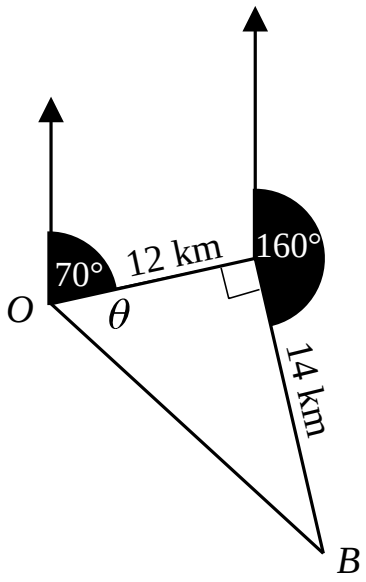
4.2 Answers

A-Level Pure Mathematics
Vectors II : Year 1 and Year 2

Answer 1

(i)



[2 marks]

(ii) $\tan\theta = \frac{14}{12} \Rightarrow \theta = 49°$

$\therefore$ Bearing of the bunker from the oasis is 119°

[2 marks]

Answer 2

(i) $\overrightarrow{YX} = -\boldsymbol{b}$

(ii) $\overrightarrow{BX} = 2\boldsymbol{a}$

(iii) $\overrightarrow{OZ} = \boldsymbol{a} + 2\boldsymbol{b}$

[3 marks]

Answer 3

(i) $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a}$

$= \begin{pmatrix} 4 \\ -2 \end{pmatrix} - \begin{pmatrix} -3 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ -1 \end{pmatrix}$ or $7\boldsymbol{i} - \boldsymbol{j}$

[2 marks]

(ii) $distance = \sqrt{7^2 + 1^2} = \sqrt{50} = 5\sqrt{2}$

or 7.07 km

[2 marks]

Answer 4

(i) $|V_C| = \sqrt{3^2 + 5^2} = \sqrt{34}$ or 5.83 kmh^{-1}

[2 marks]

(ii) Distance = Speed × Time

$= 5.83 \times 2.25$

$= 13.12$ km

[2 marks]

(iii) $\boldsymbol{V_{RC}} = V_C - \boldsymbol{V_R}$

$= \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ 1 \end{pmatrix}$ kmh^{-1} or $\boldsymbol{V_{RC}} = -5\boldsymbol{i} + \boldsymbol{j}$ kmh^{-1}

[2 marks]

(iv) $|\boldsymbol{V_{RC}}| = \sqrt{(-5)^2 + 1^2} = 5.1$ kmh^{-1} (Speed of separation)

$\text{Time} = \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{8}{5.1} = 1.57$ hours i.e. 1 hour and 34 minutes

[2 marks]

Answer 5

$$\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$$

$$\begin{pmatrix} 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} a_x \\ a_y \end{pmatrix} \times 2$$

$$2\begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \text{ms}^{-2} \quad \text{or} \quad \boldsymbol{a} = 2\boldsymbol{i} + \boldsymbol{j} \text{ ms}^{-2}$$

[4 marks]

Answer 6

(i) $d = \sqrt{(3t - 7)^2 + (6t + 1)^2}$

$d = \sqrt{9t^2 - 42t + 49 + 36t^2 + 12t + 1}$

$d = \sqrt{45t^2 - 30t + 50}$

[2 marks]

(ii) $d^2 = 45t^2 - 30t + 50$

$\frac{1}{5}d^2 = 9t^2 - 6t + 10$

$\frac{1}{5}d^2 = (9t^2 - 6t + 1) + 9$

$\frac{1}{5}d^2 = (3t - 1)^2 + 9$

$\frac{1}{5}d^2 = 9\left(t - \frac{1}{3}\right)^2 + 9$ □

[2 marks]

(iii) This is a minimum when $t = \frac{1}{3}$ seconds

[1 mark]

(iv) $\frac{1}{5}d^2 = 9 \Leftrightarrow d^2 = 45 \Rightarrow d = \sqrt{45} = 6.71$ metres

[1 mark]

Answer 7

(i) At time t, the aircraft is at $\boldsymbol{r}_A = \begin{pmatrix} 200 \\ 30 \end{pmatrix} + \begin{pmatrix} 180 \\ -120 \end{pmatrix} t$

It's due east when $30 - 120t = 0 \Rightarrow t = 0.25$ hours
i.e. When the time is 11:15

[2 marks]

(ii) Distance $= 200 + 180 \times 0.25 = 245$ km

[2 marks]

(iii) When t = 1 hour, $\boldsymbol{r}_A = \begin{pmatrix} 380 \\ -90 \end{pmatrix}$

$\therefore$ Distance $= \sqrt{380^2 + (-90)^2} = 390.5$ km

[2 marks]