6.3 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics In Algebra

6.3.1 Solutions to 6.2 Exercise

Answer 1

(i)	$D = 28$ which is + ve	$\therefore$ 2 distinct real roots
(ii)	$D = -28$ which is – ve	$\therefore$ 0 real roots
(iii)	$D = 0$	$\therefore$ 1 real root
(iv)	$D = -3$ which is – ve	$\therefore$ 0 real roots
(v)	$D = -11$ which is – ve	$\therefore$ 0 real roots
(vi)	$D = 0$	$\therefore$ 1 real root

[6 marks]

Answer 2

$$\text{No real roots implies that } D < 0$$
$$(-3)^2 - 4 \times 1 \times (-k) < 0$$
$$9 + 4k < 0$$
$$4k < -9$$
$$k < -\frac{9}{4}$$
$$k < -2.25$$

[3 marks]

Answer 3

(i)

$$\text{Two distinct real roots implies that } D > 0$$
$$k^2 - 4 \times 1 \times 4 > 0$$
$$k^2 - 16 > 0$$
$$k^2 > 16$$

[2 marks]

(ii)

$k = 5 \Rightarrow k^2 = 25$ for $x^2 + 5x + 4 = 0$ so, *y*es, this has 2 real roots

$k = 3 \Rightarrow k^2 = 9$ for $x^2 + 3x + 4 = 0$ so, no, this does not has 2 real roots

$k = -1 \Rightarrow k^2 = 1$ for $x^2 - x + 4 = 0$ so, no, this does not have 2 real roots

$k = -7 \Rightarrow k^2 = 49$ for $x^2 - 7x + 4 = 0$ so, yes, this has 2 roots

[2 marks]

(iii) $x = -3 \pm \sqrt{5}$

(Must be obtained by completing the square)

[2 marks]

Answer 4

A repeated root implies that, $D = 0$

$$(2k + 10)^2 - 4 \times 1 \times \left(k^2 + 5\right) = 0$$

$$4k^2 + 40k + 100 - 4k^2 - 20 = 0$$

$$40k + 80 = 0$$

$$k = -2$$

[3 marks]

Answer 5

A repeated root implies that, $D = 0$

$$144 - 4k^2 = 0$$

$$k^2 = 36$$

$$k = \pm 6$$

$k = 6$ as the question states that k is positive

[4 marks]

Answer 6

(a) $x^2 + 2px + (3p + 4) = 0$

Equal roots implies that, $D = 0$

$$(2p)^2 - 4 \times 1 \times (3p + 4) = 0$$

$$4p^2 - 12p - 16 = 0$$

Divide through by 4

$$p^2 - 3p - 4 = 0$$

$$(p - 4)(p - 1) = 0$$

Either $p = 4$ or $p = -1$ (Rejected as p is a positive constant)

$$\therefore \; p = 4$$

[4 marks]

(b) The equation becomes, $x^2 + 8x + 16 = 0$

$$(x + 4)^2 = 0$$

$$x = -4$$

[2 marks]

Answer 7

Obviously, $(x + 4)^2 \geqslant 0$ as any real number when squared is non-negative.

Even when $(x + 4)^2$ has its least value of zero, 1 is added on.

$\therefore \; (x + 4)^2 + 1 > 0$ That is, is always positive. ☐

[2 marks]

Answer 8

$$x^2 + 2x + 5 = \left[x^2 + 2x \right] + 5$$
$$= \left[(x + 1)^2 - 1 \right] + 5$$
$$= (x + 1)^2 + 4$$

As $(x + 1)^2$ is always greater than or equal to zero

it follows that $(x + 1)^2 + 4$ is always positive

$\therefore \; x^2 + 2x + 5$ is always positive □

[4 marks]

Answer 9

(i)

$$D = (k + 5)^2 - 4 \times 1 \times 4k$$
$$= k^2 + 10k + 25 - 16k$$
$$= k^2 - 6k + 25$$
$$= (k - 3)^2 - 9 + 25$$
$$= (k - 3)^2 + 16$$

That is $a = 3$ and $b = 16$

[3 marks]

(ii) Obviously, $(k - 3)^2 \geqslant 0$ as any real number squared is non-negative.

Even when $(k - 3)^2$ has its least value of zero, 16 is added on.

$\therefore \; (x - 3)^2 + 16 > 0$ That is, is always positive.

$\therefore \; f(x) = 0$ will always have two distinct real roots. □

[2 marks]

Answer 10

The discriminant, $D = 169$.

As this is positive, the equation will have 2 distinct real roots.

[3 marks]

Answer 11

Having two repeated roots implies that, $D = 0$

$$k^2 - 4 \times (k + 1)(k + 1) = 0$$

$$k^2 - 4\left(k^2 + 2k + 1\right) = 0$$

$$k^2 - 4k^2 - 8k - 4 = 0$$

$$-3k^2 - 8k - 4 = 0$$

$$3k^2 + 8k + 4 = 0$$

$$(3k + 2)(k + 2) = 0$$

$$\therefore \ k = -\frac{2}{3} \quad \text{or} \quad k = -2$$

[4 marks]