

4.3 Answers (4.2 Exercise)

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics In Algebra

Answer 1

$$\begin{aligned} \text{(i)} \quad & 2x^2 + 7x + 1 = 0 \\ & 2\left[x^2 + \frac{7}{2}x\right] + 1 = 0 \\ & 2\left[\left(x + \frac{7}{4}\right)^2 - \frac{49}{16}\right] + 1 = 0 \\ & 2\left(x + \frac{7}{4}\right)^2 - \frac{49}{8} + \frac{8}{8} = 0 \\ & 2\left(x + \frac{7}{4}\right)^2 - \frac{41}{8} = 0 \\ & 2\left(x + \frac{7}{4}\right)^2 = \frac{41}{8} \\ & \left(x + \frac{7}{4}\right)^2 = \frac{41}{16} \\ & x + \frac{7}{4} = \pm\sqrt{\frac{41}{16}} \Rightarrow x = -\frac{7}{4} \pm \frac{\sqrt{41}}{4} \end{aligned}$$

[4 marks]

$$\begin{aligned} \text{(ii)} \quad & 5x^2 + 7x + 1 = 0 \\ & 5\left[x^2 + \frac{7}{5}x\right] + 1 = 0 \\ & 5\left[\left(x + \frac{7}{10}\right)^2 - \frac{49}{100}\right] + 1 = 0 \\ & 5\left(x + \frac{7}{10}\right)^2 - \frac{49}{20} + \frac{20}{20} = 0 \\ & 5\left(x + \frac{7}{10}\right)^2 - \frac{29}{20} = 0 \\ & 5\left(x + \frac{7}{10}\right)^2 = \frac{29}{20} \\ & \left(x + \frac{7}{10}\right)^2 = \frac{29}{100} \\ & x + \frac{7}{10} = \pm\sqrt{\frac{29}{100}} \Rightarrow x = -\frac{7}{10} \pm \frac{\sqrt{29}}{10} \end{aligned}$$

[4 marks]

(iii)

$$3x^2 + 5x + 1 = 0$$

$$3\left[x^2 + \frac{5}{3}x\right] + 1 = 0$$

$$3\left[\left(x + \frac{5}{6}\right)^2 - \frac{25}{36}\right] + 1 = 0$$

$$3\left(x + \frac{5}{6}\right)^2 - \frac{25}{12} + \frac{12}{12} = 0$$

$$3\left(x + \frac{5}{6}\right)^2 - \frac{13}{12} = 0$$

$$3\left(x + \frac{5}{6}\right)^2 = \frac{13}{12}$$

$$\left(x + \frac{5}{6}\right)^2 = \frac{13}{36}$$

$$x + \frac{5}{6} = \pm\sqrt{\frac{13}{36}}$$

$$x = -\frac{5}{6} \pm \frac{\sqrt{13}}{6}$$

[4 marks]

(iv)

$$10x^2 + 3x - 2 = 0$$

$$10\left[x^2 + \frac{3}{10}x\right] - 2 = 0$$

$$10\left[\left(x + \frac{3}{20}\right)^2 - \frac{9}{400}\right] - 2 = 0$$

$$10\left(x + \frac{3}{20}\right)^2 - \frac{9}{40} - \frac{80}{40} = 0$$

$$10\left(x + \frac{3}{20}\right)^2 - \frac{89}{40} = 0$$

$$10\left(x + \frac{3}{20}\right)^2 = \frac{89}{40}$$

$$\left(x + \frac{3}{20}\right)^2 = \frac{89}{400}$$

$$x + \frac{3}{20} = \pm\sqrt{\frac{89}{400}}$$

$$x = -\frac{3}{20} \pm \frac{\sqrt{89}}{20}$$

[4 marks]

Answer 2

$$\begin{aligned}
5x^2 + 4x - 2 &= 0 \\
5\left[x^2 + \frac{4}{5}x\right] - 2 &= 0 \\
5\left[\left(x + \frac{2}{5}\right)^2 - \frac{4}{25}\right] - 2 &= 0 \\
5\left(x + \frac{2}{5}\right)^2 - \frac{4}{5} - \frac{10}{5} &= 0 \\
5\left(x + \frac{2}{5}\right)^2 - \frac{14}{5} &= 0 \\
5\left(x + \frac{2}{5}\right)^2 &= \frac{14}{5} \\
\left(x + \frac{2}{5}\right)^2 &= \frac{14}{25} \\
x + \frac{2}{5} &= \pm\sqrt{\frac{14}{25}} \\
x &= -\frac{2}{5} \pm \frac{\sqrt{14}}{5}
\end{aligned}$$

[4 marks]**Answer 3**

$$\begin{aligned}
2x^2 - 12x + 17 &= 0 \\
2[x^2 - 6x] + 17 &= 0 \\
2[(x - 3)^2 - 9] + 17 &= 0 \\
2(x - 3)^2 - 18 + 17 &= 0 \\
2(x - 3)^2 - 1 &= 0 \\
2(x - 3)^2 &= 1 \\
(x - 3)^2 &= \frac{1}{2} \\
x - 3 &= \pm\sqrt{\frac{1}{2}} \\
x &= 3 \pm \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
x &= 3 \pm \frac{\sqrt{2}}{2}
\end{aligned}$$

[4 marks]

Answer 4

$$2x^2 + 2x - 1 = 0$$

$$2[x^2 + x] - 1 = 0$$

$$2\left[\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}\right] - 1 = 0$$

$$2\left(x + \frac{1}{2}\right)^2 - \frac{1}{2} - \frac{2}{2} = 0$$

$$2\left(x + \frac{1}{2}\right)^2 - \frac{3}{2} = 0$$

$$2\left(x + \frac{1}{2}\right)^2 = \frac{3}{2}$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{3}{4}$$

$$x + \frac{1}{2} = \pm\sqrt{\frac{3}{4}}$$

$$x = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}$$

[4 marks]**Answer 5**

$$9x^2 + 6x - 17 = 0$$

$$9\left[x^2 + \frac{2}{3}x\right] - 17 = 0$$

$$9\left[\left(x + \frac{1}{3}\right)^2 - \frac{1}{9}\right] - 17 = 0$$

$$9\left(x + \frac{1}{3}\right)^2 - 1 - 17 = 0$$

$$9\left(x + \frac{1}{3}\right)^2 - 18 = 0$$

$$9\left(x + \frac{1}{3}\right)^2 = 18$$

$$\left(x + \frac{1}{3}\right)^2 = 2$$

$$x + \frac{1}{3} = \pm\sqrt{2}$$

$$x = -\frac{1}{3} \pm \sqrt{2}$$

[4 marks]

Answer 6

$$9x^2 - 6x - 26 = 0$$

$$9\left[x^2 - \frac{2}{3}x\right] - 26 = 0$$

$$9\left[\left(x - \frac{1}{3}\right)^2 - \frac{1}{9}\right] - 26 = 0$$

$$9\left(x - \frac{1}{3}\right)^2 - 1 - 26 = 0$$

$$9\left(x - \frac{1}{3}\right)^2 - 27 = 0$$

$$9\left(x - \frac{1}{3}\right)^2 = 27$$

$$\left(x - \frac{1}{3}\right)^2 = 3$$

$$x - \frac{1}{3} = \pm\sqrt{3}$$

$$x = \frac{1}{3} \pm \sqrt{3}$$

[4 marks]