1.3 Answers

1.3.1 Solutions (1.1 Examples)

Answer 1
(i)

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}\,t$$

$$\boldsymbol{v} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 5 \end{pmatrix} \times 4$$

$$\boldsymbol{v} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} + \begin{pmatrix} -12 \\ 20 \end{pmatrix}$$

$$\boldsymbol{v} = \begin{pmatrix} -5 \\ 26 \end{pmatrix} \text{ ms}^{-1}$$

$$\boldsymbol{v} = -5\,\boldsymbol{i} + 26\,\boldsymbol{j} \quad \text{ms}^{-1}$$

(ii)

$$\begin{aligned} speed &= |\,\boldsymbol{v}\,| \\ &= \sqrt{(-5)^2 + (26)^2} \\ &= 26.5 \text{ ms}^{-1} \end{aligned}$$

Answer 2
(i)

$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}\,t^2$$

$$\boldsymbol{s} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \times 5 + \frac{1}{2}\begin{pmatrix} 1 \\ -2 \end{pmatrix} \times 5^2$$

$$\boldsymbol{s} = \begin{pmatrix} -10 \\ 5 \end{pmatrix} + \begin{pmatrix} 12.5 \\ -25 \end{pmatrix}$$

$$\boldsymbol{s} = \begin{pmatrix} 2.5 \\ -20 \end{pmatrix}$$

(ii)

$$\begin{aligned} \boldsymbol{r} &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 2.5 \\ -20 \end{pmatrix} \\ &= \begin{pmatrix} 5.5 \\ -16 \end{pmatrix} \end{aligned}$$

or $5.5\,\boldsymbol{i} - 16\,\boldsymbol{j}$

1.3.2 Solutions (1.2 Exercise)

Answer 1

(i)

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}\,t$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} 7$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -7 \\ 14 \end{pmatrix}$$

$$= \begin{pmatrix} -4 \\ 15 \end{pmatrix}$$

$$\boldsymbol{v} = -4\,\boldsymbol{i} + 15\,\boldsymbol{j} \quad \text{ms}^{-1}$$

(ii)

$$speed = |\,\boldsymbol{v}\,|$$

$$= \sqrt{(-4)^2 + (15)^2}$$

$$= 15.5\ \text{ms}^{-1}$$

Answer 2

(i)

$$\boldsymbol{s} = \boldsymbol{u}t + \frac{1}{2}\boldsymbol{a}\,t^2$$

$$\boldsymbol{s} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \times 3 + \frac{1}{2}\begin{pmatrix} 4 \\ -1 \end{pmatrix} \times 3^2$$

$$\boldsymbol{s} = \begin{pmatrix} 9 \\ 6 \end{pmatrix} + \begin{pmatrix} 18 \\ -4.5 \end{pmatrix}$$

$$\boldsymbol{s} = \begin{pmatrix} 27 \\ 1.5 \end{pmatrix}$$

(ii)

$$\boldsymbol{r} = \begin{pmatrix} -20 \\ 2 \end{pmatrix} + \begin{pmatrix} 27 \\ 1.5 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 3.5 \end{pmatrix}$$

$$\text{or} \quad 7\,\boldsymbol{i} + 3.5\,\boldsymbol{j}$$

Answer 3

To get to the t = 2 location, move for 4 s from the t = 6 s position back down the path using the reversed velocity vector.

Note : The acceleration is zero as the question states the velocity is constant.

i.e.

$$\begin{pmatrix} -4 \\ -7 \end{pmatrix} + 4\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 8 \\ -15 \end{pmatrix}$$

which is the particle's displacement from the origin when t = 2 s

Question asks for *distance* which is the magnitude of the *displacement* vector

i.e.

$$\begin{aligned} distance &= \sqrt{8^2 + (-15)^2} \\ &= 17\ \text{m} \end{aligned}$$

Answer 4

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\begin{pmatrix} -6 \\ 1 \end{pmatrix} = \boldsymbol{u} + \begin{pmatrix} 2 \\ -5 \end{pmatrix} \times 3$$

$$\boldsymbol{u} = \begin{pmatrix} -12 \\ 16 \end{pmatrix}$$

Question requests the initial speed which is the magnitude of the velocity vector

That is,

$$\begin{aligned} u &= \sqrt{(-12)^2 + 16^2} \\ &= 20\ \text{ms}^{-1} \end{aligned}$$

Answer 5

(a) 9.43 ms^{-1}

(b) 328°

(c) Work out where particle is at t = 3 seconds

$$\begin{pmatrix} 7 \\ -10 \end{pmatrix} + \begin{pmatrix} -5 \\ 8 \end{pmatrix} \times 3 = \begin{pmatrix} -8 \\ 14 \end{pmatrix}$$

New constant velocity from this point places particle at origin after 4 s

$$\begin{pmatrix} -8 \\ 14 \end{pmatrix} + \begin{pmatrix} u \\ v \end{pmatrix} \times 4 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\therefore\ u = 2 \quad \text{and} \quad v = -3.5$$

(d) To be due south of A

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ -3.5 \end{pmatrix} \times t = \begin{pmatrix} 7 \\ \text{don't care} \end{pmatrix}$$

$$t = 3.5$$

$$\therefore \text{ Total time is 10.5 seconds}$$

Answer 6

(a) As we have no acceleration, just constant velocity...

$$velocity = \frac{displacement}{time} \qquad \left(Vector\ version\ of\ \ speed = \frac{distance}{time} \right)$$

$$= \frac{\begin{pmatrix} 21 \\ 10 \end{pmatrix} - \begin{pmatrix} 9 \\ -6 \end{pmatrix}}{4}$$

$$= \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$speed = \sqrt{3^2 + 4^2}$$

$$= 5 \ \mathrm{kmh}^{-1}$$

(b) 037°

(c)

$$\boldsymbol{s} = (\ start\ location\ ) + (velocity) \times t$$

$$= \begin{pmatrix} 9 \\ -6 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} \times t$$

$$= \begin{pmatrix} 3t + 9 \\ 4t - 6 \end{pmatrix}$$

$$= (3t + 9)\,\boldsymbol{i} + (4t - 6)\,\boldsymbol{j}$$

(d)

$$\left| \begin{pmatrix} 3t + 9 \\ 4t - 6 \end{pmatrix} - \begin{pmatrix} 18 \\ 6 \end{pmatrix} \right| = 10$$

$$\left| \begin{pmatrix} 3t - 9 \\ 4t - 12 \end{pmatrix} \right| = 10$$

$$\sqrt{(3t - 9)^2 + (4t - 12)^2} = 10$$

square both sides

$$(3t - 9)^2 + (4t - 12)^2 = 100$$

$$9t^2 - 54t + 81 + 16t^2 - 96t + 144 = 100$$

$$25t^2 - 150t + 125 = 0$$

divide through by 25

$$t^2 - 6t + 5 = 0$$

$$(t - 1)(t - 5) = 0$$

$$t = 1 \quad \text{or} \quad t = 5$$

Answer 7

(a) As we have no acceleration, just constant velocity...

$$velocity \;=\; \frac{displacement}{time} \qquad \left(Vector\ version\ of\ \ speed \;=\; \frac{distance}{time} \right)$$

$$= \frac{\begin{pmatrix} 8 \\ 11 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix}}{2.5}$$

$$= \begin{pmatrix} 2 \\ 6 \end{pmatrix}$$

$$= \; 2\,\boldsymbol{i} + 6\,\boldsymbol{j}$$

(b)

$$b \;=\; \begin{pmatrix} 3 \\ -4 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \end{pmatrix} \times \; t$$

$$= \begin{pmatrix} 2t \;+\; 3 \\ 6t \;-\; 4 \end{pmatrix}$$

$$= \; (\,2t + 3\,)\,\boldsymbol{i} \;+\; (\,6t - 4\,)\,\boldsymbol{j}$$

(c) At intercept...

$$\begin{pmatrix} 2t \;+\; 3 \\ 6t \;-\; 4 \end{pmatrix} = \begin{pmatrix} 6t \;-\; 9 \\ \lambda t \;+\; 20 \end{pmatrix}$$

First solve the upper ***i*** component line;

$$2t \;+\; 3 \;=\; 6t \;-\; 9$$

$$4t \;=\; 12$$

$$t \;=\; 3$$

Use this answer in the lower ***j*** component line;

$$6 \;\times\; 3 \;-\; 4 \;=\; 3\lambda \;+\; 20$$

$$14 \;-\; 20 \;=\; 3\lambda$$

$$\lambda \;=\; -2$$

(d) From part (a), B has speed $\sqrt{2^2 \;+\; 6^2} \;=\; \sqrt{40}$

Knowing that $\lambda = 2$, C has speed $\sqrt{6^2 \;+\; (-2)^2} \;= \sqrt{40}$

$\therefore\; B$ and C had the same speed.