

7.5 Answers

A-Level Pure Mathematics : Year 1 GCSE (Grades 8 and 9)

Algebra of Surds and Indices I

7.5.1 Solutions (7.2 Introductory Examples)

$$\begin{array}{lllll} \text{(i)} & 5 & \text{(ii)} & 3 & \text{(iii)} & \frac{1}{9} & \text{(iv)} & 8 & \text{(v)} & \frac{1}{3} \\ \text{(vi)} & \frac{1}{1000} & \text{(vii)} & 3 & \text{(viii)} & 27 & \text{(ix)} & \frac{1}{9} & \text{(x)} & 1 \end{array}$$

[5 marks]

7.5.2 Solutions to 7.3 Examples of Tricky Index Questions

Answer to Example #1

$$\begin{aligned} 81\sqrt{3} &= 3^a \\ 3^4 \times 3^{\frac{1}{2}} &= 3^a \\ 3^{4.5} &= 3^a \\ \therefore a &= 4.5 \end{aligned}$$

[2 marks]

Answer to Example #2

$$\begin{aligned} (27x^{12})^{\frac{2}{3}} &= 27^{\frac{2}{3}} \times (x^{12})^{\frac{2}{3}} \\ &= 9x^8 \end{aligned}$$

[2 marks]

Answer to Example #3

$$\begin{aligned} 2^5 \times 3^4 \times 5^3 &= 2^4 \times 3^4 \times 5^2 \times 2 \times 3 \\ \sqrt{2^5 \times 3^4 \times 5^3} &= \sqrt{2^4 \times 3^4 \times 5^2} \times \sqrt{2 \times 3} \\ &= 2^2 \times 3^2 \times 5 \times \sqrt{6} \\ &= 4 \times 9 \times 5 \times \sqrt{6} \\ &= 180\sqrt{6} \end{aligned}$$

[3 marks]

Answer to Example #4

$$\begin{aligned} \left(\frac{3x^2}{4\sqrt{y}} \right)^{-3} &= \left(\frac{4y^{\frac{1}{2}}}{3x^2} \right)^3 \\ &= \frac{64y^{\frac{3}{2}}}{27x^6} \end{aligned}$$

[3 marks]

7.5.3 Solutions (7.4 Exercise)

Answer 1

$$8h^3$$

Answer 2

$$64x^6y$$

Answer 3

$$\frac{27m^6}{125n^3}$$

Answer 4

$$\frac{b}{a}$$

[2, 3, 3, 1 marks]

Answer 5

$$\begin{aligned}\left(\frac{3\sqrt{x}}{4y^5}\right)^{-2} &= \left(\frac{4y^5}{3\sqrt{x}}\right)^2 \\ &= \frac{16y^{10}}{9x}\end{aligned}$$

[3 marks]

Answer 6

$$\begin{aligned}2^7 \times 3 \times 5^4 &= 2^6 \times 5^4 \times 2 \times 3 \\ \sqrt{2^7 \times 3 \times 5^4} &= \sqrt{2^6 \times 5^4} \times \sqrt{2 \times 3} \\ &= 2^3 \times 5^2 \times \sqrt{6} \\ &= 8 \times 25 \times \sqrt{6} \\ &= 200\sqrt{6}\end{aligned}$$

[3 marks]

Answer 7

$$\begin{aligned}8\sqrt{2} &= 2^a \\ 2^3 \times 2^{\frac{1}{2}} &= 2^a \\ 2^{3.5} &= 2^a \\ a &= 3.5\end{aligned}$$

[2 marks]

Answer 8

$$\begin{aligned}(25x^{12})^{\frac{3}{2}} &= 25^{\frac{3}{2}} \times (x^{12})^{\frac{3}{2}} \\ &= 125x^{18}\end{aligned}$$

[2 marks]

Answer 9

$$\frac{28x^5y^3}{4xy^2} = 7x^4y$$

[3 marks]

Answer 10

(a) $gh = 2^4 \times 3^2 \times 7^5$

[2 marks]

(b) $\frac{g}{h} = 2^2 \times 3^0 \times 7^{-1}$

$$a = 2, \quad b = 0, \quad c = -1$$

[3 marks]

(c) $\text{LHS} = (7 - 2\sqrt{5})(7 + 2\sqrt{5})$
 $= 49 + 14\sqrt{5} - 14\sqrt{5} - 2 \times 2 \times \sqrt{5} \times \sqrt{5}$
 $= 49 - 20$
 $= 29$
 $= \text{RHS} \quad \square$

[3 marks]

(d) $\frac{1}{\sqrt[3]{9^4}} = 3^n$
 $(9^4)^{-\frac{1}{3}} = 3^n$
 $(3^8)^{-\frac{1}{3}} = 3^n$
 $3^{-\frac{8}{3}} = 3^n$
 $n = -\frac{8}{3}$

[3 marks]

Answer 11

$$(8a^9e^6)^{\frac{1}{3}} = 2a^3e^2$$

[2 marks]

Answer 12

$$\begin{aligned} 8A &= 8 \times 2^n \times 3 \times 5^m \\ &= 2^3 \times 2^n \times 3 \times 5^m \\ &= 2^{n+3} \times 3 \times 5^m \end{aligned}$$

[2 marks]

Answer 13

$$\begin{aligned} \frac{\sqrt[4]{y}}{y} &= y^{\frac{1}{4}-1} \\ &= y^{-\frac{3}{4}} \end{aligned}$$

[2 marks]

Answer 14

$$\begin{aligned}
\left(\frac{y^5}{8x^6y^8} \right)^{-\frac{1}{3}} &= \left(\frac{8x^6y^8}{y^5} \right)^{\frac{1}{3}} \\
&= (8x^6y^3)^{\frac{1}{3}} \\
&= 2x^2y
\end{aligned}$$

[3 marks]**Answer 15**

If you were going to differentiate you'd do the following first,

$$\begin{aligned}
\frac{7\sqrt{p} - p^2}{\sqrt{p^3}} &= \frac{7p^{\frac{1}{2}}}{p^{\frac{3}{2}}} - \frac{p^2}{p^{\frac{3}{2}}} \\
&= 7p^{-1} - p^{\frac{1}{2}} \\
&= \frac{7}{p} - \sqrt{p}
\end{aligned}$$

Which is by far the most elegant answer,
Other equivalent answers are likely.

[3 marks]**Answer 16**

$$\begin{aligned}
\text{(a)} \quad (3 + \sqrt{c})(2\sqrt{c} - 3) &= 1 + k\sqrt{c} \\
6\sqrt{c} + 2c - 9 - 3\sqrt{c} &= 1 + k\sqrt{c} \\
2c - 9 + 3\sqrt{c} &= 1 + k\sqrt{c}
\end{aligned}$$

$$\text{Matching integers, } 2c - 9 = 1$$

$$2c = 10$$

$$c = 5$$

$$\text{Matching coefficients of } \sqrt{c}, \quad k = 3$$

[3 marks]**(b)**

$$\begin{aligned}
p^m &= \frac{1}{p \times \sqrt[3]{p^2}} \\
&= \frac{1}{p \times p^{\frac{2}{3}}} \\
&= p^{-\frac{5}{3}} \quad \therefore m = -\frac{5}{3}
\end{aligned}$$

[3 marks]