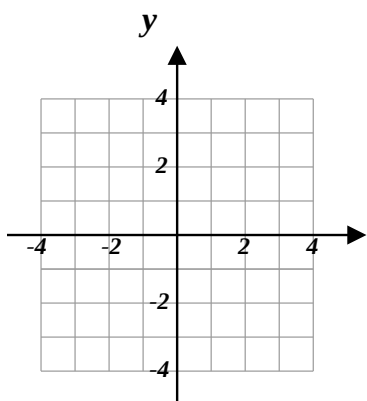
$$y = 2x - 4$$



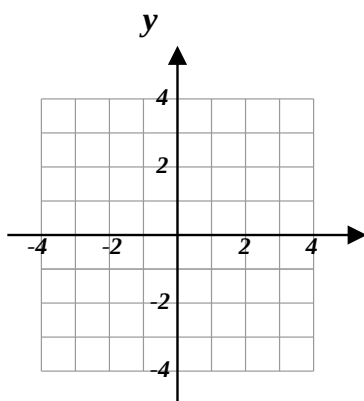
$$y = 4x + 1$$



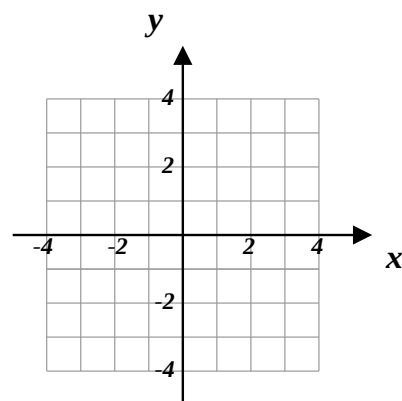
$$y = -2x - 1$$



$$y = -x + 3$$



$$y = 0.5x + 3$$

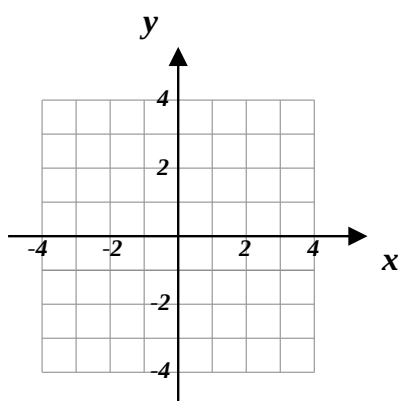


$$y = -0.5x + 3$$

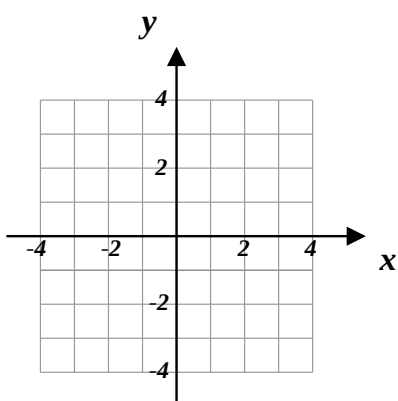
Question 2

Show, by shading on the grid, the region which satisfies these inequalities.

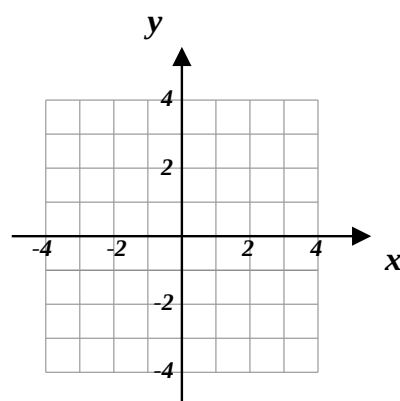
Label the region specified by the inequality with an **R**.



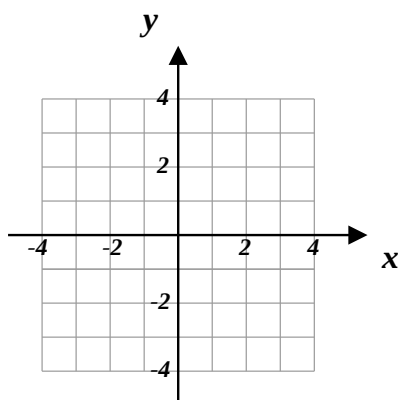
$$y < 2x + 1.5$$



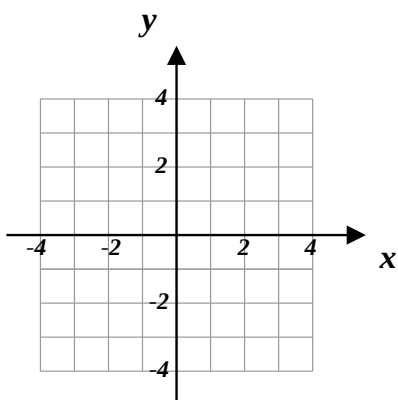
$$y \geq 0.5x - 3$$



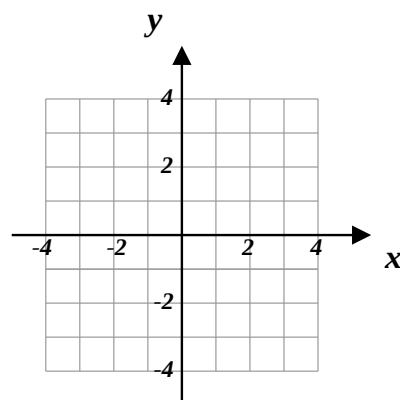
$$y \leq -x + 1$$



$$y > 2x$$



$$y \geq 1.5x - 1.5$$



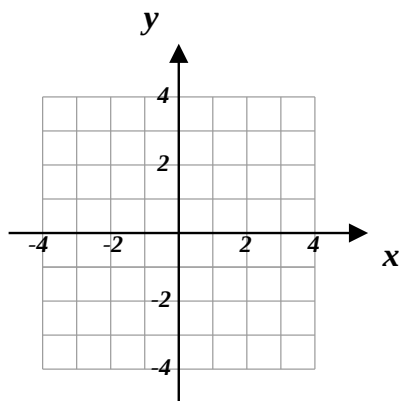
$$y \leq -3x + 4$$

Question 3

Show, by shading on the grid, the region which satisfies all three of these inequalities.

$x \geq -2$ **and** $y > x$ **and** $y < 3$

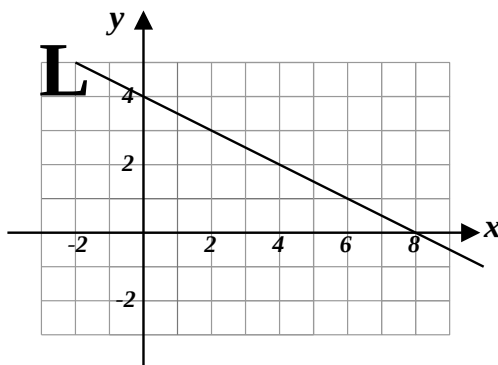
Label your region **R**.



Question 4

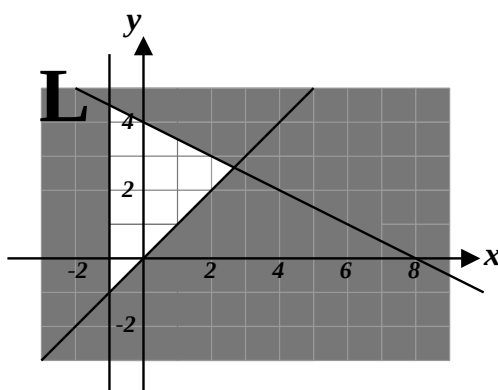
GCSE Examination Question from June 2010, Paper 4H, Q12.

- (a) Find the equation of the line **L**.



[3 marks]

- (b) Find the three inequalities that define the **unshaded** region shown in the diagram below.



[3 marks]

Question 5

GCSE Examination Question from May 2007, Paper 3H, Q13.

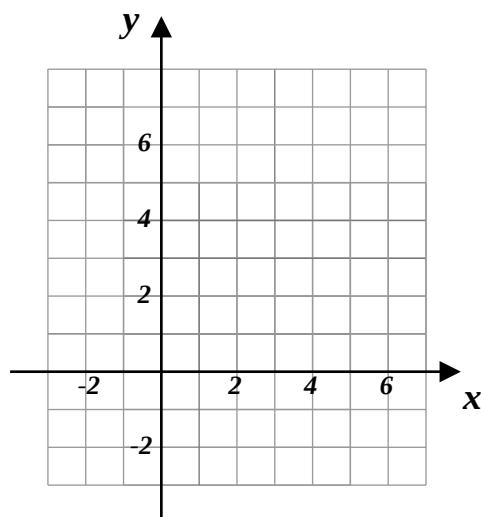
Show, by shading on the grid, the region which satisfies all three inequalities.

$$y \leq 5$$

$$y \leq 2x$$

$$y \geq x + 1$$

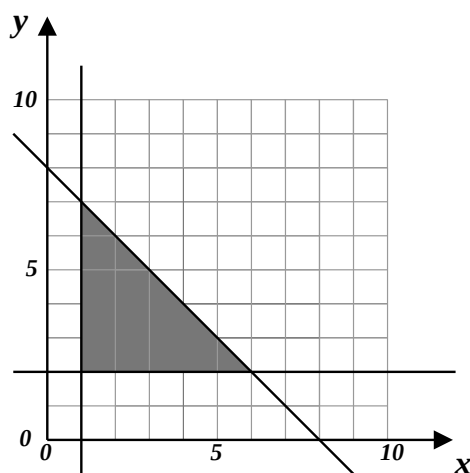
Label your region **R**.



[4 marks]

Question 6

GCSE Examination Question from November 2010, Paper 4H, Q9.

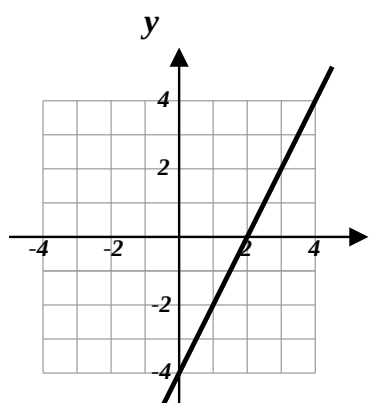


Write down the 3 inequalities that define the shaded region.

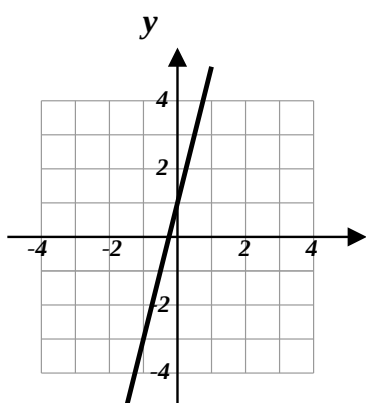
[3 marks]

8.8 Answers

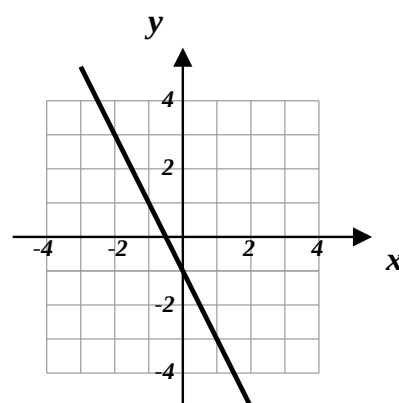
Answer 1



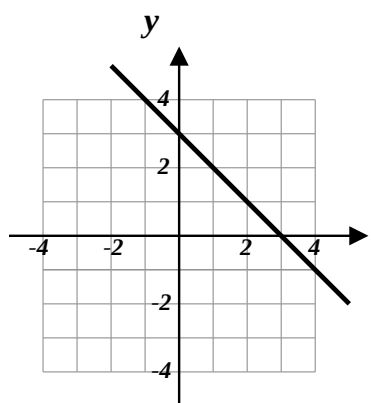
$$y = 2x - 4$$



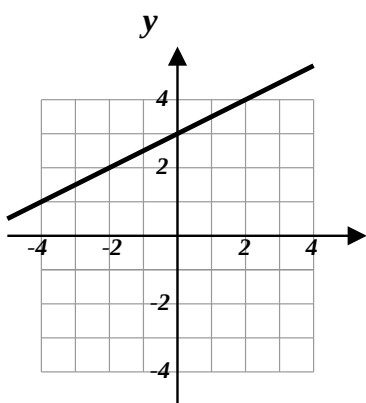
$$y = 4x + 1$$



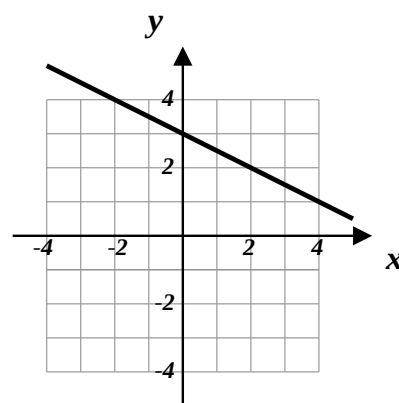
$$y = -2x - 1$$



$$y = -x + 3$$

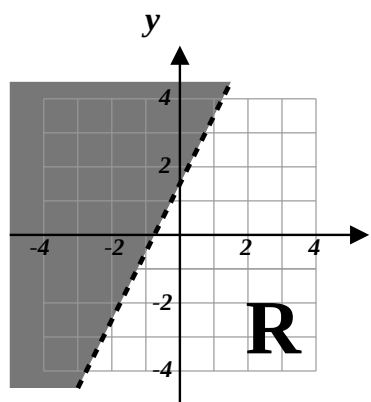


$$y = 0.5x + 3$$

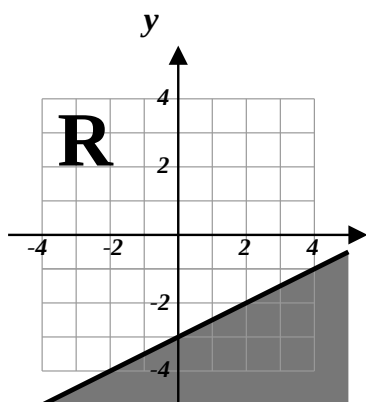


$$y = -0.5x + 3$$

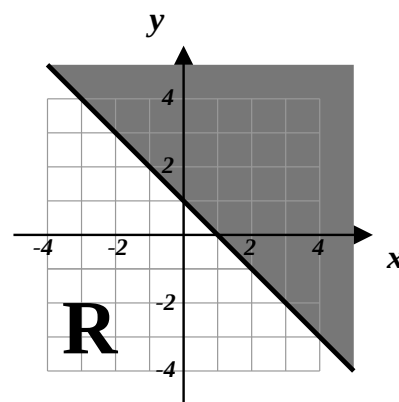
Answer 2



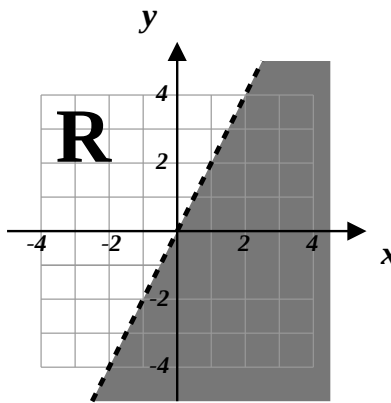
$$y < 2x + 1.5$$



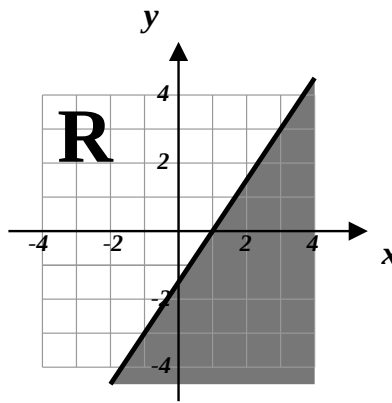
$$y \geq 0.5x - 3$$



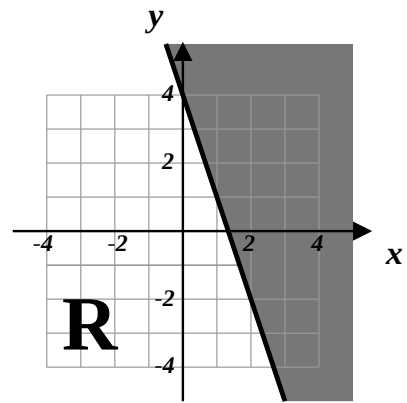
$$y \leq -x + 1$$



$$y > 2x$$

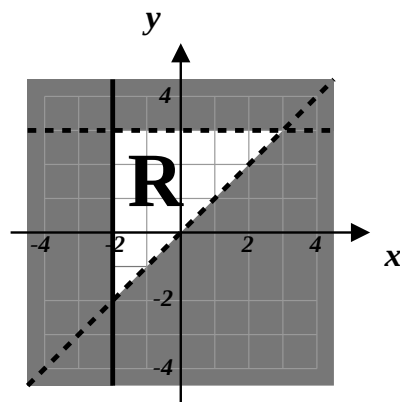


$$y \geq 1.5x - 1.5$$



$$y \leq -3x + 4$$

Answer 3



Answer 4

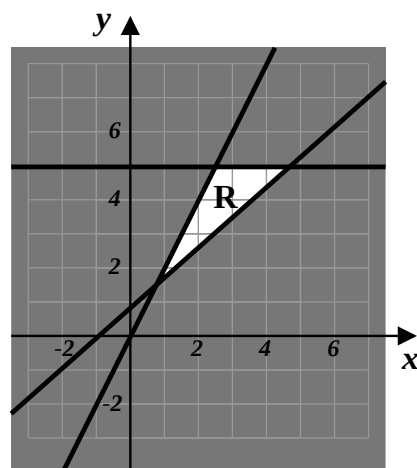
(a) $y = -0.5x + 4$

(b) $x \geq -1$

$y \geq x$

$y \leq -0.5x + 4$

Answer 5



Answer 6

$x \geq 1$

$y \geq 2$

$y \leq -x + 8$

Lesson 9

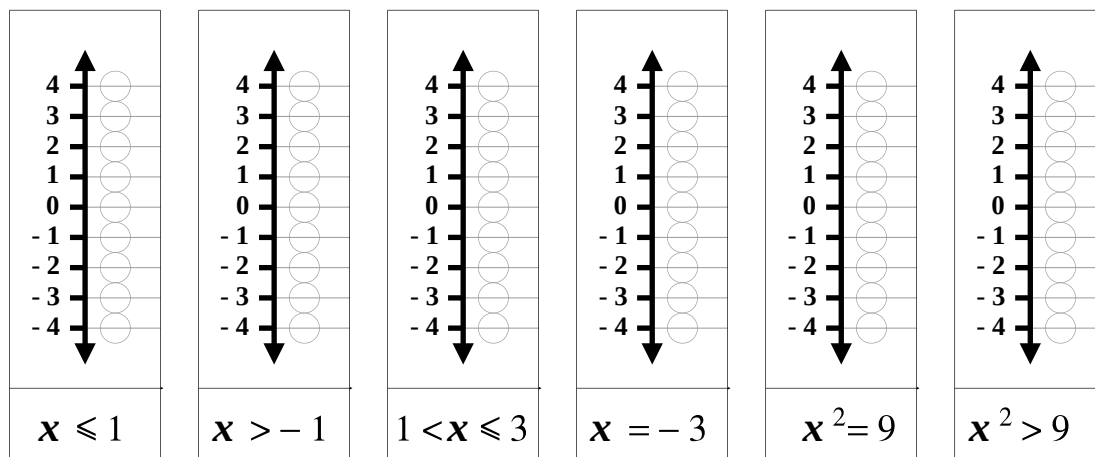
GCSE : Algebra of Inequalities

9.1 REVISION

Do NOT use a calculator

Question 1

On the number ladders below, draw the inequalities;



Question 2

Solve these inequalities;

(i) $5x + 17 < 52$

(ii) $0.8 + 2.6x > -1.8$

(iii) $-\frac{3}{8} + 2x \geq \frac{5}{8}$

(iv) $18 - 3x \leq 9$

(v) $7(x - 5) > 42$

(vi) $\frac{x}{5} + 12 < 19$

Question 3

In each of the following, list the possible values of x ;

(i) x is EVEN and $41 < x < 47$.

(ii) x is SQUARE and $45 < x < 100$.

(iii) x is INTEGER and $\pi < x < 2\pi$

Question 4

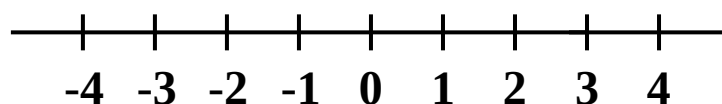
True or False ?

If y is a year in the 19th century then $1800\text{AD} \leq y < 1900\text{AD}$

Question 5

(i) Solve the inequality $-22 \leq 5x - 7 < -7$

(ii) On the number line, represent the solution to part (i).



Question 6

Solve the following inequality;

$$\frac{3x + 7}{2} \geq 44$$

Question 7

Solve these inequalities;

(i) $5 < x - 13 \leq 8$

(ii) $48 - 5x > -12$

(iii) $9x + 17 > 5x + 45$ (iv) $39 \leq 3(x + 8) < 66$

Question 8

If $x^2 < 36$ and x is a negative integer, what are the possible values of x ?

Question 9

This question is about solving the quadratic inequality

$$x^2 - 10x + 21 \leq 0$$

This is to be tackled by first plotting the graph of the related equality,

$$y = x^2 - 10x + 21$$

(a) To that end first complete the following table;

x	0	1	2	3	4	5	6	7	8	9	10
x^2				9	16					81	
$-10x$					- 40				- 80		
$+21$	21	21			21						21
y					- 3						

(b) On the grid on the opposite page, graph the equality.

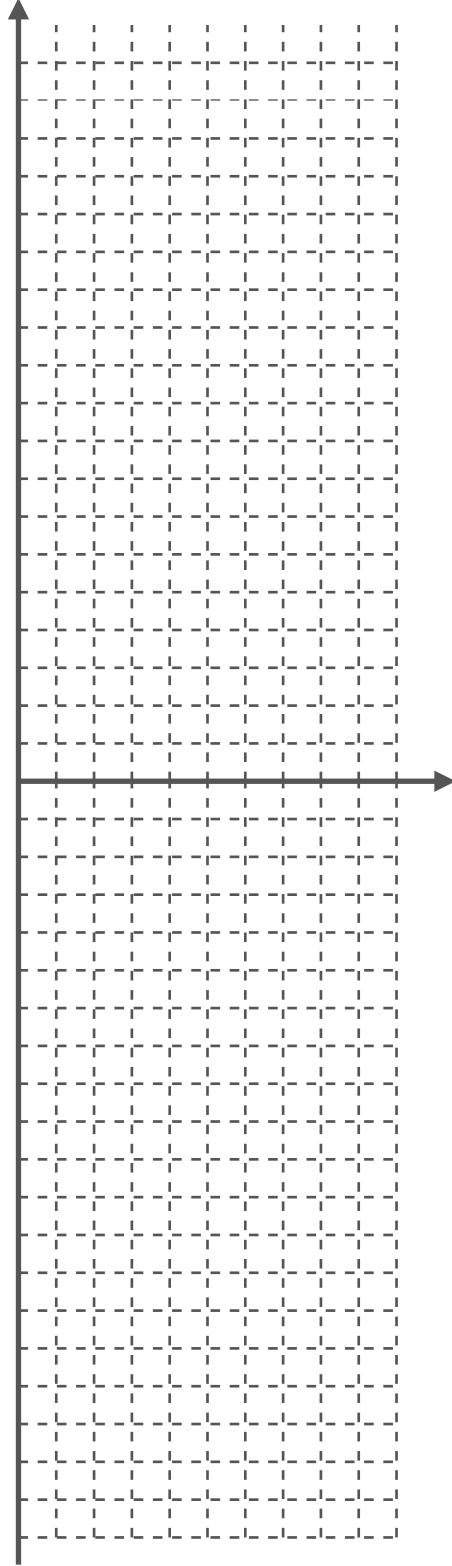
(c) From your graph, solve the quadratic inequality

$$x^2 - 10x + 21 \leq 0$$

giving your answer in the form

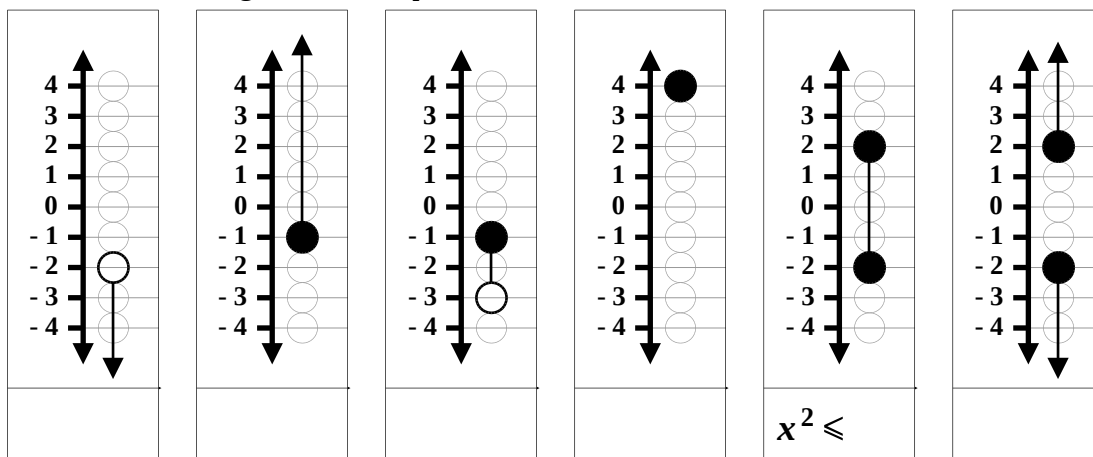
$$a \leq x \leq b$$

where a and b are numbers you must find.



Question 10

Write out in algebra the inequalities illustrated below;



Question 11

Solve the following inequality;

$$10 \leq 43 - 3x < 37$$

Question 12

Let P be the amount of petrol in my car's tank.

(a) Write down the following statements as inequalities;

(i) My car's petrol tank can hold 38 litres.

(ii) I put 24 litres of petrol in my car this morning.

(b) Combine the information of part (a) as an inequality of the form

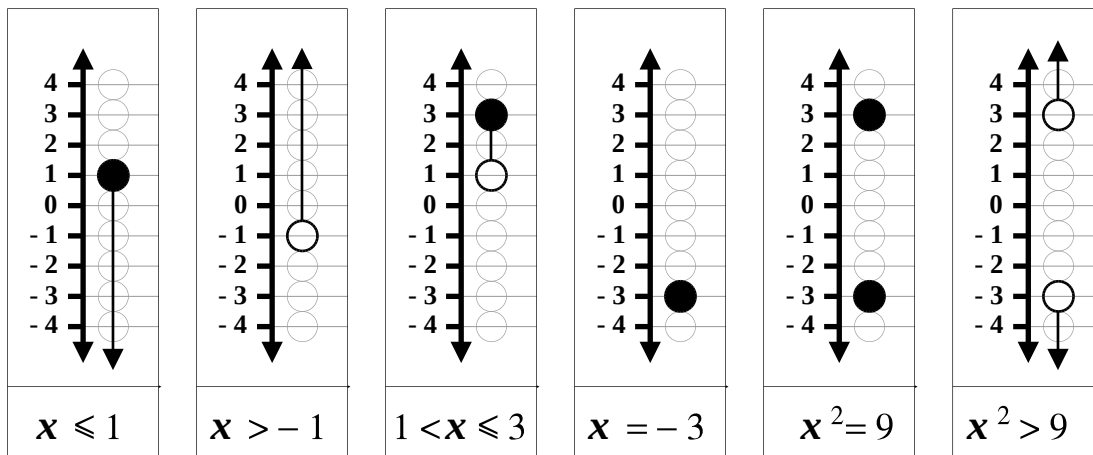
$$a \leq P \leq b$$

where a and b are numbers you must find.

NOTE : The TEST will also include several questions on inequality graphs.

9.2 Answers

Answer 1



Answer 2

- | | |
|--------------------|-----------------|
| (i) $x < 7$ | (ii) $x > -1$ |
| (iii) $x \geq 0.5$ | (iv) $x \geq 3$ |
| (v) $x > 11$ | (vi) $x < 35$ |

Answer 3

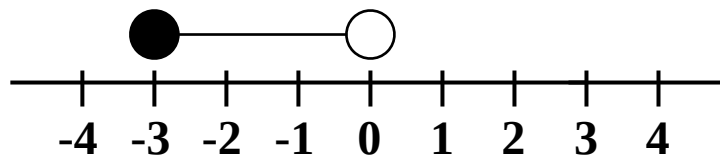
- (i) $x \in \{ 42, 44, 46 \}$
(ii) $x \in \{ 49, 64, 81 \}$
(iii) $x \in \{ 4, 5, 6 \}$

Answer 4

True

Answer 5

- (i) $-3 \leq x < 0$
(ii)



Answer 6

$x \geq 27$

Answer 7

- | | |
|----------------------|----------------------|
| (i) $18 < x \leq 21$ | (ii) $x < 12$ |
| (iii) $x > 7$ | (iv) $5 \leq x < 14$ |

Answer 8

$$x \in \{ -5, -4, -3, -2, -1 \}$$

Answer 9

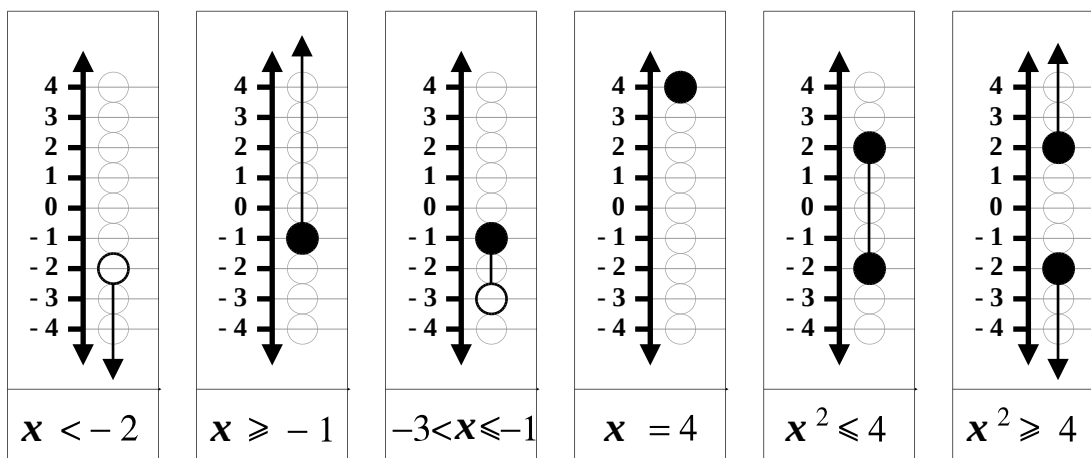
(a)

x	0	1	2	3	4	5	6	7	8	9	10
x^2	0	1	4	9	16	25	36	49	64	81	100
$-10x$	0	-10	-20	-30	-40	-50	-60	-70	-80	-90	-100
$+21$	21	21	21	21	21	21	21	21	21	21	21
y	21	12	5	0	-3	-4	-3	0	5	12	21

(b) Quadratic curve through (3, 0) and (7, 0)

(c) $3 \leq x \leq 7$

Answer 10



Answer 11

$$2 < x \leq 11$$

Answer 12

(a) (i) $P \leq 38$ or $0 \leq P \leq 38$

(ii) $P \geq 24$

(b) $24 \leq P \leq 38$

Lesson 10

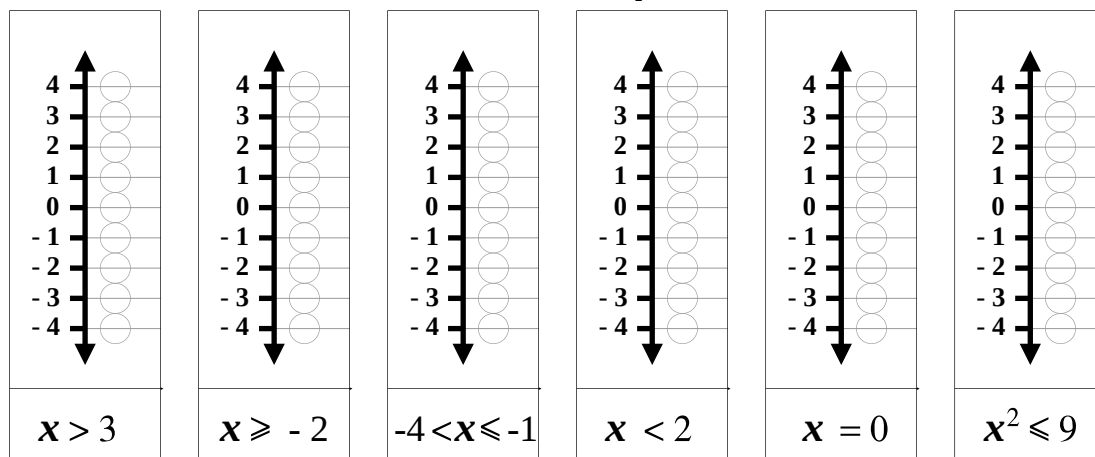
GCSE : Algebra of Inequalities

10.1 TEST

Non Calculator

Question 1

On the number ladders below, draw the inequalities;



[6 marks]

Question 2

Solve these inequalities

(i) $7x + 9 < 37$

(ii) $0.8x + 3.8 < 12.6$

(iii) $-13 + 2x \geq 19$

(iv) $4x - \frac{5}{8} \leq \frac{3}{8}$

(v) $4(x - 5) > 12$

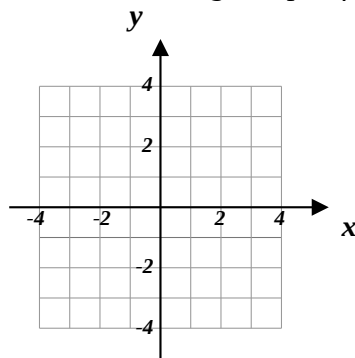
(vi) $\frac{x}{7} + 17 < 28$

[12 marks]

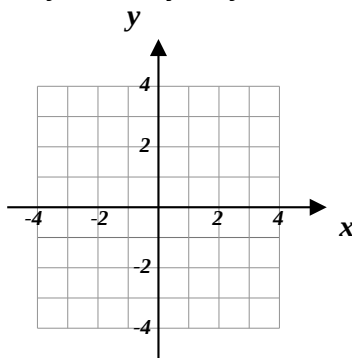
Question 3

Draw a graph for each inequality.

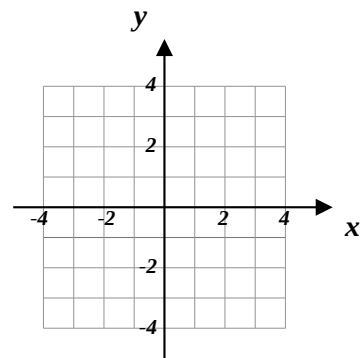
Mark the region specified by the inequality **R**.



$y > 3$



$1 \leq x < 3$



$y \geq x$

[6 marks]

Question 4

In each of the following, list the possible values of x ;

(i) x is ODD and $233 \leq x < 241$

(ii) x is SQUARE and $8.5 < x < 27.2$

(iii) x is INTEGER and $\sqrt{12} < x < \sqrt{32}$

[3 marks]

Question 5

True or false ?

If y is a year in the 17th century then $1700 \leq y < 1800$.

[1 mark]

Question 6

Let A be the number of prime numbers less than 24.

Let B be the number of square numbers less than 101.

Which one of the following statements is true ?

(a) $A > B$

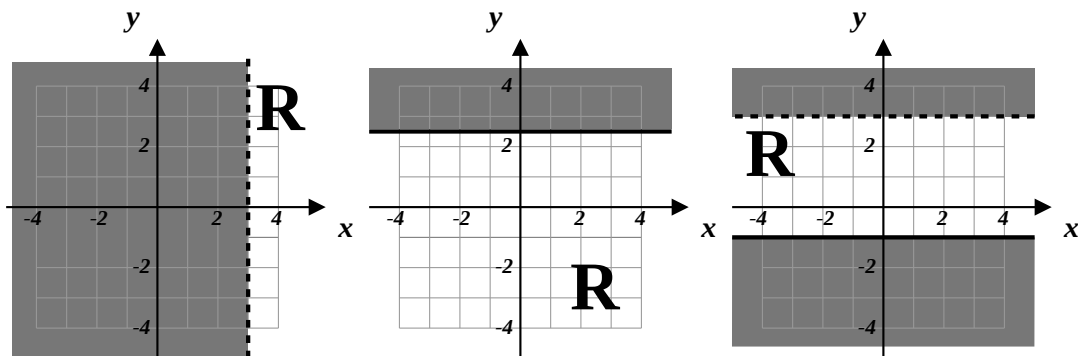
(b) $A < B$

(c) $A = B$

[2 marks]

Question 7

For each graph, write down the inequality specified by the region labelled **R**.

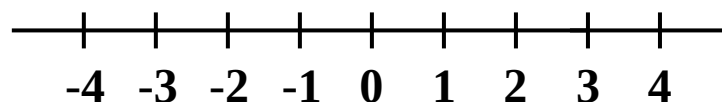


[3 marks]

Question 8

(i) Solve the inequality $-23 < 6x - 5 \leq 7$

(ii) On the number line, represent the solution to part (i).



[3 marks]

Question 9

Solve the following inequality;

$$\frac{3x + 19}{13} < 4$$

[3 marks]

Question 10

Solve the following inequalities;

(i) $\frac{4x}{3} + 7 < 15$

(ii) $101 - 7x < -4$

(iii) $7x + 59 > 3x + 87$

(iv) $42 < 3(x + 9) < 609$

[12 marks]

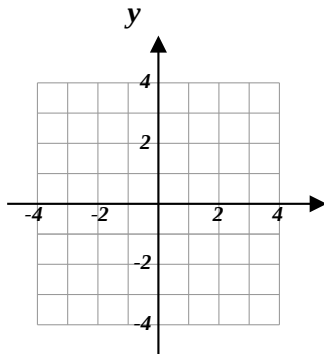
Question 11

If $x^2 < 25$ and x is a negative integer, what are the possible values of x ?

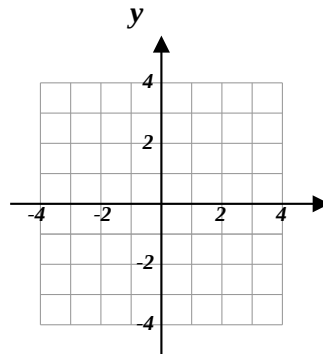
[2 marks]

Question 12

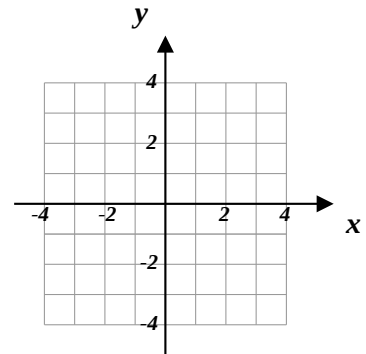
On the grids below, plot the given line.



$$y = 3x - 2$$



$$y = 0.5x + 1$$



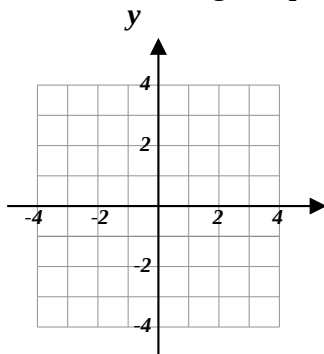
$$y = -x + 3$$

[6 marks]

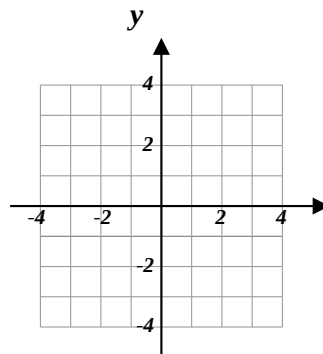
Question 13

Plot the following inequalities.

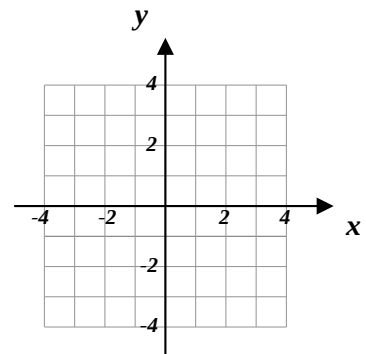
Label the region specified **R**.



$$y < 0.5x - 1$$



$$y \geq 2x + 1$$



$$y \leq -2x + 1$$

[6 marks]

Question 14

Without using a calculator, work out;

(i) $\left(\frac{1}{2}\right)^0$

(ii) $\left(\frac{1}{2}\right)^1$

(iii) $\left(\frac{1}{2}\right)^2$

(iv) $\left(\frac{1}{2}\right)^3$

(v) $\left(\frac{1}{2}\right)^4$

For $x \geq 0$, what inequality can you write that involves $\left(\frac{1}{2}\right)^x$?

[3 marks]

Question 15

GCSE exam question from May 2012, 3H, Q6.

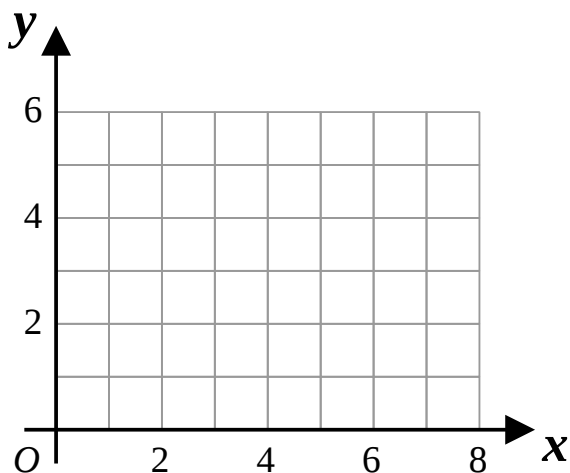
Show, by shading on the grid, the region defined by all three of the inequalities

$$x \leq 5$$

$$y \geq 3$$

$$y \leq x$$

Label your region **R**.



[3 marks]

Question 16

GCSE exam question from November 2009, 4H, Q9.

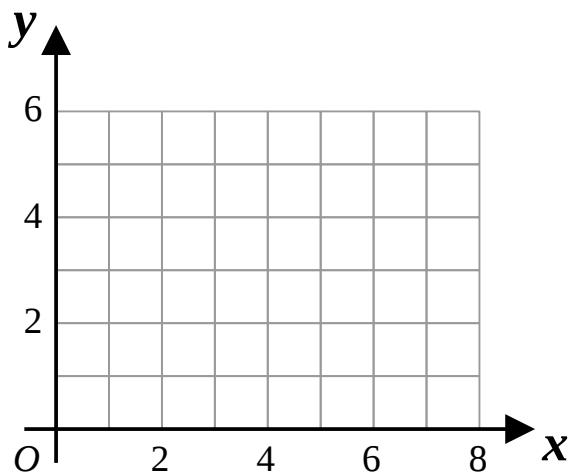
On the grid, show by shading the region which satisfies the inequalities

$$3 \leq x \leq 6$$

and

$$2 \leq y \leq 4$$

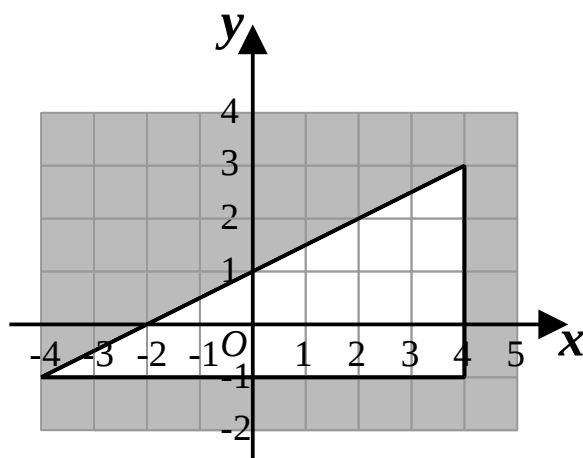
Label your region **R**.



[5 marks]

Question 17

GCSE exam question from May 2008, 4H, Q14.



Write down the three inequalities that define the **unshaded** region.

[3 marks]

Question 18

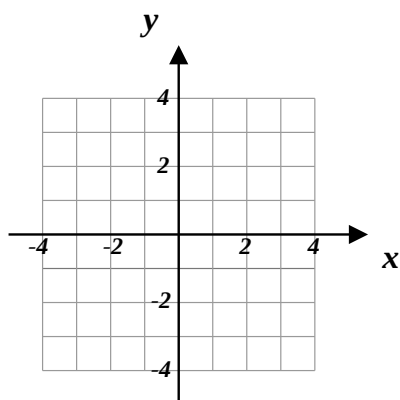
Show, by shading on the grid, the region which satisfies all three of these inequalities.

$$x \geq -1$$

$$y > x$$

$$y < 3$$

Label your region **R**.



[3 marks]

Question 19

This question is about solving the quadratic inequality

$$x^2 - 9x + 14 \leq 0$$

This is to be tackled by first plotting the graph of the related equality,

$$y = x^2 - 9x + 14$$

(a) To that end first complete the following table;

x	0	1	2	3	4	5	6	7	8	9	10
x^2				9	16					81	
$-9x$					- 36				- 72		
$+14$	14	14			14						14
y					- 6						

(b) On the grid on the opposite page, graph the equality.

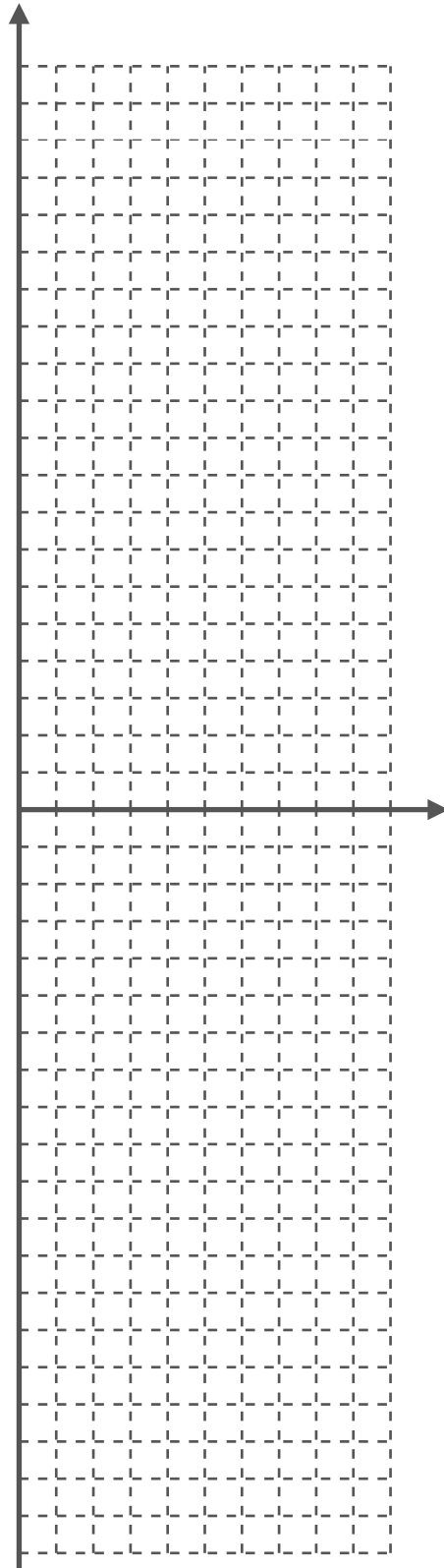
(c) From your graph, solve the quadratic inequality

$$x^2 - 9x + 14 \leq 0$$

giving your answer in the form

$$a \leq x \leq b$$

where a and b are numbers you must find.



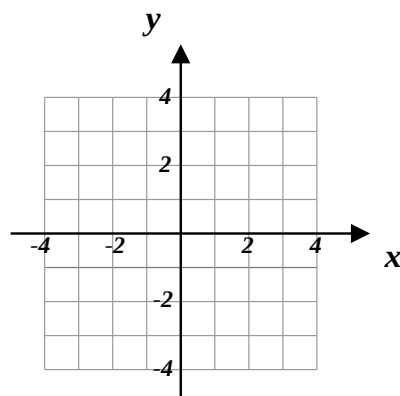
[7 marks]

Turn over for Question 20....

Question 20

Show, by shading on the grid, the region which satisfies the inequality $y^2 \leq 9$

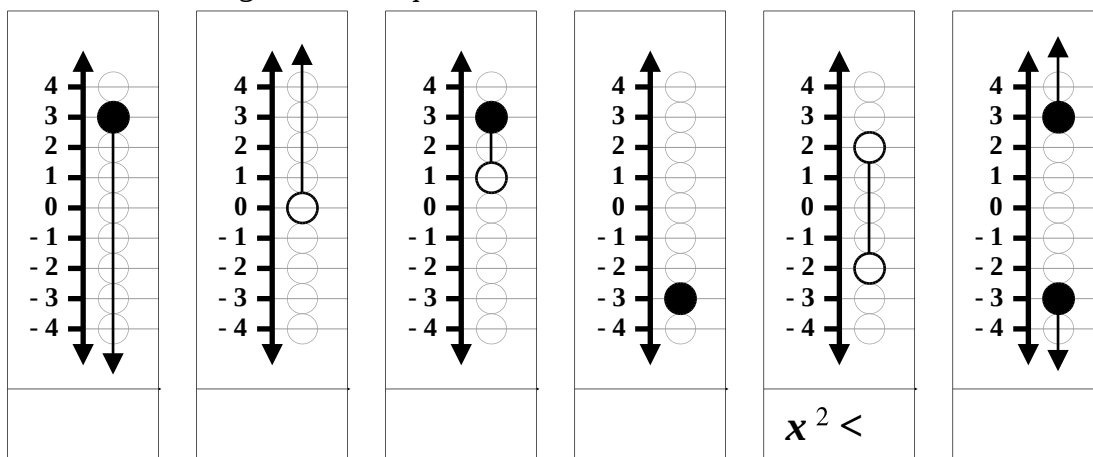
Label your region **R**.



[3 marks]

Question 21

Write out in algebra the inequalities illustrated below;



[6 marks]

Question 22

In a room are lots of boys and girls. I'm not sure exactly how many.....

Let x be the number of people originally in the room.

I get them to hold hands and leave as boy / girl pairs.

I count 156 such pairs leaving.

The people in the room is now less than 100.

(i) Write down an inequality for x from this information.

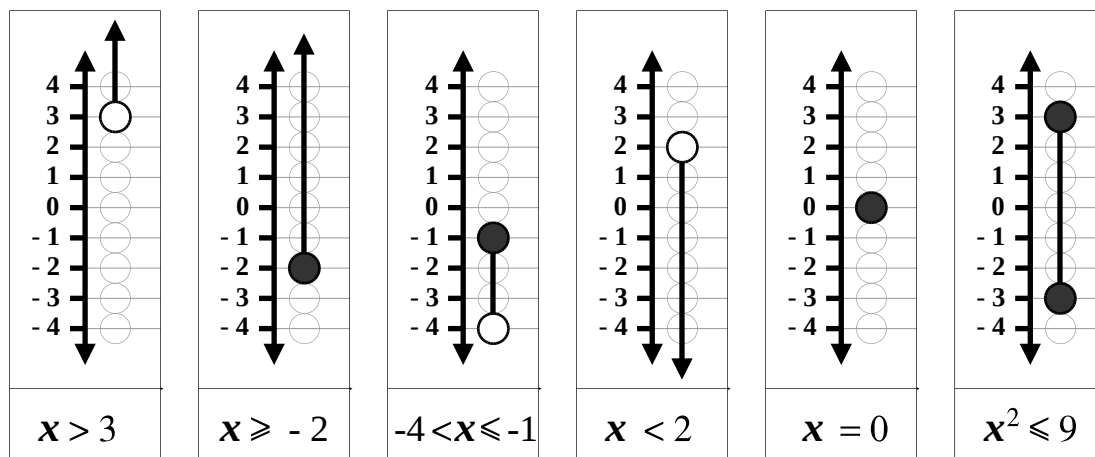
(ii) Solve your part (i) inequality.

[3 marks]

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10.2 Answers

Answer 1

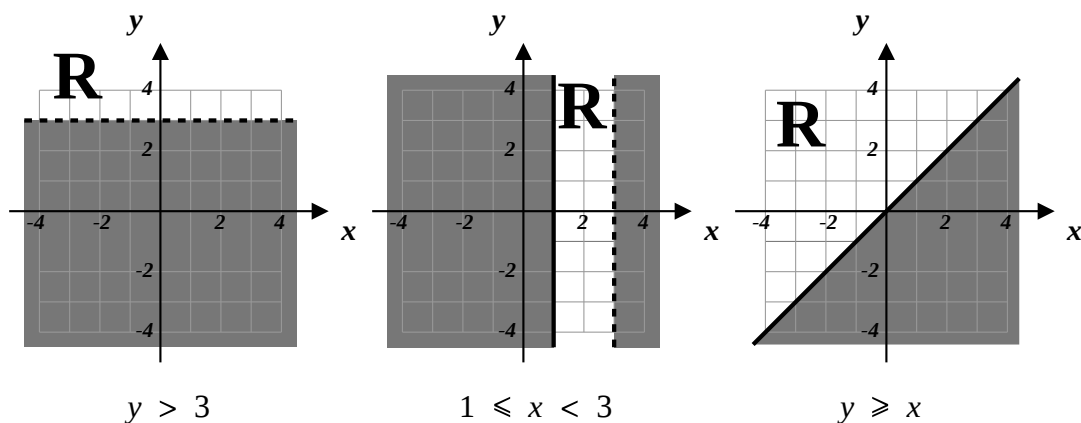


Answer 2

- (a) $x < 4$
 (c) $x \geq 16$
 (e) $x > 8$

- (b) $x < 11$
 (d) $x \leq 0.25$
 (f) $x < 77$

Answer 3



Answer 4

- (i) 233, 235, 237, 239
 (ii) 9, 16, 25
 (iii) 4, 5

Answer 5

FALSE The first century was $0 \leq y < 100$ and so on...

Answer 6

A : 2, 3, 5, 7, 11, 13, 17, 19, 23

9

B :

10

So (b) $A < B$

Answer 7

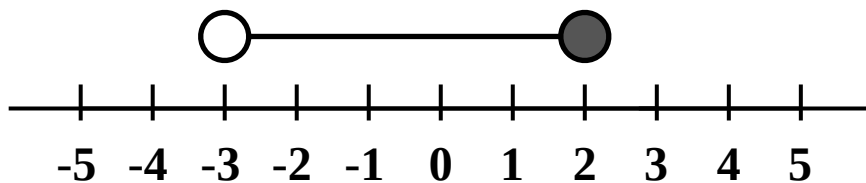
$x > 3$

$y \leq 2.5$

$-1 \leq y < 3$

Answer 8

(i) $-3 < x \leq 2$

(ii)**Answer 9**

$x < 11$

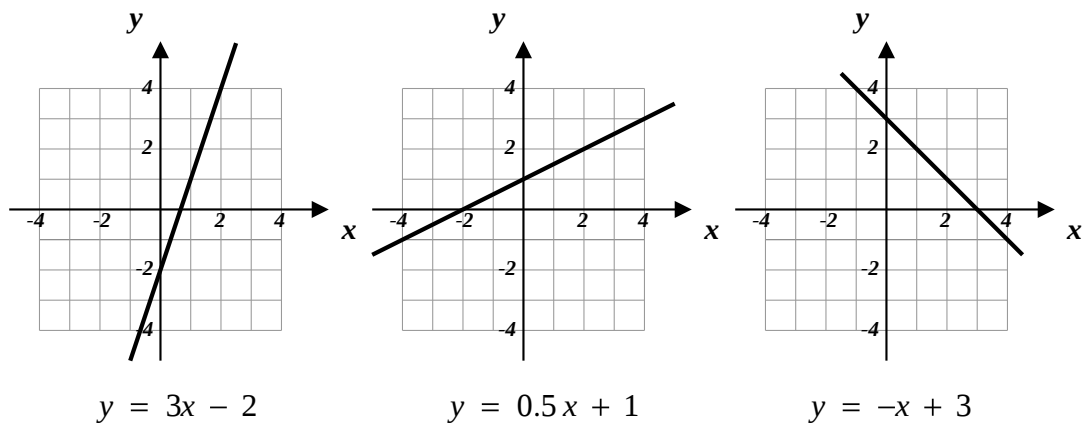
Answer 10

(i) $x < 6$

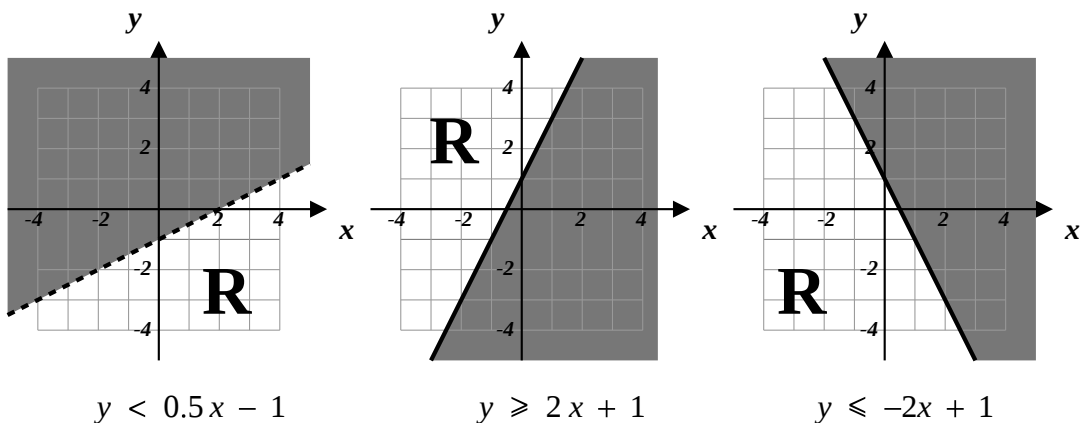
(ii) $x > 15$

(iii) $x > 7$

(iv) $5 < x < 194$

Answer 11 $-4, -3, -2, -1$ **Answer 12**

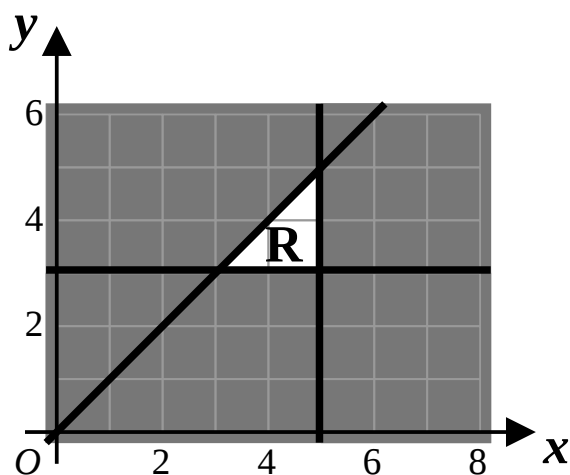
Answer 13



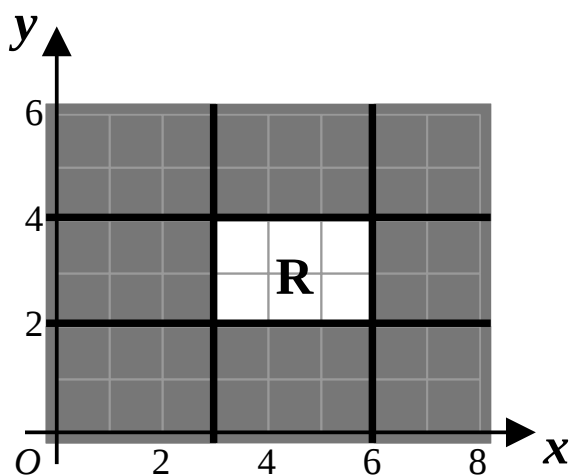
Answer 14

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots \quad 0 < \left(\frac{1}{2}\right)^x \leq 1$$

Answer 15



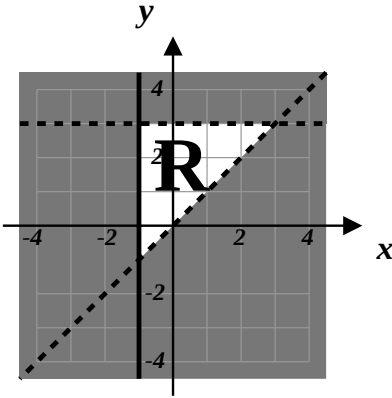
Answer 16



Answer 17

$x \leq 4$ $y \geq -1$ $y \leq \frac{1}{2}x + 1$

Answer 18



Answer 19

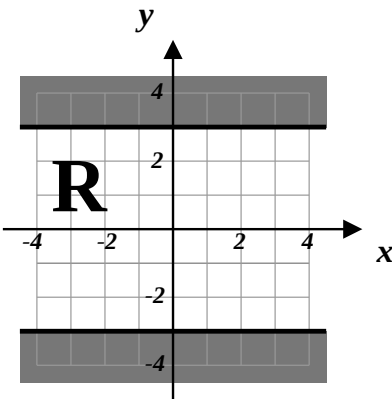
(a)

x	0	1	2	3	4	5	6	7	8	9	10
x^2	0	1	4	9	16	25	36	49	64	81	100
$-9x$	0	-9	-18	-27	-36	-45	-54	-63	-72	-81	-90
$+14$	14	14	14	14	14	14	14	14	14	14	14
y	14	6	0	-4	-6	-6	-4	0	6	14	24

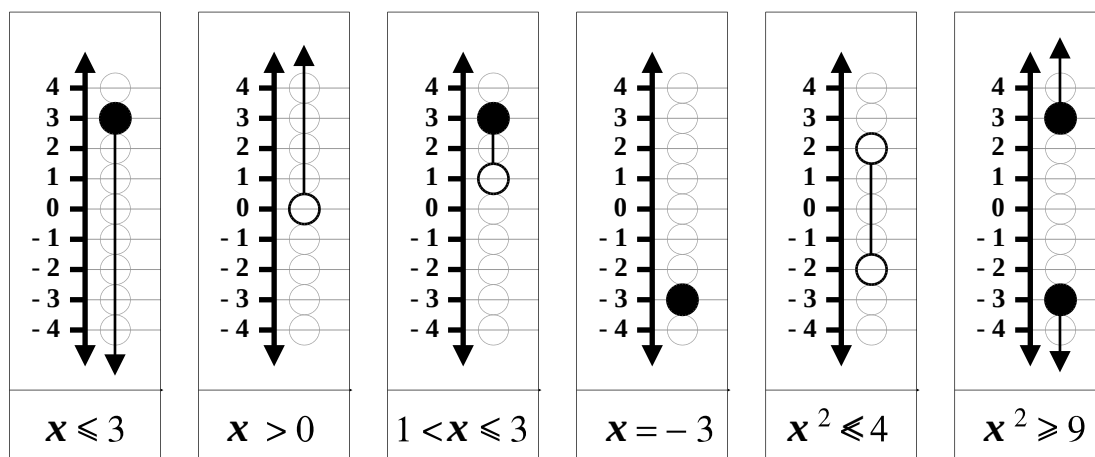
(b) Quadratic curve through (2, 0) and (7, 0)

(c) $2 \leq x \leq 7$

Answer 20



Answer 21



Answer 22

- (i) $0 < x - 156 \times 2 < 100$
(ii) $312 \leq x < 412$