5.4 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics In Algebra

5.4.1 Solutions to 5.2 Example

(i)

$$x^2 + 14x + 40 = 0$$

$$(x + 4)(x + 10) = 0$$

Either $x + 4 = 0$ in which case $x = -4$

or $x + 10 = 0$ in which case $x = -10$

[2 marks]

(ii)

$$x^2 + 14x + 40 = 0$$

$$\left[x^2 + 14x \right] + 40 = 0$$

$$\left[(x + 7)^2 - 49 \right] + 40 = 0$$

$$(x + 7)^2 - 9 = 0$$

$$(x + 7)^2 = 9$$

$$x + 7 = \pm 3$$

$$x = -7 \pm 3$$

That is, $x = -10, -4$

[2 marks]

(iii) $x^2 + 14x + 40 = 0$

$$x = \frac{-14 \pm \sqrt{14^2 - 4 \times 1 \times 40}}{2}$$

$$= \frac{-14 \pm \sqrt{196 - 160}}{2}$$

$$= \frac{-14 \pm \sqrt{36}}{2}$$

$$= \frac{-14 \pm 6}{2}$$

$$= -10, -4$$

[2 marks]

Notice:
On the graph observe the x-axis crossing points are at $(-10, 0)$ or $(-4, 0)$
Also, from the rewrite of the curve's equation

$$y = x^2 + 14x + 40$$

as

$$y = (x + 7)^2 - 9$$

it can be noticed that the minimum value is at $(-7, -9)$

5.4.2 Solutions to 5.3 Exercise

Answer 1

(i)

$$9x^2 - 16x + 7 = 0$$

$$(9x - 7)(x - 1) = 0$$

Either $9x - 7 = 0$ in which case $x = \frac{7}{9}$

or $x - 1 = 0$ in which case $x = 1$

[3 marks]

(ii)

$$14x^2 + 11x + 2 = 0$$

$$(7x + 2)(2x + 1) = 0$$

Either $7x + 2 = 0$ in which case $x = -\frac{2}{7}$

or $2x + 1 = 0$ in which case $x = -\frac{1}{2}$

[3 marks]

Answer 2

$$x^2 - 6x + 1 = 0$$

$$\left[x^2 - 6x\right] + 1 = 0$$

$$\left[(x - 3)^2 - 9\right] + 1 = 0$$

$$(x - 3)^2 - 8 = 0$$

$$(x - 3)^2 = 8$$

$$x - 3 = \pm 2\sqrt{2}$$

$$x = 3 \pm 2\sqrt{2}$$

[3 marks]

Answer 3

The roots occur when, $7x^2 - 3x - 2 = 0$

$$x = \frac{3 \pm \sqrt{3^2 - 4 \times 7 \times (-2)}}{2 \times 7}$$

$$= \frac{3 \pm \sqrt{65}}{14}$$

[3 marks]

Answer 4
Simon has found that the formula yields an expression containing the square root of a negative number which he calls "going wrong".

That is $x = \dfrac{-1 \pm \sqrt{-3}}{2}$

This means there are zero real solutions, or roots, to this equation.
The number of roots corresponds to the number of x-axis crossing points should the graph of the given quadratic equation be plotted.

So the maths is telling Simon that the graph of $y = x^2 + x + 1$ does not cross the x-axis.

[4 marks]

Answer 5

(i)	$D = 36$ which is +ve	$\therefore$ 2 solutions
(ii)	$D = -4$ which is –ve	$\therefore$ 0 solutions
(iii)	$D = 41$ which is +ve	$\therefore$ 2 solutions
(iv)	$D = 0$	$\therefore$ 1 solutions
(v)	$D = -8$ which is –ve	$\therefore$ 0 solutions
(vi)	$D = 400$ which is +ve	$\therefore$ 2 solutions

[6 marks]

Answer 6
For equal roots the discriminant must be zero.

$$D = 0$$

$$(3p)^2 - 4 \times 1 \times p = 0$$

$$9p^2 - 4p = 0$$

As told p is not zero, divide by p

$$9p - 4 = 0$$

$$p = \frac{4}{9}$$

[4 marks]

Answer 7

$$(p-2)x^2 + 8x + (p+4) = 0$$

For no real roots the discriminant of this equation must be less than zero;

$$8^2 - 4 \times (p-2) \times (p+4) < 0$$

$$64 - 4\left(p^2 + 2p - 8\right) < 0$$

$$64 - 4p^2 - 8p + 32 < 0$$

Multiply through by – 1 and reverse the inequality

$$4p^2 + 8p - 96 > 0$$

Divide through by 4

$$p^2 + 2p - 24 > 0 \quad \square$$

[3 marks]

Answer 8

$$k\left(3x^2 + 8x + 9\right) = 2 - 6x$$

$$3kx^2 + (8k+6)x + (9k-2) = 0$$

For no real roots the discriminant of this equation must be less than zero;

$$(8k+6)^2 - 4 \times 3k \times (9k-2) < 0$$

$$64k^2 + 96k + 36 - 108k^2 + 24k < 0$$

$$-44k^2 + 120k + 36 < 0$$

Multiply through by – 1 and reverse the inequality

$$44k^2 - 120k - 36 > 0$$

Divide through by 4

$$11k^2 - 30k - 9 > 0 \quad \square$$

[4 marks]

Answer 9

The line and the curve would have intersected when,

$$2px^2 - 6px + 4p = 3x - 7$$

$$2px^2 - (6p+3)x + (4p+7) = 0$$

For the line and the curve not to intersect, this must have no solutions.
In other words, its discriminant must be less than zero.

$$D < 0$$

$$(-1)^2(6p + 3)^2 - 4 \times 2p \times (4p+7) < 0$$

$$36p^2 + 36p + 9 - 32p^2 - 56p < 0$$

$$4p^2 - 20p + 9 < 0 \quad \square$$

[4 marks]

Answer 10

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

$$x^2 + (kx)^2 - 6x + 10(kx) + 9 = 0$$

$$\left(1 + k^2\right)x^2 + (10k - 6)x + 9 = 0$$

To have a single point of intersection there must be only one solution to this.

$\therefore$ The discriminant must be equal to zero.

$$(10k - 6)^2 - 4 \times \left(1 + k^2\right) \times 9 = 0$$

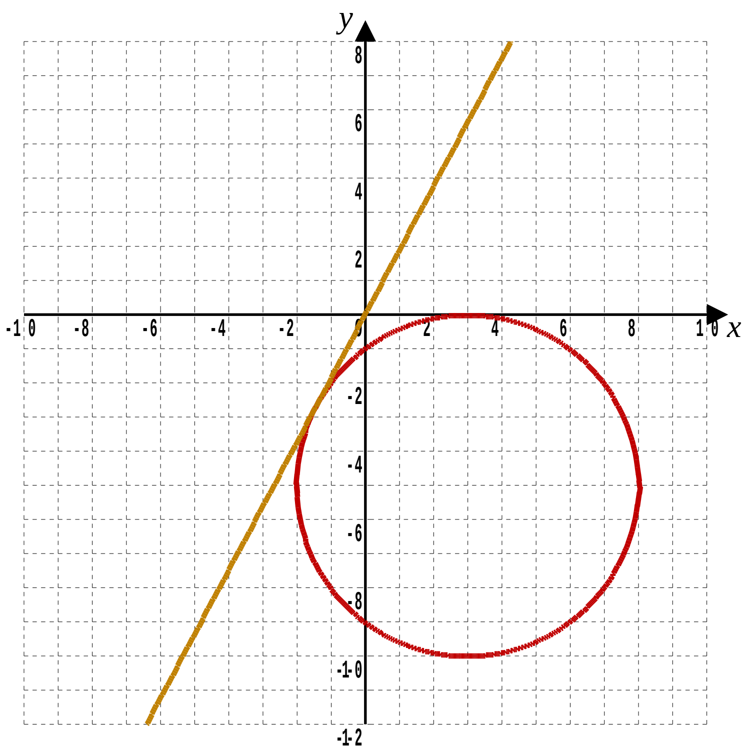
$$100k^2 - 120k + 36 - 36 - 36k^2 = 0$$

$$64k^2 - 120k = 0$$

As $k \neq 0$, divide through by k

$$64k - 120 = 0$$

$$\therefore k = \frac{15}{8}$$



The graph shows the circle plotted in red and the line $y = \frac{15}{8}x$ plotted in gold.

They intersect at a single point