11.2 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics in Algebra

Answer 1

(a) $-4 \leqslant 2y < 6$

divide through by 2

$-2 \leqslant y < 3$

$\therefore y \in \{-2, -1, 0, 1, 2\}$

[2 marks]

(b) $7t - 3 \leqslant 2t + 31$

$5t \leqslant 34$

$t \leqslant \dfrac{34}{5}$

[2 marks]

Answer 2

(a) $25a^4c^7d + 45a^9c^3h = 5a^4c^3\left(5c^4d + 9a^5h\right)$

[2 marks]

(b)

$$\begin{aligned}(2x+5)^2 &= (2x+3)(2x-1)\\ 4x^2 + 20x + 25 &= 4x^2 + 4x - 3\\ 16x &= -28\\ x &= -\frac{28}{16}\\ &= -\frac{7}{4}\end{aligned}$$

[3 marks]

Answer 3

(a) $3x - 7 > 3 - x$

$4x > 10$

$x > \frac{5}{2}$

[2 marks]

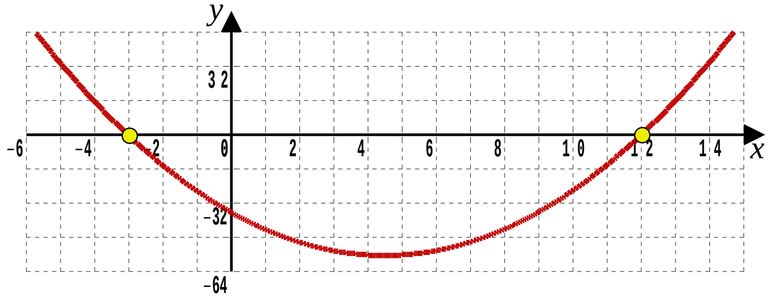
(b) $x^2 - 9x \leqslant 36$

$x^2 - 9x - 36 \leqslant 0$

Finding the critical values...

$(x + 3)(x - 12) = 0$

$x = -3$ or $x = 12$



From the graph, the solution to the inequality $x^2 - 9x \leqslant 36$ is $-3 \leqslant x \leqslant 12$

[3 marks]

(c) $\frac{5}{2} < x \leqslant 12$

[1 mark]

Answer 4

$$x^2 + 4 = 6x$$
$$\left[x^2 - 6x\right] + 4 = 0$$
$$\left[(x-3)^2 - 9\right] + 4 = 0$$
$$(x-3)^2 = 5$$
$$x - 3 = \pm\sqrt{5}$$
$$x = 3 \pm \sqrt{5}$$

[2 marks]

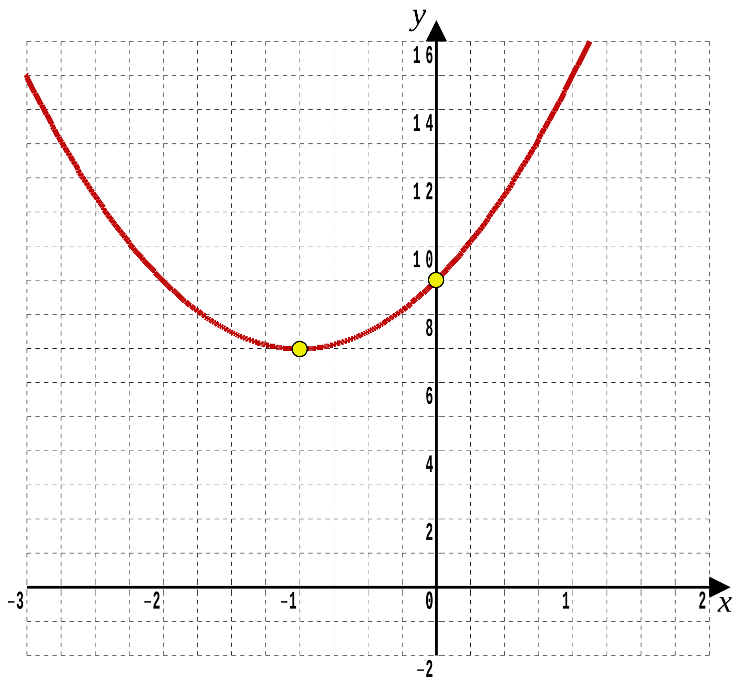
Answer 5

(a)

$$f(x) = 2x^2 + 4x + 9$$
$$= 2\left[x^2 + 2x\right] + 9$$
$$= 2\left[(x+1)^2 - 1\right] + 9$$
$$= 2(x+1)^2 - 2 + 9$$
$$= 2(x+1)^2 + 7$$
$$\therefore \quad a = 2,\ b = 1 \text{ and } c = 7$$

[3 marks]

(b)



The only point of coordinate axes intersection is at $(0,\ 9)$

There is a minimum turning point at $(-1,\ 7)$

[3 marks]

Answer 6

$$7 - 12x - 2x^2 = (-2)\left[\, x^2 + 6x \,\right] + 7$$
$$= (-2)\left[\, (x+3)^2 - 9 \,\right] + 7$$
$$= (-2)(x+3)^2 + 18 + 7$$
$$= 25 + (-2)(x+3)^2$$
$$\therefore\ a = 25,\ b = -2 \text{ and } c = 3$$

[3 marks]

Answer 7

$$x^2 + (k-3)x + (3-2k) = 0$$

(a) To have 2 distinct real roots the discriminant must be positive.

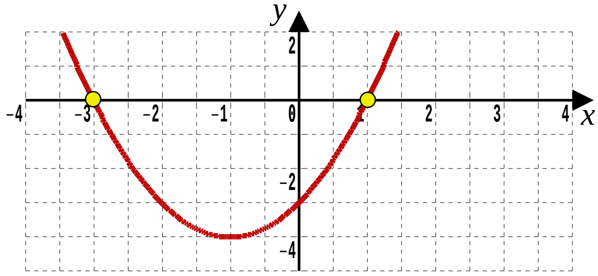
$$D > 0$$
$$(k-3)^2 - 4 \times 1 \times (3-2k) > 0$$
$$k^2 - 6k + 9 - 12 + 8k > 0$$
$$k^2 + 2k - 3 > 0 \qquad \square$$

[3 marks]

(b) Finding the critical values...

$$(k+3)(k-1) = 0$$
$$\text{Either } k = -3 \text{ or } k = 1$$

Let $y = k^2 + 2k - 3$



From the graph, the solution to the inequality $k^2 + 2k - 3 > 0$

is $k < -3$ or $k > 1$

[4 marks]

Answer 8

(a) Starting with the equations,

$$y = x - 4$$
$$2x^2 - xy = 8$$

$$2x^2 - x(x - 4) = 8$$
$$2x^2 - x^2 + 4x - 8 = 0$$
$$x^2 + 4x - 8 = 0 \qquad \square$$

[2 marks]

(b)

$$\left[x^2 + 4x\right] - 8 = 0$$
$$\left[(x + 2)^2 - 4\right] - 8 = 0$$
$$(x + 2)^2 = 12$$
$$x + 2 = \pm 2\sqrt{3}$$
$$x = -2 \pm 2\sqrt{3}$$
$$\therefore a = -2 \text{ and } b = 2$$

Using $y = x - 4$ then gives the solutions as,

$$\left(-2 - 2\sqrt{3}, -6 - 2\sqrt{3}\right) \text{ or } \left(-2 + 2\sqrt{3}, -6 + 2\sqrt{3}\right)$$

[5 marks]

Answer 9

(a) Starting with the equations,

$$y = 2x + k$$
$$x^2 + y^2 = 5$$

$$x^2 + (2x + k)^2 = 5$$
$$x^2 + 4x^2 + 4kx + k^2 = 5$$
$$5x^2 + 4kx + \left(k^2 - 5\right) = 0 \qquad \square$$

[2 marks]

(b) A tangent requires that there be one distinct root.
This in turn requires that the discriminant equal zero.

$$(4k)^2 - 4 \times 5 \times \left(k^2 - 5\right) = 0$$
$$16k^2 - 20k^2 + 100 = 0$$
$$4k^2 = 100$$
$$k = \pm 5$$

[2 marks]

Answer 10

(a)

$$x^2 + 2kx - 7 = 0$$

$$\left[x^2 + 2kx \right] - 7 = 0$$

$$\left[(x + k)^2 - k^2 \right] - 7 = 0$$

$$(x + k)^2 = 7 + k^2$$

$$x + k = \pm \sqrt{7 + k^2}$$

$$x = -k \pm \sqrt{7 + k^2}$$

[4 marks]

(b) The discriminant of $x^2 + 2kx - 7 = 0$ is,

$$(2k)^2 - 4 \times 1 \times (-7) = 4k^2 + 28$$

For all real values of k, $k^2 \geqslant 0$

Even when k^2 has its minimum value of 0, the discriminant, $4k^2 + 28$, is positive and so the roots of $x^2 + 2kx - 7$ are always real and different. □

[2 marks]

(c) From part (a) with $k = \sqrt{2}$, $x = -\sqrt{2} \pm \sqrt{7 + (\sqrt{2})^2}$

$\therefore$ The exact roots are $x = \pm 3 - \sqrt{2}$

[2 marks]

Answer 11

(a) $2x + 2(x + 20) < 300$

$$4x + 40 < 300$$

$$4x < 260$$

$$x < 65$$

[2 marks]

(b) $x(x + 20) > 4800$

$$x^2 + 20x - 4800 > 0$$

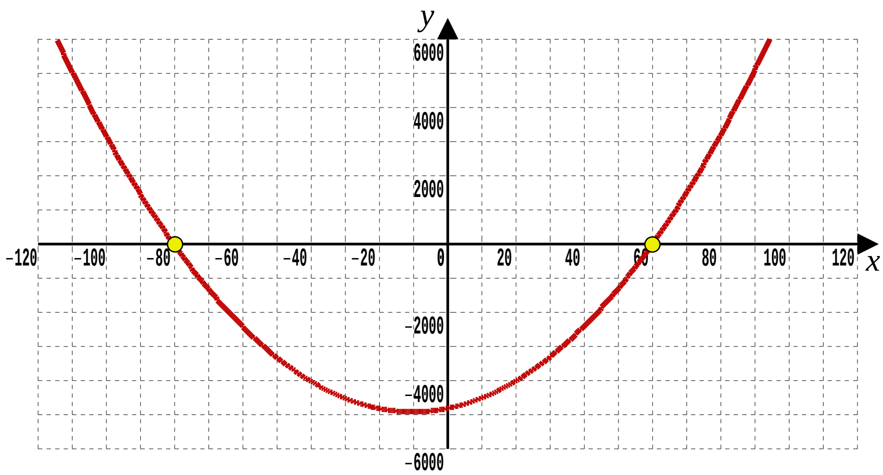
[2 marks]

(c) Finding the critical values for the inequality of part (b)...

$$(x + 80)(x - 60) = 0$$

Either $x = -80$ or $x = 60$

Let $y = x^2 + 20x - 4800$



From the graph, the solution to the inequality $x^2 + 20x - 4800 > 0$

is $x < -80$ or $x > 60$

In combination with part (a), and the fact that x has to be positive,

the set of possible values for x is thus, $60 < x < 65$

[4 marks]