$\sqrt{S \cdot U \cdot R \cdot D \cdot S}$

&

$I^{i} \cdot N^{n} \cdot D^{d} \cdot I^{i} \cdot C^{c} \cdot E^{e} \cdot S^{s}$

Surds and Indices

Lesson 1

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

1.1 Square Roots without a calculator

Example #1
Without using a calculator, find $\sqrt{2704}$

[3 marks]

Example #2
The following number is too big for my calculator;

$$11^{102} \times 13^{284}$$

Even so, square root this number, writing the answer in index form

[2 marks]

1.2 Two Quick Questions

(i) Without using a calculator, making your method clear, determine,

$$\sqrt{3969}$$

[3 marks]

(ii) The following number is too big for my calculator;

$$17^{38} \times 19^{74}$$

Even so, square root this number, writing the answer in index form

[2 marks]

1.3 Square Free

Any number which is not prime can be written as a unique product of primes.
For example,

$$120 = 2^3 \times 3 \times 5$$

Mathematicians talk of the *decomposition* of 120 into a product of primes.

There is another decomposition of 120 that is useful. It revolves around identifying the biggest square number that will divide into 120 exactly.

Reminder: Square Numbers = { 1, 4, 9, 16, 25, 36, ... }

As 4 is the biggest square number that divides into 120 we can write,

$$120 = 4 \times 30$$

Notice that no square number, other than 1, will divide into 30.
Thus 30 is termed square-free or, more succinctly, □ Free.

In summary, our new decomposition takes an integer that is not □ Free and expresses it as a square number multiplied by a square free number.
i.e.

$$Not\ \square\ Free = \square \times \square\ Free$$

Example : Without using a calculator, write $\sqrt{120}$ in the form $a\sqrt{p}$
where a and p are integers
and p is □ FREE.

[2 marks]

1.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 26

Question 1

Without using a calculator, write each of the following in the form $a\sqrt{p}$

where a and p are integers

and p is □ FREE.

(i) $\sqrt{8}$

(ii) $\sqrt{3^3}$

(iii) $\sqrt{48}$

(iv) $\sqrt{98}$

(v) $\sqrt{2^3 \times 7}$

(vi) $\sqrt{2 \times 11^2}$

(vii) $\sqrt{2^8 \times 5}$

[7 marks]

Question 2

Which of the following are ☐ FREE ?

(i) 5 **(ii)** 5^2 **(iii)** 5^3

(iv) 5^4 **(v)** 2×3 **(vi)** $2^3 \times 3$

[2 marks]

Question 3

The following number is too big for my calculator;

$$5^{52} \times 7 \times 13^{95}$$

Even so, square root this number, writing the answer in in the form $a\sqrt{p}$

where a and p are integers, that may be written in index form

and p is ☐ FREE.

[3 marks]

Question 4

Without using a calculator, write each of the following in the form $a\sqrt{p}$

where a and p are integers

and p is ☐ FREE.

(i) $\sqrt{504}$ **(ii)** $\sqrt{1452}$

(iii) $\sqrt{2 \times 3^3 \times 5^3}$ **(iv)** $\sqrt{2 \times 3^5 \times 11}$

[2, 3, 2, 3 marks]

Question 5

Without using a calculator, determine the cube root of 1728

i.e. $\sqrt[3]{1728}$

[4 marks]

1.5 Answers

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

1.5.1 Solution to 1.1 Example #1)

A division ladder;

2	2704
2	1352
2	676
2	338
13	169
13	13
	1

Thus;

$$2704 = 2^4 \times 13^2 \quad \text{from the division ladder}$$

$$\therefore \sqrt{2704} = \sqrt{2^4 \times 13^2}$$

$$= \left(2^4 \times 13^2\right)^{\frac{1}{2}}$$

$$= 2^2 \times 13$$

$$= 52$$

[3 marks]

1.5.2 Solution to 1.1 Example #2

$$\sqrt{11^{102} \times 13^{284}} = \left(11^{102} \times 13^{284}\right)^{\frac{1}{2}}$$

$$= 11^{51} \times 13^{142}$$

[2 marks]

1.5.3 Solution to 1.2 Exercise

(i) 63

[3 marks]

(ii) $17^{19} \times 19^{37}$

[2 marks]

1.5.4 Solution to 1.3 Example

$$\sqrt{120} = \sqrt{4} \times \sqrt{30}$$

$$= 2\sqrt{30}$$

[2 marks]

1.5.5 Solutions to 1.4 Exercise

Answer 1

(i) $8 = 4 \times 2$
$\therefore \sqrt{8} = 2\sqrt{2}$

(ii) $\sqrt{3^2 \times 3} = 3\sqrt{3}$

(iii) $48 = 16 \times 3$
$\therefore \sqrt{48} = 4\sqrt{3}$

(iv) $98 = 49 \times 2$
$\therefore \sqrt{98} = 7\sqrt{2}$

(v) $\sqrt{2^2 \times 2 \times 7} = 2\sqrt{14}$

(vi) $\therefore \sqrt{11^2 \times 2} = 11\sqrt{2}$

(vii)

$$\sqrt{2^8 \times 5} = 2^4\sqrt{5} = 16\sqrt{5}$$

[7 marks]

Answer 2

Only **(i)** and **(v)** are □ FREE

[2 marks]

Answer 3

$$\sqrt{5^{52} \times 13^{94} \times 7 \times 13} = 5^{26} \times 13^{47} \times \sqrt{7 \times 13}$$

[3 marks]

Answer 4

(i) $504 = 36 \times 14$
$\therefore \sqrt{504} = 6\sqrt{14}$

(ii) $1452 = 484 \times 3$
$\therefore \sqrt{1452} = 22\sqrt{3}$

(iii) $15\sqrt{30}$

(iv) $9\sqrt{66}$

[2, 3, 2, 3 marks]

Answer 5

12

[4 marks]

1.6 Homework

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 24

Do NOT use a calculator

Question 1

Use the fact that $405 = 3^4 \times 5$ to write $\sqrt{405}$ in the form $a\sqrt{p}$

where a and p are integers
and p is ☐ FREE. (And also, in this case, prime)

[2 marks]

Question 2

Write each of the following in the form $a\sqrt{p}$

where a & p are integers
and p is ☐FREE. (And also, in these cases, prime)

(i) $\sqrt{44}$ **(ii)** $\sqrt{50}$

(iii) $\sqrt{32}$ **(iv)** $\sqrt{99}$

(v) $\sqrt{200}$ **(vi)** $\sqrt{162}$

[6 marks]

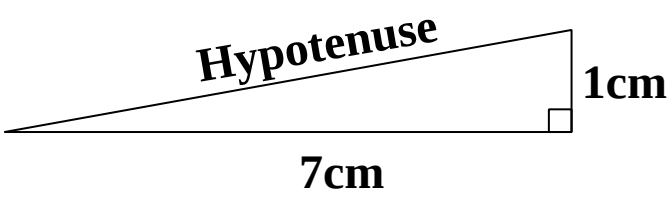
Question 3

Find the ***exact*** length of the hypotenuse of a right-angled Δ of base 7 cm and height 1 cm.

Write your answer in the form $a\sqrt{p}$

where a and p are integers

and p is □FREE. (and also, in this case, prime)



[3 marks]

Question 4

Find the ***exact*** length of the hypotenuse of a right angled Δ of base 6 cm and height 2 cm.

Write your answer in the form $a\sqrt{f}$

where a and f are integers

and f is □ FREE. (Note: f is not prime)

[3 marks]

Question 5

Find the ***exact*** length of the hypotenuse of a right-angled Δ of base 10 cm and height 4 cm.

Write your answer in the form $a\sqrt{p}$

where a and p are integers

and p is $\square$ FREE. (In this case, p is prime)

[3 marks]

Question 6

Find the ***exact*** length of the hypotenuse of a right-angled Δ of base 13 cm and height 9 cm.

Write your answer in the form $a\sqrt{f}$

where a and f are integers

and f is $\square$ FREE. (Note: f is not prime)

[3 marks]

Question 7

Find the ***exact*** length of the hypotenuse of a right-angled Δ of base 15 cm and height 9 cm.

Write your answer in the form $a\sqrt{f}$

where a and f are integers

and f is □ FREE. (Note: f is not prime)

[4 marks]

DECLARATION:

I, _____, being of sound body and brain,
do declare that I have not used a calculator in answering any of these questions.

Nor did I look to the left nor the right, at my neighbours answers.
(which would have been wrong anyway)

Furthermore, I love maths.

Signed: _____

1.7 Answers

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

1.7.1 Solutions (1.6 Homework)

Answer 1
$9\sqrt{5}$

[2 marks]

Answer 2
(i) $2\sqrt{11}$ **(ii)** $5\sqrt{2}$
(iii) $4\sqrt{2}$ **(iv)** $3\sqrt{11}$
(v) $10\sqrt{2}$ **(vi)** $9\sqrt{2}$

[6 marks]

Answer 3
$5\sqrt{2}$

Answer 4
$2\sqrt{10}$

[3, 3 marks]

Answer 5
$2\sqrt{29}$

Answer 6
$5\sqrt{10}$

[3, 3 marks]

Answer 7
$3\sqrt{34}$

[4 marks]

Lesson 2

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

2.1 SURD Manipulation

There are benefits in 'using your wits' to try and spot 'short cuts' and 'clever moves' to avoid long-winded and pedantic solutions.

However, if you need to put in lots of steps to be certain of getting a question right, put them in - it's often quicker to scribble down a few extra lines rather than agonise over a tricky step in your mind.

Also keep in mind that there are often a couple of different methods that could be employed - use what you are comfortable with, which may differ from what your neighbour is doing.

Example

Without using a calculator, calculate $2\sqrt{15} \times 4\sqrt{10}$

Write your answer in the form $a\sqrt{b}$

where a and b are integers

and b is as small as possible.

[2 marks]

2.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 70

Do NOT use a calculator

Question 1

Calculate each of the following, writing your answers in the form $a\sqrt{b}$
where a and b are integers
and b is as small as possible.

(i) $3\sqrt{6} \times 7\sqrt{21}$

[2 marks]

(ii) $5\sqrt{14} \times 2\sqrt{10}$

[2 marks]

(iii) $10\sqrt{22} \times 4\sqrt{6}$

[2 marks]

(iv) $3\sqrt{10} \times 4\sqrt{55}$

[2 marks]

Question 2

Simplify:

(i) $\sqrt{12}$ **(ii)** $\sqrt{20}$ **(iii)** $\sqrt{8}$

(iv) $\sqrt{3^2 \times 5}$ **(v)** $\sqrt{2^4 \times 3}$ **(vi)** $\sqrt{242}$

(vii) $\sqrt{75}$ **(viii)** $\sqrt{6 \times 27}$ **(ix)** $\sqrt{147}$

(x) $\sqrt{5^3}$ **(xi)** $\sqrt{567}$ **(xii)** $\sqrt{8 \times 14}$

[12 marks]

Question 3

Simplify:

(i) $5\sqrt{18}$ **(ii)** $2\sqrt{30 \times 10}$ **(iii)** $5\sqrt{54}$

(iv) $3\sqrt{2^3 \times 10}$ **(v)** $2\sqrt{2^2 \times 5 \times 35}$ **(vi)** $2\sqrt{245}$

[6 marks]

Question 4

Simplify:

(i) $\frac{\sqrt{44}}{2}$ **(ii)** $\frac{\sqrt{24}}{2}$ **(iii)** $\frac{\sqrt{200}}{5}$

(iv) $\frac{\sqrt{243}}{3}$ **(v)** $\frac{\sqrt{288}}{4}$ **(vi)** $\frac{\sqrt{450}}{3}$

[6 marks]

Question 5

Simplify:

(i) $\frac{\sqrt{98}}{\sqrt{2}}$ **(ii)** $\frac{\sqrt{500}}{\sqrt{5}}$ **(iii)** $\frac{\sqrt{63}}{\sqrt{7}}$

[3 marks]

Question 6

Simplify:

(i) $\frac{\sqrt{10}}{\sqrt{5}}$ **(ii)** $\frac{\sqrt{22}}{\sqrt{11}}$ **(iii)** $\frac{\sqrt{56}}{\sqrt{7}}$

[3 marks]

Question 7

Simplify:

(i) $3\sqrt{75} + 2\sqrt{12}$

[2 marks]

(ii) $2\sqrt{18} + \sqrt{2^3 \times 5^2} - \sqrt{72}$

[2 marks]

(iii) $5\sqrt{20} + 3\sqrt{45} - 4\sqrt{80}$

[2 marks]

(iv) $6\sqrt{6} - \sqrt{24} + 3\sqrt{2 \times 3 \times 7^2}$

[2 marks]

Question 8
Simplify:

(i) $5\sqrt{63}$ **(ii)** $7\sqrt{200}$

(iii) $10\sqrt{216}$ **(iv)** $2\sqrt{90}$

(v) $11\sqrt{3 \times 5^3}$ **(vi)** $12\sqrt{2^5 \times 3^2}$

[6 marks]

Question 9
Write each of the following in the form $a + b\sqrt{c}$ for integer a, b and c.
Furthermore, c is to be square free.
[Your answers to **Question 8** will be helpful...]

(i) $\frac{18 + 5\sqrt{63}}{3}$ **(ii)** $\frac{14 + 7\sqrt{200}}{14}$

(iii) $\frac{45 + 10\sqrt{216}}{15}$ **(iv)** $\frac{12 + 2\sqrt{90}}{3}$

(v) $\frac{30 - 11\sqrt{375}}{5}$ **(vi)** $\frac{36 + 12\sqrt{288}}{9}$

[12 marks]

Question 10

Write in the form $a + b\sqrt{c}$ where a, b and c are integers.
Furthermore, c, is to be □ Free.

(i) $\dfrac{200 + 3\sqrt{1000}}{5}$

(ii) $\dfrac{-64 + 16\sqrt{88}}{32}$

(iii) $\dfrac{2 + 3\sqrt{156}}{2}$

[6 marks]

2.3 Answers

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

2.3.1 Solutions (2.1 Example)

Method 1 : Less Mathematical

$$\begin{aligned} & 2\sqrt{15} \times 4\sqrt{10} \\ &= 2 \times \sqrt{3} \times \sqrt{5} \times 4 \times \sqrt{2} \times \sqrt{5} \\ &= 2 \times 4 \times (\sqrt{5} \times \sqrt{5}) \times \sqrt{3} \times \sqrt{2} \\ &= 8 \times 5 \times \sqrt{3} \times \sqrt{2} \\ &= 40\sqrt{6} \end{aligned}$$

Method 2 : More Mathematical

$$\begin{aligned} & 2\sqrt{15} \times 4\sqrt{10} \\ &= 8 \times \sqrt{150} \\ &= 8 \times \sqrt{25 \times 6} \\ &= 8 \times 5 \times \sqrt{6} \\ &= 40\sqrt{6} \end{aligned}$$

[2 marks]

2.3.2 Solutions to 2.2 Exercise

Answer 1

(i) $63\sqrt{14}$ **(ii)** $20\sqrt{35}$
(iii) $80\sqrt{33}$ **(iv)** $60\sqrt{22}$

[8 marks]

Answer 2

(i)	$2\sqrt{3}$	**(ii)**	$2\sqrt{5}$	**(iii)**	$2\sqrt{2}$
(iv)	$3\sqrt{5}$	**(v)**	$4\sqrt{3}$	**(vi)**	$11\sqrt{2}$
(vii)	$5\sqrt{3}$	**(viii)**	$9\sqrt{2}$	**(ix)**	$7\sqrt{3}$
(x)	$5\sqrt{5}$	**(xi)**	$9\sqrt{7}$	**(xii)**	$4\sqrt{7}$

[12 marks]

Answer 3

(i)	$15\sqrt{2}$	**(ii)**	$20\sqrt{3}$	**(iii)**	$15\sqrt{6}$
(iv)	$12\sqrt{5}$	**(v)**	$20\sqrt{7}$	**(vi)**	$14\sqrt{5}$

[6 marks]

Answer 4

(i)	$\sqrt{11}$	**(ii)**	$\sqrt{6}$	**(iii)**	$2\sqrt{2}$
(iv)	$3\sqrt{3}$	**(v)**	$3\sqrt{2}$	**(vi)**	$5\sqrt{2}$

[6 marks]

Answer 5

(i) 7 **(ii)** 10 **(iii)** 3

[3 marks]

Answer 6

(i) $\sqrt{2}$ **(ii)** $\sqrt{2}$ **(iii)** $2\sqrt{2}$

[3 marks]

Answer 7

(i) $19\sqrt{3}$

(ii) $10\sqrt{2}$

(iii) $3\sqrt{5}$

(iv) $25\sqrt{6}$

[8 marks]

Answer 8

(i) $15\sqrt{7}$ **(ii)** $70\sqrt{2}$

(iii) $60\sqrt{6}$ **(iv)** $6\sqrt{10}$

(v) $55\sqrt{15}$ **(vi)** $144\sqrt{2}$

[6 marks]

Answer 9

(i) $6 + 5\sqrt{7}$ **(ii)** $1 + 5\sqrt{2}$

(iii) $3 + 4\sqrt{6}$ **(iv)** $4 + 2\sqrt{10}$

(v) $6 - 11\sqrt{15}$ **(vi)** $4 + 16\sqrt{2}$

[12 marks]

Answer 10

(i) $40 + 6\sqrt{10}$

(ii) $-2 + \sqrt{22}$

(iii) $1 + 3\sqrt{39}$

[6 marks]

Lesson 3

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

3.1 Rationalising the Denominator #1

Mathematicians' dislike fractions which have a square root in the denominator.
There are standard techniques for manipulating such fractions to remove the offending square root from the denominator.
This may well result in a square root in the numerator, but this is considered fine !

Example:

Rationalise the denominator of $\frac{4\sqrt{3}}{\sqrt{5}}$

[1 mark]

3.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable
Marks Available : 35

Do NOT use a calculator

Question 1

Rationalise the denominators of the following fractions;

(i) $\frac{20}{\sqrt{5}}$ **(ii)** $\frac{28}{\sqrt{7}}$ **(iii)** $\frac{24\sqrt{3}}{\sqrt{2}}$

(iv) $\frac{12}{\sqrt{3}}$ **(v)** $\frac{5}{\sqrt{13}}$ **(vi)** $\frac{14\sqrt{3}}{\sqrt{2}}$

(vii) $\frac{55}{\sqrt{11}}$ **(viii)** $\frac{1}{\sqrt{2}}$ **(ix)** $\frac{15\sqrt{2}}{\sqrt{15}}$

[9 marks]

Question 2

By multiplying both numerator and denominator by $\sqrt{2}$ rationalise the denominator of $\frac{5}{8\sqrt{2}}$

[1 mark]

Question 3

Rationalise the denominators of the following fractions, simplifying the result.

(i) $\frac{52}{3\sqrt{13}}$ **(ii)** $\frac{48}{5\sqrt{6}}$ **(iii)** $\frac{7}{3\sqrt{15}}$

(iv) $\frac{11}{12\sqrt{3}}$ **(v)** $\frac{6\sqrt{3}}{7\sqrt{2}}$ **(vi)** $\frac{3}{\sqrt{7}}$

(vii) $\frac{44}{5\sqrt{11}}$ **(viii)** $\frac{14}{\sqrt{2}}$ **(ix)** $\frac{28}{3\sqrt{14}}$

[9 marks]

Question 4

Show that, $\frac{3}{\sqrt{2}} + \frac{5}{\sqrt{3}} = \frac{9\sqrt{2} + 10\sqrt{3}}{6}$

[2 marks]

Question 5

Simplify, $\frac{1}{\sqrt{5}} + \frac{4\sqrt{5}}{5}$

[2 marks]

Question 6

Simplify, $\frac{7}{\sqrt{6}} + \frac{\sqrt{3}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}}$

[3 marks]

Question 7

A-Level Examination Question from January 2019, C12, Q2 (Edexcel)

Given $y = 2^x$, express each of the following in terms of y.

Write each expression in its simplest form.

(a) 2^{2x}

[1 mark]

(b) 2^{x+3}

[1 mark]

(c) $\frac{1}{4^{2x-3}}$

[2 marks]

Question 8

A-Level Examination Question from June 2018, C12, Q4 (Edexcel)

Given that,

$$y = \frac{64x^6}{25}, \quad x > 0$$

express each of the following in the form $k x^n$ where k and n are constants.

(a) $y^{-\frac{1}{2}}$

[3 marks]

(b) $(25y)^{\frac{2}{3}}$

[2 marks]

3.3 Answers

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

3.3.1 Solutions (3.1 Introductory Example)

Multiply both numerator and denominator by $\sqrt{5}$

$$= \frac{4\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{4\sqrt{15}}{5} \qquad \left(0.8\sqrt{15} \right)$$

[2 marks]

3.3.2 Solutions (3.2 Exercise)

Answer 1

(i) $4\sqrt{5}$ **(ii)** $4\sqrt{7}$ **(iii)** $12\sqrt{6}$

(iv) $4\sqrt{3}$ **(v)** $\frac{5\sqrt{13}}{13}$ **(vi)** $7\sqrt{6}$

(vii) $5\sqrt{11}$ (viii) $\frac{\sqrt{2}}{2}$ (ix) $\sqrt{30}$

[9 marks]

Answer 2

$\frac{5\sqrt{2}}{16}$

[1 mark]

Answer 3

(i) $\frac{4\sqrt{13}}{3}$ **(ii)** $\frac{8\sqrt{6}}{5}$ **(iii)** $\frac{7\sqrt{15}}{45}$

(iv) $\frac{11\sqrt{3}}{36}$ **(v)** $\frac{3\sqrt{6}}{7}$ **(vi)** $\frac{3\sqrt{7}}{7}$

(vii) $\frac{4\sqrt{11}}{5}$ **(viii)** $7\sqrt{2}$ **(ix)** $\frac{2\sqrt{14}}{3}$

[9 marks]

Answer 4

proof

Answer 5

$\sqrt{5}$

Answer 6

$2\sqrt{6}$

[2, 2, 3 marks]

Answer 7

(a) y^2

[1 mark]

(b) $8y$

[1 mark]

(c) $\dfrac{64}{y^4}$

[2 marks]

Answer 8

(a)

$$y^{-\frac{1}{2}} = \left(\frac{64\,x^6}{25} \right)^{-\frac{1}{2}}$$

$$= \left(\frac{25}{64\,x^6} \right)^{\frac{1}{2}}$$

$$= \frac{5}{8\,x^3}$$

$$= \frac{5}{8}\,x^{-3}$$

[3 marks]

(b)

$$(25\,y)^{\frac{2}{3}} = 16\,x^4$$

[2 marks]

Lesson 4

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

4.1 Surds from Quadratics

When solving quadratic equations, the exact answers can be numbers of the form,

$$x = a + b\sqrt{c}$$

A consequence of this is that the arithmetic of such numbers becomes of interest.

Example #1

Find the exact solutions of the quadratic equation $x^2 - 2x - 11 = 0$

[3 marks]

Example #2

Expand the brackets and simplify; $\left(7 + 2\sqrt{5}\right)\left(4 + 3\sqrt{5}\right)$

[3 marks]

4.2 You Try

Expand the brackets, $\left(3 + \sqrt{7}\right)\left(2 + \sqrt{7}\right)$

Once done, check your answer with mine, over the page.

[2 marks]

4.3 You Try Answer

$$(3+\sqrt{7})(2+\sqrt{7}) = 6 + 3\sqrt{7} + 2\sqrt{7} + 7$$
$$= 13 + 5\sqrt{7}$$

[2 marks]

4.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 50

Do NOT use a calculator

Question 1

Expand the brackets and simplify each of the following;

(i) $(6+\sqrt{2})(3+\sqrt{2})$

(ii) $(5+\sqrt{13})(4+\sqrt{13})$

(iii) $(7+3\sqrt{2})(5+\sqrt{2})$

(iv) $(2+\sqrt{3})(1+5\sqrt{3})$

[8 marks]

Question 2

Find the exact solutions of the quadratic equation $x^2 - 2x - 1 = 0$

[3 marks]

Question 3

Expand the brackets and simplify each of the following;

(i) $\left(4 + 7\sqrt{3}\right)\left(5 + 2\sqrt{3}\right)$

(ii) $\left(7 + 3\sqrt{2}\right)^2$

(iii) $\left(6 + \sqrt{5}\right)\left(3 - 2\sqrt{5}\right)$

(iv) $\left(6 - 5\sqrt{3}\right)^2$

[12 marks]

Question 4

Express $\sqrt{80} + \frac{30}{\sqrt{5}}$ in the form $c\sqrt{5}$ where c is an integer

[2 marks]

Question 5

Express $\sqrt{50} + \sqrt{3} \times \sqrt{6} - \frac{14}{\sqrt{2}}$ in as simple a form as possible.

[3 marks]

Question 6

By using the result "a difference of two squares", or otherwise, simplify;

(i) $\left(20 + 3\sqrt{7}\right)\left(20 - 3\sqrt{7}\right)$

[2 marks]

(ii) $\left(11 + 2\sqrt{3}\right)\left(11 - 2\sqrt{3}\right)$

[2 marks]

Question 7

Expand the brackets and simplify;

(i) $(x + 1) (x + 2) (x + 3)$

[3 marks]

(ii) $(x + 2) (x + 5) (x - 3)$

[3 marks]

Question 8

Expand the brackets and simplify;

(i) $\left(1+\sqrt{5}\right)\left(2+\sqrt{5}\right)\left(3+\sqrt{5}\right)$

[4 marks]

(ii) $\left(4+\sqrt{2}\right)\left(1+\sqrt{2}\right)\left(5-\sqrt{2}\right)$

[4 marks]

Question 9

Expand the brackets and simplify,

$\left(1 + 4\sqrt{3}\right)\left(5 + \sqrt{3}\right)\left(2 + 5\sqrt{3}\right)$

[4 marks]

4.5 Answers

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

4.5.1 Solutions to 4.1 Introductory Examples

Example #1

$$x^2 - 2x - 11 = 0$$

$$x = \frac{2 \pm \sqrt{2^2 - 4 \times 1 \times (-11)}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{48}}{2}$$

$$= \frac{2 \pm 2\sqrt{12}}{2}$$

$$= 1 \pm \sqrt{12}$$

$$= 1 \pm 2\sqrt{3}$$

[3 marks]

Example #2

$$(7 + 2\sqrt{5})(4 + 3\sqrt{5}) = 28 + 21\sqrt{5} + 8\sqrt{5} + 6 \times 5$$

$$= 58 + 29\sqrt{5}$$

[3 marks]

4.5.2 Solutions (4.2 Exercise)

Answer 1

(i) $(6 + \sqrt{2})(3 + \sqrt{2}) = 18 + 6\sqrt{2} + 3\sqrt{2} + 2$

$$= 20 + 9\sqrt{2}$$

[2 marks]

(ii) $(5 + \sqrt{13})(4 + \sqrt{13}) = 20 + 5\sqrt{13} + 4\sqrt{13} + 13$

$$= 33 + 9\sqrt{13}$$

[2 marks]

(iii) $(7 + 3\sqrt{2})(5 + \sqrt{2}) = 35 + 7\sqrt{2} + 15\sqrt{2} + 6$

$$= 41 + 22\sqrt{2}$$

[2 marks]

(iv) $(2 + \sqrt{3})(1 + 5\sqrt{3}) = 2 + 10\sqrt{3} + \sqrt{3} + 15$

$$= 17 + 11\sqrt{3}$$

[2 marks]

Answer 2

$$x^2 - 2x - 1 = 0$$

$$x = \frac{2 \pm \sqrt{2^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

$$= \frac{2 \pm 2\sqrt{2}}{2}$$

$$= 1 \pm \sqrt{2}$$

[3 marks]

Answer 3

(i) $\left(4 + 7\sqrt{3}\right)\left(5 + 2\sqrt{3}\right) = 20 + 8\sqrt{3} + 35\sqrt{3} + 42$

$$= 62 + 43\sqrt{3}$$

[3 marks]

(ii) $\left(7 + 3\sqrt{2}\right)^2 = \left(7 + 3\sqrt{2}\right)\left(7 + 3\sqrt{2}\right)$

$$= 49 + 21\sqrt{2} + 21\sqrt{2} + 18$$

$$= 67 + 42\sqrt{2}$$

[3 marks]

(iii) $\left(6 + \sqrt{5}\right)\left(3 - 2\sqrt{5}\right) = 18 - 12\sqrt{5} + 3\sqrt{5} - 10$

$$= 8 - 9\sqrt{5}$$

[3 marks]

(iv) $\left(6 - 5\sqrt{3}\right)^2 = \left(6 - 5\sqrt{3}\right)\left(6 - 5\sqrt{3}\right)$

$$= 36 - 30\sqrt{3} - 30\sqrt{3} + 75$$

$$= 111 - 60\sqrt{3}$$

[3 marks]

Answer 4

$$\sqrt{80} + \frac{30}{\sqrt{5}} = \sqrt{16 \times 5} + \frac{30}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

$$= 4\sqrt{5} + 6\sqrt{5}$$

$$= 10\sqrt{5}$$

[2 marks]

Answer 5

$$\sqrt{50} + \sqrt{3} \times \sqrt{6} - \frac{14}{\sqrt{2}} = 5\sqrt{2} + 3\sqrt{2} - 7\sqrt{2}$$
$$= \sqrt{2}$$

[3 marks]

Answer 6

(i) $(20 + 3\sqrt{7})(20 - 3\sqrt{7}) = 400 - 60\sqrt{7} + 60\sqrt{7} - 63$
$$= 337$$

[2 marks]

(ii) $(11 + 2\sqrt{3})(11 - 2\sqrt{3}) = 121 - 22\sqrt{3} + 22\sqrt{3} - 12$
$$= 109$$

[2 marks]

Answer 7

(i) $(x + 1)(x + 2)(x + 3) = (x^2 + 3x + 2)(x + 3)$
$$= x^3 + 6x^2 + 11x + 6$$

[3 marks]

(ii) $(x + 2)(x + 5)(x - 3) = (x^2 + 7x + 10)(x - 3)$
$$= x^3 + 4x^2 - 11x - 30$$

[3 marks]

Answer 8

(i) $(1 + \sqrt{5})(2 + \sqrt{5})(3 + \sqrt{5}) = (7 + 3\sqrt{5})(3 + \sqrt{5})$
$$= 36 + 16\sqrt{5}$$

[4 marks]

(ii) $(4 + \sqrt{2})(1 + \sqrt{2})(5 - \sqrt{2}) = (6 + 5\sqrt{2})(5 - \sqrt{2})$
$$= 20 + 19\sqrt{2}$$

[4 marks]

Answer 9

$$(1 + 4\sqrt{3})(5 + \sqrt{3})(2 + 5\sqrt{3}) = (17 + 21\sqrt{3})(2 + 5\sqrt{3})$$
$$= 349 + 127\sqrt{3}$$

[4 marks]

Lesson 5

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

5.1 Rationalising the Denominator #2

Two observations, made previously, are these;

Firstly, when solving quadratic equations, numbers of the form $a + b\sqrt{c}$ are often the result. Secondly, it was noted that mathematicians' consider any occurrence of a square root in a denominator undesirable.
These two observations collide when asked to rationalise the denominator of an expression of the form

$$\frac{m}{a + b\sqrt{c}} \text{ where } m, a, b, \text{ and } c \text{ are integers, } c > 0$$

5.2 The Key Idea

The previous tactic of multiplying both numerator and denominator by something that clears the square root from the denominator is still the correct strategy, but what might that something be ?
It's not as straight forward as before as the following attempt using $\sqrt{c}$ shows;

$$\frac{m}{a + b\sqrt{c}} = \frac{m}{a + b\sqrt{c}} \times \frac{\sqrt{c}}{\sqrt{c}}$$

$$= \frac{m\sqrt{c}}{a\sqrt{c} + bc} \text{ which still has a square root in the denominator.}$$

The key idea is to set up a difference of two squares.

The Rationaliser Rule

To rationalise the denominator of

$$\frac{m}{(a \pm b\sqrt{c})} \text{ where } m, a, b, \text{ and } c \text{ are integers, } c > 0$$

multiply both numerator and denominator by

$$(a \mp b\sqrt{c})$$

Proof

$$\frac{m}{(a \pm b\sqrt{c})} = \frac{m}{(a \pm b\sqrt{c})} \times \frac{(a \mp b\sqrt{c})}{(a \mp b\sqrt{c})}$$

$$= \frac{m(a - b\sqrt{c})}{a^2 - b^2 c}$$

Which has no square roots in the denominator. □

5.3 Example

Consider the fraction $\frac{21}{5 + 3\sqrt{2}}$ which has a square root in its denominator.

(i) Expand the brackets, $(5 + 3\sqrt{2})(5 - 3\sqrt{2})$

[2 marks]

(ii) Hence, or otherwise, rationalise the denominator of $\frac{21}{5 + 3\sqrt{2}}$

[3 marks]

Note the technique of NOT multiplying out the numerator until first cancelling it against the denominator thus avoiding large number mental arithmetic.

5.2 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 45

Do NOT use a calculator

Question 1

This question is about rationalising the denominator of

$$\frac{12}{3+\sqrt{5}}$$

Step 1 : Expand the brackets : $(3+\sqrt{5})(3-\sqrt{5})$

[2 marks]

Step 2: Multiply both numerator and denominator by $3-\sqrt{5}$
That is, calculate and simplify,

$$\frac{12}{(3+\sqrt{5})} \times \frac{(3-\sqrt{5})}{(3-\sqrt{5})}$$

[3 marks]

Question 2

This question is about rationalising the denominator of

$$\frac{132}{9 + 4\sqrt{3}}$$

Step 1 : Expand the brackets : $\left(9 + 4\sqrt{3}\right)\left(9 - 4\sqrt{3}\right)$

[2 marks]

Step 2 : Multiply both numerator and denominator by $9 - 4\sqrt{3}$

That is, calculate and simplify,

$$\frac{132}{(9 + 4\sqrt{3})} \times \frac{(9 - 4\sqrt{3})}{(9 - 4\sqrt{3})}$$

[3 marks]

Question 3

Rationalise the denominator of, $\frac{6}{2 + \sqrt{2}}$

[4 marks]

Question 4

This question is about rationalising the denominator of

$$\frac{60}{7 + 3\sqrt{5}}$$

Step 1 : Expand the brackets : $(7 + 3\sqrt{5})(7 - 3\sqrt{5})$

[2 marks]

Step 2 : Multiply both numerator and denominator by $7 - 3\sqrt{5}$

[3 marks]

Question 5

Rationalise the denominator, writing your answer in the form $a + b\sqrt{c}$

$$\frac{\sqrt{7}}{8 - 3\sqrt{7}}$$

[4 marks]

Question 6

(i) Expand the brackets : $\left(7 + 4\sqrt{3}\right)^2$

[2 marks]

(ii) Expand the brackets : $\left(7 + 4\sqrt{3}\right)\left(7 - 4\sqrt{3}\right)$

[2 marks]

(iii) Hence, rationalise the denominator of $\dfrac{7 + 4\sqrt{3}}{7 - 4\sqrt{3}}$

[2 marks]

Question 7

GCSE Examination Question, June 2018, Q21 (b)

Express $\dfrac{2}{\sqrt{3} - 1}$ in the form $p + \sqrt{q}$ where p and q are integers

Show your working clearly.

[2 marks]

Question 8

Rationalise the denominator of, $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

[4 marks]

Question 9

Rationalise the denominator of $\frac{100\sqrt{5}}{7 + 2\sqrt{11}}$

Write your answer in the form $a\sqrt{5} + b\sqrt{55}$

[5 marks]

Question 10

Rationalise the denominator of $\dfrac{46}{5\sqrt{2} + 3\sqrt{3}}$

Write your answer in the form $a\sqrt{2} + b\sqrt{3}$

[5 marks]

5.3 Answers

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

5.3.1 Solution to 5.1 Introductory Example

Step 1 :

$$(5 + 3\sqrt{2})(5 - 3\sqrt{2}) = 25 - 15\sqrt{2} + 15\sqrt{2} - 18$$
$$= 7$$

[2 marks]

Step 2 :

$$\frac{21}{(5 + 3\sqrt{2})} = \frac{21}{(5 + 3\sqrt{2})} \times \frac{(5 - 3\sqrt{2})}{(5 - 3\sqrt{2})}$$
$$= \frac{21(5 - 3\sqrt{2})}{7}$$
$$= 3(5 - 3\sqrt{2})$$
$$= 15 - 9\sqrt{2}$$

[3 marks]

5.3.2 Solutions to 5.2 Exercise

Answer 1
Step 1 :

$$(3 + \sqrt{5})(3 - \sqrt{5}) = 9 - 3\sqrt{5} + 3\sqrt{5} - 5$$
$$= 4$$

[2 marks]

Step 2:

$$\frac{12}{(3 + \sqrt{5})} = \frac{12}{(3 + \sqrt{5})} \times \frac{(3 - \sqrt{5})}{(3 - \sqrt{5})}$$
$$= \frac{12(3 - \sqrt{5})}{4}$$
$$= 3(3 - \sqrt{5})$$
$$= 9 - 3\sqrt{5}$$

[3 marks]

Answer 2

Step 1 :

$$(9 + 4\sqrt{3})(9 - 4\sqrt{3}) = 81 - 36\sqrt{3} + 36\sqrt{3} - 48$$
$$= 33$$

[2 marks]

Step 2:

$$\frac{132}{(9 + 4\sqrt{3})} = \frac{132}{(9 + 4\sqrt{3})} \times \frac{(9 - 4\sqrt{3})}{(9 - 4\sqrt{3})}$$
$$= \frac{132(9 - 4\sqrt{3})}{33}$$
$$= 4(9 - 4\sqrt{3})$$
$$= 36 - 16\sqrt{3}$$

[3 marks]

Answer 3

$$\frac{6}{2 + \sqrt{2}} = \frac{6}{2 + \sqrt{2}} \times \frac{2 - \sqrt{2}}{2 - \sqrt{2}}$$
$$= \frac{6(2 - \sqrt{2})}{2}$$
$$= 3(2 - \sqrt{2})$$
$$= 6 - 3\sqrt{2}$$

[4 marks]

Answer 4

Step 1 :

$$(7 + 3\sqrt{5})(7 - 3\sqrt{5}) = 49 - 21\sqrt{5} + 21\sqrt{5} - 45$$
$$= 4$$

[2 marks]

Step 2:

$$\frac{60}{(7 + 3\sqrt{5})} = \frac{60}{(7 + 3\sqrt{5})} \times \frac{(7 - 3\sqrt{5})}{(7 - 3\sqrt{5})}$$
$$= \frac{60(7 - 3\sqrt{5})}{4}$$
$$= 15(7 - 3\sqrt{5})$$
$$= 105 - 45\sqrt{5}$$

[3 marks]

Answer 5

Step 1 :

$$(8 + 3\sqrt{7})(8 - 3\sqrt{7}) = 64 - 24\sqrt{7} + 24\sqrt{7} - 63$$
$$= 1$$

Step 2:

$$\frac{\sqrt{7}}{(8 - 3\sqrt{7})} = \frac{\sqrt{7}}{(8 - 3\sqrt{7})} \times \frac{(8 + 3\sqrt{7})}{(8 + 3\sqrt{7})}$$
$$= \frac{\sqrt{7}\,(8 + 3\sqrt{7})}{1}$$
$$= 21 + 8\sqrt{7}$$

[4 marks]

Answer 6

(i) $(7 + 4\sqrt{3})(7 + 4\sqrt{3}) = 49 + 28\sqrt{3} + 28\sqrt{3} + 48$
$= 97 + 56\sqrt{3}$

[2 marks]

(ii) $(7 + 4\sqrt{3})(7 - 4\sqrt{3}) = 49 - 28\sqrt{3} + 28\sqrt{3} - 48$
$= 1$

[2 marks]

(iii) $\frac{(7 + 4\sqrt{3})}{(7 - 4\sqrt{3})} = \frac{(7 + 4\sqrt{3})}{(7 - 4\sqrt{3})} \times \frac{(7 + 4\sqrt{3})}{(7 + 4\sqrt{3})}$
$= 97 + 56\sqrt{3}$

[2 marks]

Answer 7

$$\frac{2}{(\sqrt{3} - 1)} = \frac{2}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$$
$$= \frac{2(\sqrt{3} + 1)}{2}$$
$$= \sqrt{3} + 1$$

[2 marks]

Answer 8

$$\frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})} = \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})} \times \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})}$$
$$= \frac{3 + \sqrt{6} + \sqrt{6} + 2}{1}$$
$$= 5 + 2\sqrt{6}$$

[4 marks]

Answer 9

$$\frac{100\sqrt{5}}{(7+2\sqrt{11})} = \frac{100\sqrt{5}}{(7+2\sqrt{11})} \times \frac{(7-2\sqrt{11})}{(7-2\sqrt{11})}$$

$$= \frac{100\sqrt{5}\,(7-2\sqrt{11})}{49-14\sqrt{11}+14\sqrt{11}-44}$$

$$= 20\sqrt{5}\,(7-2\sqrt{11})$$

$$= 140\sqrt{5} - 40\sqrt{55}$$

[5 marks]

Answer 10

$$\frac{46}{(5\sqrt{2}+3\sqrt{3})} = \frac{46}{(5\sqrt{2}+3\sqrt{3})} \times \frac{(5\sqrt{2}-3\sqrt{3})}{(5\sqrt{2}-3\sqrt{3})}$$

$$= \frac{46(5\sqrt{2}-3\sqrt{3})}{50-15\sqrt{6}+15\sqrt{6}-27}$$

$$= \frac{46(5\sqrt{2}-3\sqrt{3})}{50-27}$$

$$= \frac{46(5\sqrt{2}-3\sqrt{3})}{23}$$

$$= 2(5\sqrt{2}-3\sqrt{3})$$

$$= 10\sqrt{2} - 6\sqrt{3})$$

[5 marks]

Lesson 6

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

6.1 Surd arithmetic work-out

Example

Show how to calculate $2\sqrt{15} \times 4\sqrt{10}$ without using a calculator.
Write your answer in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

[2 marks]

6.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 50

Do NOT use a calculator

Question 1

Show how to calculate each of the following without using a calculator.
Write your answers in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

(i) $5\sqrt{6} \times 11\sqrt{21}$

(ii) $5\sqrt{14} \times 3\sqrt{10}$

(iii) $10\sqrt{22} \times 4\sqrt{6}$

(iv) $3\sqrt{10} \times 4\sqrt{55}$

[8 marks]

Question 2

Show how to rationalise the denominator of the following without using a calculator. However, check your answer is correct by using a calculator.

(i) $\dfrac{3}{2\sqrt{3}}$ **(ii)** $\dfrac{3}{4\sqrt{5}}$

(iii) $\dfrac{21}{\sqrt{7}}$ **(iv)** $\dfrac{18}{11\sqrt{3}}$

[8 marks]

Question 3

Show how to rationalise the denominator of the following without using a calculator. However, check your answer is correct by using a calculator.

(i) $\dfrac{1}{2-\sqrt{3}}$ **(ii)** $\dfrac{1}{\sqrt{5}+\sqrt{3}}$

[3, 3 marks]

Question 4

(i) Write 252 as a product of primes.
You may use the factorise button on your calculator, marked FACT

[1 mark]

(ii) If $\sqrt{252} = x\sqrt{7}$ deduce the value of the integer x

[2 marks]

Question 5

(i) Write 882 as a product of primes.
You may use the factorise button on your calculator, marked FACT

[1 mark]

(ii) If $\sqrt{882} = y\sqrt{2}$ deduce the value of the integer y

[2 marks]

Question 6

Combine your answers from questions 4 and 5 to determine $\sqrt{252} \times \sqrt{882}$

[2 marks]

Question 7

Show how to rationalise the denominator of the following without using a calculator. However, check your answer is correct by using a calculator.

(i) $\dfrac{3}{2\sqrt{5}+1}$

[4 marks]

(ii) $\dfrac{\sqrt{2}}{3\sqrt{2}-1}$

[4 marks]

Question 8

(i) Write 2541 as a product of primes
You may use the factorise button on your calculator

[1 mark]

(ii) Write 3024 as a product of primes
You may use the factorise button on your calculator

[1 mark]

(iii) Show how to combine your answers to parts **(i)** and **(ii)** to determine the value of $\sqrt{2541} + \sqrt{3024}$

[2 marks]

Question 9

(i) Write 3146 as a product of primes
You may use the factorise button on your calculator

[1 mark]

(ii) Write 936 as a product of primes
You may use the factorise button on your calculator

[1 mark]

(iii) Show how to combine your answers to parts **(i)** and **(ii)** to determine the value of $\sqrt{3146} - \sqrt{936}$

[2 marks]

Question 10

Write in the form $a + b\sqrt{c}$ where a, b and c are integers.

Show clear working that demonstrates that you did not use a calculator.

$$\frac{75 + 3\sqrt{6000}}{5}$$

HINT : $\dfrac{a + b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$

[4 marks]

6.3 Answers

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

6.3.1 Solutions (6.1 Introductory Example)

$$2\sqrt{15} \times 4\sqrt{10} = 2\sqrt{3}\ \sqrt{5}\ 4\ \sqrt{2}\ \sqrt{5}$$
$$= 40\sqrt{6}$$

[2 marks]

6.3.2 Solutions (6.2 Exercise)

Answer 1

(i) $165\sqrt{14}$ **(ii)** $30\sqrt{35}$
(iii) $80\sqrt{33}$ **(iv)** $60\sqrt{22}$

[8 marks]

Answer 2

(i) $\dfrac{\sqrt{3}}{2}$ **(ii)** $\dfrac{3\sqrt{5}}{20}$

(iii) $3\sqrt{7}$ **(iv)** $\dfrac{6\sqrt{3}}{11}$

[8 marks]

Answer 3

(i) $2 + \sqrt{3}$ **(ii)** $\dfrac{\sqrt{5} - \sqrt{3}}{2}$

[3, 3 marks]

Answer 4

(i) $252 = 2^2\, 3^2\, 7$
(ii) $6\sqrt{7}$ That is, $x = 6$

Answer 5

(i) $882 = 2\, 3^2\, 7^2$
(ii) $21\sqrt{2}$ That is, $y = 21$

[3, 3 marks]

Answer 6

$126\sqrt{14}$

[2 marks]

Answer 7

(i) $\dfrac{6\sqrt{5} - 3}{19}$ **(ii)** $\dfrac{6 + \sqrt{2}}{17}$

[4, 4 marks]

Answer 8

(i) $2541 = 3\,7\,11^2$ $\therefore \sqrt{2541} = 11\sqrt{21}$

(ii) $3024 = 2^4\,3^3\,7$ $\therefore \sqrt{3024} = 12\sqrt{21}$

(iii) $23\sqrt{21}$

[1, 1, 2 marks]

Answer 9

(i) $3146 = 2\,11^2\,13$ $\therefore \sqrt{3146} = 11\sqrt{26}$

(ii) $936 = 2^3\,3^2\,13$ $\therefore \sqrt{936} = 6\sqrt{26}$

(iii) $5\sqrt{26}$

[1, 1, 2 marks]

Answer 10

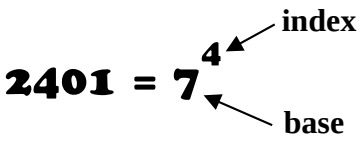
$15 + 12\sqrt{15}$

[4 marks]

Lesson 7

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

7.1 Index Manipulations



7.2 Introductory Examples

Without using a calculator, write down the values of

(i) $25^{\frac{1}{2}}$ **(ii)** $27^{\frac{1}{3}}$ **(iii)** 3^{-2} **(iv)** $4^{\frac{3}{2}}$ (v) $9^{-\frac{1}{2}}$

(vi) $100^{-\frac{3}{2}}$ **(vii)** $81^{\frac{1}{4}}$ **(viii)** $81^{\frac{3}{4}}$ **(ix)** $81^{-\frac{1}{2}}$ **(x)** 81^{0}

[5 marks]

7.3 Examples Of Tricky Index Questions

Example #1

Given that $81\sqrt{3} = 3^{a}$ find the value of the real valued constant a

[2 marks]

Example #2

Simplify $\left(27x^{12}\right)^{\frac{2}{3}}$

[2 marks]

Example #3

Without using a calculator, and making your method clear, find the square root of

$$2^5 \times 3^4 \times 5^3$$

Writing your answer in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

[3 marks]

Example #4

Simplify, $\left(\frac{3x^2}{4\sqrt{y}} \right)^{-3}$

[3 marks]

7.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 45

Do NOT use a calculator

Question 1

Simplify, $(2h)^3$

[2 marks]

Question 2

Simplify, $\left(8x^3\sqrt{y}\right)^2$

[3 marks]

Question 3

Simplify, $\left(\frac{3m^2}{5n} \right)^3$

[3 marks]

Question 4

Simplify, $\left(\frac{a}{b} \right)^{-1}$

[1 mark]

Question 5

Simplify, $\left(\frac{3\sqrt{x}}{4y^5} \right)^{-2}$

[3 marks]

Question 6

Without using a calculator, and making your method clear, find the square root of

$$2^7 \times 3 \times 5^4$$

Writing your answer in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

[3 marks]

Question 7

Given that, $8\sqrt{2} = 2^a$ find the value of the real constant a

[2 marks]

Question 8

Simplify, $\left(25x^{12}\right)^{\frac{3}{2}}$

[2 marks]

Question 9

GCSE Examination Question, May 2016, Paper 3H, Q7 (d)

Simplify fully, $\dfrac{28x^5y^3}{4xy^2}$

[3 marks]

Question 10

GCSE Examination Question, January 2017, Paper 4H, Q16

$g = 2^3 \times 3 \times 7^2$ $\qquad$ $h = 2 \times 3 \times 7^3$

(a) Express gh as a product of powers of its prime factors
Simplify your answer

[2 marks]

(b) Find the value of a, the value of b and the value of c given that,

$$\frac{g}{h} = 2^a \times 3^b \times 7^c$$

[3 marks]

(c) Show that, $\left(7 - 2\sqrt{5}\right)\left(7 + 2\sqrt{5}\right) = 29$
Show your working clearly.

[3 marks]

(d) Work out the exact value of n, given that, $\dfrac{1}{\sqrt[3]{9^4}} = 3^n$

[3 marks]

Question 11

GCSE Examination Question, June 2016, Paper 4H, Q15 (b)

Simplify, $\left(8\,a^9\,e^6\right)^{\frac{1}{3}}$

[2 marks]

Question 12

GCSE Examination Question, May 2019, Paper 1H, Q10

$$A = 2^n \times 3 \times 5^m$$

Write 8*A* as a product of powers of its prime factors.

[2 marks]

Question 13

GCSE Examination Question, May 2019, Paper 1H, Q12(c)

Write the following in the form y^b where *b* is a fraction

Write $\frac{\sqrt[4]{y}}{y}$ in the form y^b where *b* is a fraction

[2 marks]

Question 14

GCSE Examination Question, January 2017, Paper 3H(R), Q11 (c)

Simplify, $\left(\frac{y^5}{8\,x^6\,y^8}\right)^{-\frac{1}{3}}$

[3 marks]

Question 15

GCSE Examination Question, January 2017, Paper 3H(R), Q18

Given that *p* is a prime number, rationalise the denominator of

$$\frac{7\sqrt{p} - p^2}{\sqrt{p^3}}$$

Simplify your answer.

[3 marks]

Question 16

GCSE Examination Question, January 2017, Paper 4H(R), Q20

(a) $\left(3 + \sqrt{c}\right)\left(2\sqrt{c} - 3\right) = 1 + k\sqrt{c}$ where c and k are prime numbers.

Find the value of c and the value of k.

[3 marks]

(b)

$$p^m = \frac{1}{p \times \sqrt[3]{p^2}}$$

Find the value of m.

[3 marks]

7.5 Answers

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

7.5.1 Solutions (7.2 Introductory Examples)

(i) 5 **(ii)** 3 **(iii)** $\frac{1}{9}$ **(iv)** 8 **(v)** $\frac{1}{3}$

(vi) $\frac{1}{1000}$ **(vii)** 3 **(viii)** 27 **(ix)** $\frac{1}{9}$ **(x)** 1

[5 marks]

7.5.2 Solutions to 7.3 Examples of Tricky Index Questions

Answer to Example #1

$$\begin{aligned} 81\sqrt{3} &= 3^a \\ 3^4 \times 3^{\frac{1}{2}} &= 3^a \\ 3^{4.5} &= 3^a \\ \therefore \quad a &= 4.5 \end{aligned}$$

[2 marks]

Answer to Example #2

$$\begin{aligned} \left(27x^{12}\right)^{\frac{2}{3}} &= 27^{\frac{2}{3}} \times \left(x^{12}\right)^{\frac{2}{3}} \\ &= 9x^8 \end{aligned}$$

[2 marks]

Answer to Example #3

$$\begin{aligned} 2^5 \times 3^4 \times 5^3 &= 2^4 \times 3^4 \times 5^2 \times 2 \times 3 \\ \sqrt{2^5 \times 3^4 \times 5^3} &= \sqrt{2^4 \times 3^4 \times 5^2} \times \sqrt{2 \times 3} \\ &= 2^2 \times 3^2 \times 5 \times \sqrt{6} \\ &= 4 \times 9 \times 5 \times \sqrt{6} \\ &= 180\sqrt{6} \end{aligned}$$

[3 marks]

Answer to Example #4

$$\begin{aligned} \left(\frac{3x^2}{4\sqrt{y}}\right)^{-3} &= \left(\frac{4y^{\frac{1}{2}}}{3x^2}\right)^3 \\ &= \frac{64y^{\frac{3}{2}}}{27x^6} \end{aligned}$$

[3 marks]

7.5.3 Solutions (7.4 Exercise)

Answer 1 **Answer 2** **Answer 3** **Answer 4**

$8h^3$ $64x^6y$ $\frac{27m^6}{125n^3}$ $\frac{b}{a}$

[2, 3, 3, 1 marks]

Answer 5

$$\left(\frac{3\sqrt{x}}{4y^5}\right)^{-2} = \left(\frac{4y^5}{3\sqrt{x}}\right)^2$$
$$= \frac{16y^{10}}{9x}$$

[3 marks]

Answer 6

$$2^7 \times 3 \times 5^4 = 2^6 \times 5^4 \times 2 \times 3$$
$$\sqrt{2^7 \times 3 \times 5^4} = \sqrt{2^6 \times 5^4} \times \sqrt{2 \times 3}$$
$$= 2^3 \times 5^2 \times \sqrt{6}$$
$$= 8 \times 25 \times \sqrt{6}$$
$$= 200\sqrt{6}$$

[3 marks]

Answer 7

$$8\sqrt{2} = 2^a$$
$$2^3 \times 2^{\frac{1}{2}} = 2^a$$
$$2^{3.5} = 2^a$$
$$a = 3.5$$

[2 marks]

Answer 8

$$\left(25x^{12}\right)^{\frac{3}{2}} = 25^{\frac{3}{2}} \times \left(x^{12}\right)^{\frac{3}{2}}$$
$$= 125x^{18}$$

[2 marks]

Answer 9

$$\frac{28x^5y^3}{4xy^2} = 7x^4y$$

[3 marks]

Answer 10

(a) $gh = 2^4 \times 3^2 \times 7^5$

[2 marks]

(b) $\frac{g}{h} = 2^2 \times 3^0 \times 7^{-1}$

$a = 2, \qquad b = 0, \qquad c = -1$

[3 marks]

(c) $\text{LHS} = \left(7 - 2\sqrt{5}\right)\left(7 + 2\sqrt{5}\right)$

$= 49 + 14\sqrt{5} - 14\sqrt{5} - 2 \times 2 \times \sqrt{5} \times \sqrt{5}$

$= 49 - 20$

$= 29$

$= \text{RHS} \qquad \square$

[3 marks]

(d) $\frac{1}{\sqrt[3]{9^4}} = 3^n$

$\left(9^4\right)^{-\frac{1}{3}} = 3^n$

$\left(3^8\right)^{-\frac{1}{3}} = 3^n$

$3^{-\frac{8}{3}} = 3^n$

$n = -\frac{8}{3}$

[3 marks]

Answer 11

$\left(8\,a^9\,e^6\right)^{\frac{1}{3}} = 2\,a^3\,e^2$

[2 marks]

Answer 12

$8A = 8 \times 2^n \times 3 \times 5^m$

$= 2^3 \times 2^n \times 3 \times 5^m$

$= 2^{n+3} \times 3 \times 5^m$

[2 marks]

Answer 13

$\frac{\sqrt[4]{y}}{y} = y^{\frac{1}{4}-1}$

$= y^{-\frac{3}{4}}$

[2 marks]

Answer 14

$$\left(\frac{y^5}{8x^6y^8} \right)^{-\frac{1}{3}} = \left(\frac{8x^6y^8}{y^5} \right)^{\frac{1}{3}}$$
$$= \left(8x^6y^3 \right)^{\frac{1}{3}}$$
$$= 2x^2y$$

[3 marks]

Answer 15

If you were going to differentiate you'd do the following first,

$$\frac{7\sqrt{p} - p^2}{\sqrt{p^3}} = \frac{7p^{\frac{1}{2}}}{p^{\frac{3}{2}}} - \frac{p^2}{p^{\frac{3}{2}}}$$
$$= 7p^{-1} - p^{\frac{1}{2}}$$
$$= \frac{7}{p} - \sqrt{p}$$

Which is by far the most elegant answer,
Other equivalent answers are likely.

[3 marks]

Answer 16

(a)
$$\left(3 + \sqrt{c}\right)\left(2\sqrt{c} - 3\right) = 1 + k\sqrt{c}$$
$$6\sqrt{c} + 2c - 9 - 3\sqrt{c} = 1 + k\sqrt{c}$$
$$2c - 9 + 3\sqrt{c} = 1 + k\sqrt{c}$$

Matching integers, $2c - 9 = 1$

$$2c = 10$$
$$c = 5$$

Matching coefficients of $\sqrt{c}$, $k = 3$

[3 marks]

(b)
$$p^m = \frac{1}{p \times \sqrt[3]{p^2}}$$
$$= \frac{1}{p \times p^{\frac{2}{3}}}$$
$$= p^{-\frac{5}{3}} \qquad \therefore m = -\frac{5}{3}$$

[3 marks]

Lesson 8

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

8.1 Revision

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 56

Do NOT use a calculator

Question 1
GCSE Examination Question, June 2016, Paper 4H, Q5

(a) Simplify $m^5 \times m^2$

[1 mark]

(b) Simplify $\dfrac{c^{11}}{c^3}$

[1 mark]

(c) Simplify $\left(a^5\right)^3$

[1 mark]

(d) Expand and simplify $4\left(2x + 3\right) + 2\left(x + 5\right)$

[2 marks]

Question 2
A-Level Examination Question from June 2009, C1, Q1 (Edexcel)

(a) Simplify

$$\left(3\sqrt{7}\right)^2$$

[1 mark]

(b) Showing the non-calculator method used, simplify

$$\left(8 + \sqrt{5}\right)\left(2 - \sqrt{5}\right)$$

[3 marks]

Question 3

A-Level Examination Question from January 2007, C1, Q2 (Edexcel)

(a) Show how to express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer, without the use of a calculator.

[1 mark]

(b) Show how to express $\left(2 - \sqrt{3}\right)^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found, without the use of a calculator.

[3 marks]

Question 4

Evaluate each of the following without the use of a calculator.

Write your answers in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

(i) $5\sqrt{21} \times \sqrt{45}$ **(ii)** $2\sqrt{70} \times \sqrt{90}$

[4 marks]

Question 5

A-Level Examination Question from May 2014, C1, Q2 (Edexcel)

(a) Write down the value of $32^{\frac{1}{5}}$

[1 mark]

(b) Simplify fully, $\left(32x^5\right)^{-\frac{2}{5}}$

[3 marks]

Question 6

(i) Write 726 as a product of primes.
You may use the factorise button on your calculator.

[1 mark]

(ii) If $\sqrt{726} = v\sqrt{6}$ find v

[1 mark]

Question 7

(i) Write 1350 as a product of primes
You may use the factorise button on your calculator

[1 mark]

(ii) If $\sqrt{1350} = w\sqrt{6}$ find w

[1 mark]

Question 8

Using your answers from questions 6 and 7, or otherwise, find $\sqrt{726} + \sqrt{1350}$

[1 mark]

Question 9

A-Level Examination Question from May 2006, C1, Q6 (Edexcel)

(a) Demonstrate how to expand and simplify without the use of a calculator

$$\left(4 + \sqrt{3}\right)\left(4 - \sqrt{3}\right)$$

[2 marks]

(b) Making a non-calculator method clear, express

$$\frac{26}{4 + \sqrt{3}}$$

in the form $a + b\sqrt{3}$, where a and b are integers.

[2 marks]

Question 10

GCSE Examination Question, June 2016, Q19 edited

Simplify,

$$\left(7 + 2\sqrt{50}\right)\left(5 - 2\sqrt{2}\right)$$

Give your answer in the form $a + b\sqrt{2}$, where a and b are integers.

Show your working clearly.

[3 marks]

Question 11

Rationalise the denominator, $\dfrac{1 + \sqrt{3}}{2 - \sqrt{3}}$

[4 marks]

Question 12

A-Level Examination Question from June 2009, C1, Q2 (Edexcel)

Given that

$$32\sqrt{2} = 2^a$$

find the value of a.

[3 marks]

Question 13

Without using a calculator, and making your method clear, find the square root of

$$2^7 \times 3^2 \times 5^3$$

Writing your answer in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

[3 marks]

Question 14

A-Level Examination Question from January 2012, C1, Q2 (Edexcel)

(a) Without using a calculator, show how to simplify

$$\sqrt{32} + \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$, where a is an integer.

[2 marks]

(b) Simplify

$$\frac{\sqrt{32} + \sqrt{18}}{3 + \sqrt{2}}$$

giving your answer in the form $b\sqrt{2} + c$, where b and c are integers.
Show full working.

[4 marks]

Question 15

GCSE Examination Question, January 2016, Paper 3H, Q4 (d)

Simplify fully

$$\frac{36\,k^3\,m^4}{30\,k^5\,m}$$

[2 marks]

Question 16

GCSE Examination Question, January 2016, Paper 4H, Q17 (a)

Simplify

$$\left(\frac{8\,e^6}{f^{12}} \right)^{\frac{1}{3}}$$

[2 marks]

Question 17

GCSE Examination Question from January 2016, Paper 4H, Q25

$y = 16 \times 10^{8k}$ where k is an integer.

Find an expression, in terms of k for $y^{\frac{5}{4}}$

Give your answer in standard form.

[3 marks]

8.2 Brief Solutions

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

Answer 1

(a) m^7 **(b)** c^8 **(c)** a^{15} **(d)** $10x + 22$

[1, 1, 1, 2 marks]

Answer 2

(a) 63 **(b)** $11 - 6\sqrt{5}$

[1, 3 marks]

Answer 3

(a) $6\sqrt{3}$ **(b)** $7 - 4\sqrt{3}$

[1, 3 marks]

Answer 4

(i) $15\sqrt{105}$ **(ii)** $60\sqrt{7}$

[2, 2 marks]

Answer 5

(a) 2 **(b)** $0.25x^{-2}$

[1, 3 marks]

Answer 6

(i) $726 = 2\,3\,11^2$

(ii) $11\sqrt{6}$ i.e. $v = 11$

Answer 7

(i) $1350 = 2\,3^3\,5^2$

(ii) $15\sqrt{6}$ i.e. $w = 15$

[2, 2 marks]

Answer 8

$26\sqrt{6}$

[1 mark]

Answer 9

(a) 13 **(b)** $8 - 2\sqrt{3}$

[2, 2 marks]

Answer 10

$-5 + 36\sqrt{2}$ Unedited question wanted $-5 + 12\sqrt{18}$ which is silly !

[3 marks]

Answer 11

$5 + 3\sqrt{3}$

[4 marks]

Answer 12

$\frac{11}{2}$

[3 marks]

Answer 13

$120\sqrt{10}$

[3 marks]

Answer 14

(a) $7\sqrt{2}$ **(b)** $3\sqrt{2} - 2$

[2, 4 marks]

Answer 15

$$\frac{6\,m^3}{5\,k^2}$$

[2 marks]

Answer 16

$$\frac{2\,e^2}{f^4}$$

[2 marks]

Answer 17

$3.2 \times 10^{10k+1}$

[3 marks]

Lesson 9

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

9.1 TEST

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 71

Do NOT use a calculator

Question 1

GCSE Examination Question from June 2017, Paper 4H, Q10

(a) Simplify, $e^8 \times e^7$

[1 mark]

(b) Simplify fully, $\dfrac{12\,g^{10}}{3\,g^2}$

[2 marks]

(c) Write down the value of m^0

[1 mark]

(d) Simplify fully, $\left(27\,x^6\right)^{\frac{2}{3}}$

[2 marks]

Question 2

Simplify, $\left(3\sqrt{7}\right)^2$

[1 mark]

Question 3

A-Level Examination Question from January 2009, C1, Q3 (Edexcel)
Showing the non-calculator method used, expand and simplify

$$\left(\sqrt{7}+2\right)\left(\sqrt{7}-2\right)$$

[2 marks]

Question 4

A-Level Examination Question from May 2007, C1, Q1 (Edexcel)

Demonstrate how to simplify without using a calculator,

$$\left(3 + \sqrt{5}\right)\left(3 - \sqrt{5}\right)$$

[2 marks]

Question 5

A-Level Examination Question from January 2010, C1, Q2 (Edexcel)

(a) Expand and simplify,

$$\left(7 + \sqrt{5}\right)\ \left(3 - \sqrt{5}\right)$$

[3 marks]

(b) Making your non-calculator method clear, express

$$\frac{7 + \sqrt{5}}{3 + \sqrt{5}}$$

in the form $a + b\sqrt{5}$, where a and b are integers.

[3 marks]

Question 6

Evaluate each of the following without the use of a calculator.

Write your answers in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

(i) $3\sqrt{15} \times 7\sqrt{10}$

(ii) $3\sqrt{5^2 \times 7} \times 2\sqrt{5 \times 7^2}$

[4 marks]

Question 7

(i) Write 980 as a product of primes.
You may use the factorise button on your calculator.

[1 mark]

(ii) If $\sqrt{980} = m\sqrt{5}$ find m

[1 mark]

Question 8

(i) Write 1125 as a product of primes
You may use the factorise button on your calculator

[1 mark]

(ii) If $\sqrt{1125} = n\sqrt{5}$ find n

[1 mark]

Question 9

Using your answers from questions 7 and 8, or otherwise, find $\sqrt{980} + \sqrt{1125}$

[1 mark]

Question 10

A-Level Examination Question from January 2008, C1, Q2 (Edexcel)

(a) Write down the value of

$$16^{\frac{1}{4}}$$

[1 mark]

(b) Simplify

$$\left(16x^{12}\right)^{\frac{3}{4}}$$

[2 marks]

Question 11

A-level Examination Question from May 2013, C1, Q1 (Edexcel)

Simplify

$$\frac{7+\sqrt{5}}{\sqrt{5}-1}$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are integers.

Show full working.

[4 marks]

Question 12

Making your non-calculator method clear, rationalise the denominator and simplify;

(i) $\dfrac{12}{\sqrt{6}}$ **(ii)** $\dfrac{14\sqrt{3}}{5\sqrt{7}}$

[4 marks]

Question 13

Given that $64\sqrt{2} = 2^a$ find the value of the constant a

[3 marks]

Question 14

Without using a calculator, and making your method clear, find the square root of

$$2^7 \times 3^3 \times 5^2$$

Writing your answer in the form $a\sqrt{b}$ where a and b are integers and b is $\square$ free.

[3 marks]

Question 15

A-Level Examination Question from May 2010, C1, Q1 (edited) (Edexcel)

Making your method clear, write

$$\sqrt{75} - \sqrt{27}$$

in the form $k\sqrt{x}$, where k and x are integers.

[2 marks]

Question 16

A-level C1 Examination Question, May 2012, Q3 (edited)

Giving detail of your reasoning, show that

$$\frac{2}{\sqrt{12} - \sqrt{8}}$$

can be written in the form $\sqrt{a} + \sqrt{b}$ where a and b are integers

[5 marks]

Question 17

GCSE Examination Question from January 2018, 3H, Q20

(a) $\sqrt{50} + \sqrt{128} - \sqrt{200} = n\sqrt{2}$ where n is an integer

Find the value of n

Show each stage of your working.

[2 marks]

Given that a is a prime number,

(b) simplify

$$\frac{5\sqrt{a} + a}{10\sqrt{a}}$$

Give your answer in the form $x + y\sqrt{a}$ where x and y are fractions.

Show your working clearly.

[2 marks]

Question 18

A-level Examination Question from May 2013, C1, Q3 (Edexcel)

(a) Find the value of $8^{\frac{5}{3}}$

[2 marks]

(b) Simplify fully, $\dfrac{\left(2x^{\frac{1}{2}}\right)^3}{4x^2}$

[3 marks]

Question 19

A-Level Examination Question from May 2014, Q6

(a) Write $\sqrt{80}$ in the form $c\sqrt{5}$, where c is a positive constant.

[1 mark]

A rectangle R has a length of $\left(1 + \sqrt{5}\right)$ cm and an area of $\sqrt{80}$ cm^2

(b) Calculate the width of R in cm.

Express your answer in the form $p + q\sqrt{5}$, where p and q are integers to be found.

[4 marks]

Question 20

A-Level Examination Question from May 2016, C1, Q3 (edited) (Edexcel)

(a) Showing the reasons why, simplify,

$$\sqrt{50} - \sqrt{18}$$

giving your answer in the form $a\sqrt{2}$ where a is an integer

[2 marks]

(b) Hence, or otherwise, simplify

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}}$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$

Show full working.

[3 marks]

Question 21

A-Level Examination Question from May 2016, C1, Q2 (Edexcel)

Express 9^{3x+1} in the form 3^{y} giving y in the form $ax + b$ where a and b are constants.

[2 marks]

9.2 Answers

A-Level Pure Mathematics : Year 1
GCSE (Grades 8 and 9)
Algebra of Surds and Indices I

The test has 71 marks available

Answer 1

(a) e^{15} **(b)** $4g^{8}$ **(c)** 1 **(d)** $9x^{4}$

[1, 2, 1, 2 marks]

Answer 2

63

[1 mark]

Answer 3

(Question states "Showing non-calculator method used")

$$\left(\sqrt{7}+2\right)\left(\sqrt{7}-2\right) = 7 + 2\sqrt{7} - 2\sqrt{7} - 4$$
$$= 3$$

[2 marks]

Answer 4

(Question states "Demonstrate how to simplify without using a calculator")

$$\left(3+\sqrt{5}\right)\left(3+\sqrt{5}\right) = 9 + 3\sqrt{5} - 3\sqrt{5} - 5$$
$$= 4$$

[2 marks]

Answer 5

(a) Must expand, then simplify,

$$\left(7+\sqrt{5}\right)\left(3-\sqrt{5}\right) = 21 + 3\sqrt{5} - 7\sqrt{5} - 5$$
$$= 16 - 4\sqrt{5}$$

[3 marks]

(b) Must make non-calculator method clear,

$$\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{16 - 4\sqrt{5}}{9 + 3\sqrt{5} - 3\sqrt{5} - 5}$$
$$= \frac{4\left(4-\sqrt{5}\right)}{4}$$
$$= 4 - \sqrt{5}$$
$$\therefore \quad a = 4 \qquad b = -1$$

[3 marks]

Answer 6

Must demonstrate a non-calculator method

(i) $3\sqrt{15} \times 7\sqrt{10} = 3 \times \sqrt{3} \times \sqrt{5} \times 7 \times \sqrt{2} \times \sqrt{5}$

$= 3 \times 5 \times 7 \times \sqrt{2} \times \sqrt{3}$

$= 105\sqrt{6}$

[2 marks]

(ii) $3\sqrt{5^2 \times 7} \times 2\sqrt{5 \times 7^2} = 3 \times 5 \times \sqrt{7} \times 2 \times \sqrt{5} \times 7$

$= 210\sqrt{35}$

[2 marks]

Answer 7

(i) $980 = 2^2 \times 5 \times 7^2$

[1 mark]

(ii) $\sqrt{980} = \sqrt{2^2 \times 5 \times 7^2}$

$= 2 \times \sqrt{5} \times 7$

$= 14\sqrt{5}$

$\therefore \ m = 14$

[1 mark]

Answer 8

(i) $1125 = 3^2 \times 5^3$

[1 mark]

(ii) $\sqrt{1125} = \sqrt{3^2 \times 5^3}$

$= \sqrt{3^2 \times 5^2 \times 5}$

$= 3 \times 5 \times \sqrt{5}$

$= 15\sqrt{5}$

$\therefore \ n = 15$

[1 mark]

Answer 9

$\sqrt{980} + \sqrt{1125} = 14\sqrt{5} + 15\sqrt{5}$

$= 29\sqrt{5}$

[1 mark]

Answer 10

(a) 2 **(b)** $8x^9$

[1, 2 marks]

Answer 11

$$\frac{7+\sqrt{5}}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{7\sqrt{5}+5+7+\sqrt{5}}{5-\sqrt{5}+\sqrt{5}-1}$$

$$= \frac{12+8\sqrt{5}}{4}$$

$$= \frac{4\left(3+2\sqrt{5}\right)}{4}$$

$$= 3+2\sqrt{5}$$

[4 marks]

Answer 12

Must make non-calculator method clear...

(i)

$$\frac{12}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{12\sqrt{6}}{6}$$

$$= 2\sqrt{6}$$

[2 marks]

(ii)

$$\frac{14\sqrt{3}}{5\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{14\sqrt{21}}{5 \times 7}$$

$$= \frac{2\sqrt{21}}{35}$$

[2 marks]

Answer 13

$$64\sqrt{2} = 2^a$$

$$2^6 \times 2^{\frac{1}{2}} = 2^a$$

$$2^{6.5} = 2^a$$

$$a = 6.5$$

[3 marks]

Answer 14

Must make non-calculator method clear...

$$\sqrt{2^7 \times 3^3 \times 5^2} = \sqrt{2^6 \times 2 \times 3^2 \times 3 \times 5^2}$$

$$= 2^3 \times \sqrt{2} \times 3 \times \sqrt{3} \times 5$$

$$= 120\sqrt{6}$$

[3 marks]

Answer 15

$$\sqrt{75} - \sqrt{27} = \sqrt{25 \times 3} - \sqrt{9 \times 3}$$

$$= 5\sqrt{3} - 3\sqrt{3}$$

$$= 2\sqrt{3}$$

[2 marks]

Answer 16

$$\frac{2}{\sqrt{12}-\sqrt{8}} = \frac{2}{\sqrt{4\times 3}-\sqrt{4\times 2}}$$
$$= \frac{2}{2\sqrt{3}-2\sqrt{2}}$$
$$= \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$
$$= \frac{\sqrt{3}+\sqrt{2}}{3-\sqrt{3}\sqrt{2}+\sqrt{3}\sqrt{2}-2}$$
$$= \sqrt{3}+\sqrt{2}$$

[5 marks]

Answer 17

(a) $\sqrt{50}+\sqrt{128}-\sqrt{200} = \sqrt{25\times 2}+\sqrt{64\times 2}-\sqrt{100\times 2}$
$= 5\sqrt{2}+8\sqrt{2}-10\sqrt{2}$
$= 3\sqrt{2}$

[2 marks]

(b) $\frac{5\sqrt{a}+a}{10\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} = \frac{5a+a\sqrt{a}}{10a}$
$= \frac{a(5+\sqrt{a})}{10a}$
$= \frac{5+\sqrt{a}}{10}$
$= \frac{1}{2}+\frac{1}{10}\sqrt{a}$
$\therefore x = 0.5$ and $y = 0.1$

[2 marks]

Answer 18

(a) 32 **(b)** $2x^{-0.5}$

[2, 3 marks]

Answer 19

(a) $\sqrt{80} = \sqrt{16\times 5}$
$= 4\sqrt{5}$
$\therefore c = 4$

[1 mark]

(b)

$$\begin{aligned} \text{Area } R &= l \times w \\ \sqrt{80} &= \left(1 + \sqrt{5}\right) \times w \\ w &= \frac{\sqrt{80}}{\left(1 + \sqrt{5}\right)} \times \frac{\left(1 - \sqrt{5}\right)}{\left(1 - \sqrt{5}\right)} \\ &= \frac{4\sqrt{5}\left(1 - \sqrt{5}\right)}{1 - \sqrt{5} + \sqrt{5} - 5} \\ &= \frac{4\sqrt{5} - 20}{-4} \\ &= \frac{-4\left(5 - \sqrt{5}\right)}{-4} \\ &= 5 - \sqrt{5} \end{aligned}$$

[4 marks]

Answer 20

(a)

$$\begin{aligned} \sqrt{50} - \sqrt{18} &= \sqrt{25 \times 2} - \sqrt{9 \times 2} \\ &= 5\sqrt{2} - 3\sqrt{2} \\ &= 2\sqrt{2} \\ \therefore \quad a &= 2 \end{aligned}$$

[2 marks]

(b)

$$\begin{aligned} \frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}} &= \frac{12\sqrt{3}}{2\sqrt{2}} \\ &= \frac{12\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{12\sqrt{6}}{4} \\ &= 3\sqrt{6} \end{aligned}$$

[3 marks]

Answer 21

$$\begin{aligned} 9^{3x+1} &= 3^y \\ \left(3^2\right)^{3x+1} &= 3^y \\ 3^{6x+2} &= 3^y \\ \therefore \quad y &= 6x + 2 \end{aligned}$$

[2 marks]