4.5 Answers

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

4.5.1 Solutions to 4.1 Introductory Examples

Example #1

$$x^2 - 2x - 11 = 0$$

$$x = \frac{2 \pm \sqrt{2^2 - 4 \times 1 \times (-11)}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{48}}{2}$$

$$= \frac{2 \pm 2\sqrt{12}}{2}$$

$$= 1 \pm \sqrt{12}$$

$$= 1 \pm 2\sqrt{3}$$

[3 marks]

Example #2

$$(7 + 2\sqrt{5})(4 + 3\sqrt{5}) = 28 + 21\sqrt{5} + 8\sqrt{5} + 6 \times 5$$

$$= 58 + 29\sqrt{5}$$

[3 marks]

4.5.2 Solutions (4.2 Exercise)

Answer 1

(i) $(6 + \sqrt{2})(3 + \sqrt{2}) = 18 + 6\sqrt{2} + 3\sqrt{2} + 2$

$$= 20 + 9\sqrt{2}$$

[2 marks]

(ii) $(5 + \sqrt{13})(4 + \sqrt{13}) = 20 + 5\sqrt{13} + 4\sqrt{13} + 13$

$$= 33 + 9\sqrt{13}$$

[2 marks]

(iii) $(7 + 3\sqrt{2})(5 + \sqrt{2}) = 35 + 7\sqrt{2} + 15\sqrt{2} + 6$

$$= 41 + 22\sqrt{2}$$

[2 marks]

(iv) $(2 + \sqrt{3})(1 + 5\sqrt{3}) = 2 + 10\sqrt{3} + \sqrt{3} + 15$

$$= 17 + 11\sqrt{3}$$

[2 marks]

Answer 2

$$x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{2^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$= 1 \pm \sqrt{2}$$

[3 marks]

Answer 3

(i) $\left(4 + 7\sqrt{3}\right)\left(5 + 2\sqrt{3}\right) = 20 + 8\sqrt{3} + 35\sqrt{3} + 42$

$$= 62 + 43\sqrt{3}$$

[3 marks]

(ii) $\left(7 + 3\sqrt{2}\right)^2 = \left(7 + 3\sqrt{2}\right)\left(7 + 3\sqrt{2}\right)$

$$= 49 + 21\sqrt{2} + 21\sqrt{2} + 18$$

$$= 67 + 42\sqrt{2}$$

[3 marks]

(iii) $\left(6 + \sqrt{5}\right)\left(3 - 2\sqrt{5}\right) = 18 - 12\sqrt{5} + 3\sqrt{5} - 10$

$$= 8 - 9\sqrt{5}$$

[3 marks]

(iv) $\left(6 - 5\sqrt{3}\right)^2 = \left(6 - 5\sqrt{3}\right)\left(6 - 5\sqrt{3}\right)$

$$= 36 - 30\sqrt{3} - 30\sqrt{3} + 75$$

$$= 111 - 60\sqrt{3}$$

[3 marks]

Answer 4

$$\sqrt{80} + \frac{30}{\sqrt{5}} = \sqrt{16 \times 5} + \frac{30}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= 4\sqrt{5} + 6\sqrt{5}$$

$$= 10\sqrt{5}$$

[2 marks]

Answer 5

$$\sqrt{50} + \sqrt{3} \times \sqrt{6} - \frac{14}{\sqrt{2}} = 5\sqrt{2} + 3\sqrt{2} - 7\sqrt{2}$$
$$= \sqrt{2}$$

[3 marks]

Answer 6

(i) $(20 + 3\sqrt{7})(20 - 3\sqrt{7}) = 400 - 60\sqrt{7} + 60\sqrt{7} - 63$
$$= 337$$

[2 marks]

(ii) $(11 + 2\sqrt{3})(11 - 2\sqrt{3}) = 121 - 22\sqrt{3} + 22\sqrt{3} - 12$
$$= 109$$

[2 marks]

Answer 7

(i) $(x + 1)(x + 2)(x + 3) = (x^2 + 3x + 2)(x + 3)$
$$= x^3 + 6x^2 + 11x + 6$$

[3 marks]

(ii) $(x + 2)(x + 5)(x - 3) = (x^2 + 7x + 10)(x - 3)$
$$= x^3 + 4x^2 - 11x - 30$$

[3 marks]

Answer 8

(i) $(1 + \sqrt{5})(2 + \sqrt{5})(3 + \sqrt{5}) = (7 + 3\sqrt{5})(3 + \sqrt{5})$
$$= 36 + 16\sqrt{5}$$

[4 marks]

(ii) $(4 + \sqrt{2})(1 + \sqrt{2})(5 - \sqrt{2}) = (6 + 5\sqrt{2})(5 - \sqrt{2})$
$$= 20 + 19\sqrt{2}$$

[4 marks]

Answer 9

$$(1 + 4\sqrt{3})(5 + \sqrt{3})(2 + 5\sqrt{3}) = (17 + 21\sqrt{3})(2 + 5\sqrt{3})$$
$$= 349 + 127\sqrt{3}$$

[4 marks]