

2.2 Answers

GCSE Mathematics Non-Right Angled Trigonometry

	$\sin x$	$\cos x$	$\tan x$	$\sin^2 x$	$\cos^2 x$	$\frac{\sin x}{\cos x}$	$\cos^2 x + \sin^2 x$
0°	0.000	1.000	0.000	0.000	1.000	0.000	1.000
10°	0.174	0.985	0.176	0.030	0.970	0.177	1.000
20°	0.342	0.940	0.364	0.117	0.883	0.364	1.000
30°	0.500	0.866	0.577	0.250	0.750	0.577	1.000
40°	0.643	0.766	0.839	0.413	0.587	0.839	1.000
50°	0.766	0.643	1.192	0.587	0.413	1.191	1.000
60°	0.866	0.500	1.732	0.750	0.250	1.732	1.000
70°	0.940	0.342	2.747	0.883	0.117	2.749	1.000
80°	0.985	0.174	5.672	0.970	0.030	5.661	1.000
90°	1.000	0.000	- - -	1.000	0.000	- - -	1.000

The numerical investigation suggests that the following may be true;

$$\frac{\sin x}{\cos x} = \tan x \quad \text{provided} \quad \cos x \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

These are, in fact, true. However, no amount of numerical evidence can **prove** they are true. For that a watertight argument is needed that gives a reason for **why** they must be true. Such a reason is called **a mathematical proof**.

An accessible proof of the above (for $0^\circ < x < 90^\circ$) involves the definitions of $\sin x$, $\cos x$ and $\tan x$ in terms of a triangle's *opposite*, *adjacent* and *hypotenuse* and the use of the *theorem of Pythagoras*. An extension activity would be to try to find it (or look it up).