

5.3 Answers

A-Level Pure Mathematics : Year 1 GCSE (Grades 8 and 9)

Algebra of Surds and Indices I

5.3.1 Solution to 5.1 Introductory Example

Step 1 :

$$\begin{aligned}(5 + 3\sqrt{2})(5 - 3\sqrt{2}) &= 25 - 15\sqrt{2} + 15\sqrt{2} - 18 \\ &= 7\end{aligned}$$

[2 marks]

Step 2 :

$$\begin{aligned}\frac{21}{(5 + 3\sqrt{2})} &= \frac{21}{(5 + 3\sqrt{2})} \times \frac{(5 - 3\sqrt{2})}{(5 - 3\sqrt{2})} \\ &= \frac{21(5 - 3\sqrt{2})}{7} \\ &= 3(5 - 3\sqrt{2}) \\ &= 15 - 9\sqrt{2}\end{aligned}$$

[3 marks]

5.3.2 Solutions to 5.2 Exercise

Answer 1

Step 1 :

$$\begin{aligned}(3 + \sqrt{5})(3 - \sqrt{5}) &= 9 - 3\sqrt{5} + 3\sqrt{5} - 5 \\ &= 4\end{aligned}$$

[2 marks]

Step 2:

$$\begin{aligned}\frac{12}{(3 + \sqrt{5})} &= \frac{12}{(3 + \sqrt{5})} \times \frac{(3 - \sqrt{5})}{(3 - \sqrt{5})} \\ &= \frac{12(3 - \sqrt{5})}{4} \\ &= 3(3 - \sqrt{5}) \\ &= 9 - 3\sqrt{5}\end{aligned}$$

[3 marks]

Answer 2**Step 1 :**

$$\begin{aligned}(9 + 4\sqrt{3})(9 - 4\sqrt{3}) &= 81 - 36\sqrt{3} + 36\sqrt{3} - 48 \\ &= 33\end{aligned}$$

[2 marks]**Step 2:**

$$\begin{aligned}\frac{132}{(9 + 4\sqrt{3})} &= \frac{132}{(9 + 4\sqrt{3})} \times \frac{(9 - 4\sqrt{3})}{(9 - 4\sqrt{3})} \\ &= \frac{132(9 - 4\sqrt{3})}{33} \\ &= 4(9 - 4\sqrt{3}) \\ &= 36 - 16\sqrt{3}\end{aligned}$$

[3 marks]**Answer 3**

$$\begin{aligned}\frac{6}{2 + \sqrt{2}} &= \frac{6}{2 + \sqrt{2}} \times \frac{2 - \sqrt{2}}{2 - \sqrt{2}} \\ &= \frac{6(2 - \sqrt{2})}{2} \\ &= 3(2 - \sqrt{2}) \\ &= 6 - 3\sqrt{2}\end{aligned}$$

[4 marks]**Answer 4****Step 1 :**

$$\begin{aligned}(7 + 3\sqrt{5})(7 - 3\sqrt{5}) &= 49 - 21\sqrt{5} + 21\sqrt{5} - 45 \\ &= 4\end{aligned}$$

[2 marks]**Step 2:**

$$\begin{aligned}\frac{60}{(7 + 3\sqrt{5})} &= \frac{60}{(7 + 3\sqrt{5})} \times \frac{(7 - 3\sqrt{5})}{(7 - 3\sqrt{5})} \\ &= \frac{60(7 - 3\sqrt{5})}{4} \\ &= 15(7 - 3\sqrt{5}) \\ &= 105 - 45\sqrt{5}\end{aligned}$$

[3 marks]

Answer 5

Step 1 :

$$\begin{aligned}(8 + 3\sqrt{7})(8 - 3\sqrt{7}) &= 64 - 24\sqrt{7} + 24\sqrt{7} - 63 \\ &= 1\end{aligned}$$

Step 2:

$$\begin{aligned}\frac{\sqrt{7}}{(8 - 3\sqrt{7})} &= \frac{\sqrt{7}}{(8 - 3\sqrt{7})} \times \frac{(8 + 3\sqrt{7})}{(8 + 3\sqrt{7})} \\ &= \frac{\sqrt{7}(8 + 3\sqrt{7})}{1} \\ &= 21 + 8\sqrt{7}\end{aligned}$$

[4 marks]

Answer 6

$$\begin{aligned}\text{(i)} \quad (7 + 4\sqrt{3})(7 + 4\sqrt{3}) &= 49 + 28\sqrt{3} + 28\sqrt{3} + 48 \\ &= 97 + 56\sqrt{3}\end{aligned}$$

[2 marks]

$$\begin{aligned}\text{(ii)} \quad (7 + 4\sqrt{3})(7 - 4\sqrt{3}) &= 49 - 28\sqrt{3} + 28\sqrt{3} - 48 \\ &= 1\end{aligned}$$

[2 marks]

$$\begin{aligned}\text{(iii)} \quad \frac{(7 + 4\sqrt{3})}{(7 - 4\sqrt{3})} &= \frac{(7 + 4\sqrt{3})}{(7 - 4\sqrt{3})} \times \frac{(7 + 4\sqrt{3})}{(7 + 4\sqrt{3})} \\ &= 97 + 56\sqrt{3}\end{aligned}$$

[2 marks]

Answer 7

$$\begin{aligned}\frac{2}{(\sqrt{3} - 1)} &= \frac{2}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} \\ &= \frac{2(\sqrt{3} + 1)}{2} \\ &= \sqrt{3} + 1\end{aligned}$$

[2 marks]

Answer 8

$$\begin{aligned}\frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})} &= \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})} \times \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})} \\ &= \frac{3 + \sqrt{6} + \sqrt{6} + 2}{1} \\ &= 5 + 2\sqrt{6}\end{aligned}$$

[4 marks]

Answer 9

$$\begin{aligned}
\frac{100\sqrt{5}}{(7+2\sqrt{11})} &= \frac{100\sqrt{5}}{(7+2\sqrt{11})} \times \frac{(7-2\sqrt{11})}{(7-2\sqrt{11})} \\
&= \frac{100\sqrt{5}(7-2\sqrt{11})}{49-14\sqrt{11}+14\sqrt{11}-44} \\
&= 20\sqrt{5}(7-2\sqrt{11}) \\
&= 140\sqrt{5} - 40\sqrt{55}
\end{aligned}$$

[5 marks]**Answer 10**

$$\begin{aligned}
\frac{46}{(5\sqrt{2}+3\sqrt{3})} &= \frac{46}{(5\sqrt{2}+3\sqrt{3})} \times \frac{(5\sqrt{2}-3\sqrt{3})}{(5\sqrt{2}-3\sqrt{3})} \\
&= \frac{46(5\sqrt{2}-3\sqrt{3})}{50-15\sqrt{6}+15\sqrt{6}-27} \\
&= \frac{46(5\sqrt{2}-3\sqrt{3})}{50-27} \\
&= \frac{46(5\sqrt{2}-3\sqrt{3})}{23} \\
&= 2(5\sqrt{2}-3\sqrt{3}) \\
&= 10\sqrt{2} - 6\sqrt{3}
\end{aligned}$$

[5 marks]