3.4 Answers

3.4.1 Solutions (3.2 The Near Miss problem)

(i)

$$d = \sqrt{(t-4)^2 + (t-2)^2}$$

$$d = \sqrt{t^2 - 8t + 16 + t^2 - 4t + 4}$$

$$d = \sqrt{2t^2 - 12t + 20}$$

(ii)

$$d^2 = 2t^2 - 12t + 20$$

$$\frac{1}{2}d^2 = \left[t^2 - 6t\right] + 10$$

$$\frac{1}{2}d^2 = \left[(t-3)^2 - 9\right] + 10$$

$$\frac{1}{2}d^2 = (t-3)^2 + 1$$

(iii) The smallest value of $\frac{1}{2}d^2$ occurs when $(t-3)^2 = 0$

i.e. When $t = 3$ seconds

(iv) $\boldsymbol{r} = (t-4)\,\boldsymbol{i} + (t-2)\,\boldsymbol{j}$

when $t = 3$ becomes $\boldsymbol{r} = -\boldsymbol{i} + \boldsymbol{j}$

Distance $= \sqrt{2}$

$= 1.414$

3.4.2 Solutions (3.3 Exercise)

Answer 1

(i) $\overrightarrow{PQ} = 5\,\boldsymbol{i} + 12\,\boldsymbol{j}$ **(ii)** 13 ms^{-1} **(iii)** 40 seconds

Answer 2

(i)

$$\boldsymbol{s} = \boldsymbol{u}\,t + \frac{1}{2}\boldsymbol{a}\,t^2$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} \times 3 + \frac{1}{2}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \times 9$$

$$= \begin{pmatrix} 9 \\ 3 \end{pmatrix} + \begin{pmatrix} -4.5 \\ 4.5 \end{pmatrix}$$

$$= \begin{pmatrix} 4.5 \\ 7.5 \end{pmatrix}$$

So position is $20.5\,\boldsymbol{i} + 0.5\,\boldsymbol{j}$

(ii)

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$
$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \end{pmatrix} t$$
$$= \begin{pmatrix} 3 - t \\ 1 + t \end{pmatrix}$$
$$\boldsymbol{v} = (3 - t)\,\boldsymbol{i} + (1 + t)\,\boldsymbol{j}$$

(iii) We move in the direction $\boldsymbol{i} + \boldsymbol{j}$ when $3 - t = 1 + t$
Solving this gives $t = 1$ second

Answer 3

(i) When $t = 5$, $\boldsymbol{r} = 0\,\boldsymbol{i} + 6\,\boldsymbol{j}$ $\therefore$ Distance from origin is 6 m.

(ii)

$$d = \sqrt{(t - 5)^2 + (t + 1)^2}$$
$$d = \sqrt{t^2 - 10t + 25 + t^2 + 2t + 1}$$
$$d = \sqrt{2t^2 - 8t + 26}$$

(iii)

$$30 = \sqrt{2t^2 - 8t + 26}$$
$$900 = 2t^2 - 8t + 26$$
$$2t^2 - 8t - 874 = 0$$
$$t^2 - 4t - 437 = 0$$
$$(t - 23)(t + 19) = 0$$
$$\therefore t = 23 \text{ seconds}$$

Answer 4

(i) $\boldsymbol{r} = 100\,\boldsymbol{i} + 60\,\boldsymbol{j} + (-10\,\boldsymbol{i} + 8\,\boldsymbol{j})\,t$
$\boldsymbol{r} = (100 - 10t)\,\boldsymbol{i} + (60 + 8t)\,\boldsymbol{j}$
Due north requires $100 - 10t = 0$ in which case $t = 10$ hours
$\therefore$ Time is 10 pm

(ii) Distance = $60 + 8 \times 10$
= 140 km

(iii) At 1 pm, $t = 1$, $\boldsymbol{r} = (100 - 10)\,\boldsymbol{i} + (60 + 8)\,\boldsymbol{j}$
Distance = $\sqrt{90^2 + 68^2}$
= 112.8 km

Answer 5

(i) $\overrightarrow{AB} = \boldsymbol{i} - 2\boldsymbol{j}$

(ii) $\boldsymbol{R_A} = (\boldsymbol{i} - 3\boldsymbol{j}) + (\boldsymbol{i} + 3\boldsymbol{j})\,t$
$= \boldsymbol{i} - 3\boldsymbol{j} + t\,\boldsymbol{i} + 3t\,\boldsymbol{j}$
$= (1 + t)\,\boldsymbol{i} + (-3 + 3t)\,\boldsymbol{j}$

$\boldsymbol{R_B} = (-3\boldsymbol{i} + 5\boldsymbol{j}) + (2\boldsymbol{i} + \boldsymbol{j})\,t$
$= -3\boldsymbol{i} + 5\boldsymbol{j} + 2t\,\boldsymbol{i} + t\,\boldsymbol{j}$
$= (-3 + 2t)\,\boldsymbol{i} + (5 + t)\,\boldsymbol{j}$

(iii) $\boldsymbol{R_{AB}} = \boldsymbol{R_B} - \boldsymbol{R_A}$
$= (-4 + t)\,\boldsymbol{i} + (8 - 2t)\,\boldsymbol{j}$

(iv)

$$d = \sqrt{(4 - t)^2 + (8 - 2t)^2}$$
$$d = \sqrt{t^2 - 8t + 16 + 4t^2 - 32t + 64}$$
$$d = \sqrt{5t^2 - 40t + 80}$$
$$d^2 = 5t^2 - 40t + 80$$
$$\frac{1}{5}d^2 = t^2 - 8t + 16$$
$$\frac{1}{5}d^2 = (t - 4)^2$$

This is a minimum for when $t = 4$ seconds

(v) When $t = 4$, $d = 0$ i.e. The skaters collide !