

8.3 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics in Algebra

8.3.1 Solutions to 8.1 “Together” Examples

(i) $5x + 2 \leq 9$
 $5x \leq 7$
 $x \leq \frac{7}{5}$

(ii) $7 - 3x \geq 13$
 $-3x \geq 6$
 $x \leq -2$

[1, 2 marks]

(iii) $-2 < 4x + 3 \leq 9$
 $-5 < 4x \leq 6$
 $-\frac{5}{4} < x \leq \frac{6}{4}$

(iv) $\frac{4}{x} < \frac{5}{9}$
 $\frac{x}{4} > \frac{9}{5}$
 $x > \frac{36}{5}$

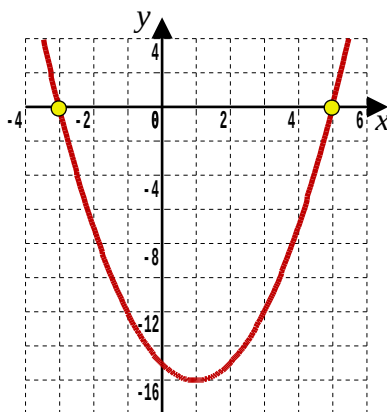
[2, 2 marks]

(v) $x^2 - 2x - 15 < 0$

Finding the critical values...

$$(x - 5)(x + 3) = 0$$

$$x = -3 \text{ or } x = 5$$



From the graph, the solution to the inequality $x^2 - 2x - 15 < 0$ is $-3 < x < 5$

EXTRA :

Had the inequality been $x^2 - 2x - 15 > 0$ the solutions, from the same graph, would have been $x < -3$ or $x > 5$

[3 marks]

8.3.2 Solutions to 8.2 Exercise

Answer 1

(i) $7x - 3 > 25$

$$7x > 28$$

$$x > 4$$

(ii) $13 - 4x < 11$

$$-4x < -2$$

$$x > 0.5$$

[1, 2 marks]

(iii) $3x - 5 > 19 - 5x$

$$8x > 24$$

$$x > 3$$

(iv) $3(2x + 3) - 4(x - 2) < 11$

$$6x + 9 - 4x + 8 < 11$$

$$2x < -6$$

$$x < -3$$

[2, 2 marks]

(v) $17 + 7(x - 6) > 45$

$$17 + 7x - 42 > 45$$

$$7x > 70$$

$$x > 10$$

(vi) $13 < 5x + 3 < 21$

$$10 < 5x < 18$$

$$2 < x < \frac{18}{5}$$

[2, 2 marks]

(vii) $-3 < 5 - 4x \leq 25$

$$-8 < -4x \leq 20$$

$$-2 < -x \leq 5$$

$$2 > x \geq -5$$

$$-5 \leq x < 2$$

(viii) $\frac{10}{x} < \frac{5}{12}$

$$\frac{x}{10} > \frac{12}{5}$$

$$x > 24$$

[2, 2 marks]

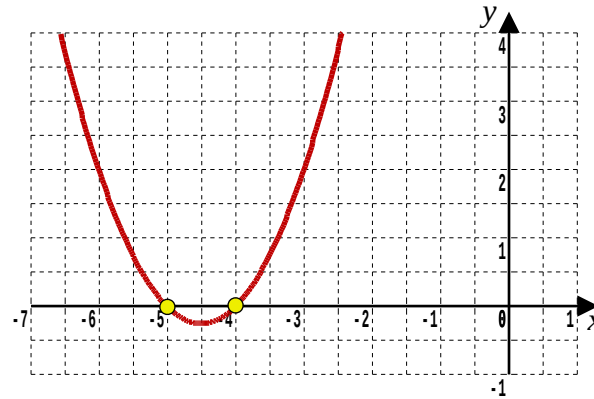
Answer 2

(i) $x^2 + 9x + 20 \leq 0$

Finding the critical values...

$$(x + 5)(x - 4) = 0$$

$$x = -5 \text{ or } x = -4$$



From the graph, the solution to the inequality $x^2 + 9x + 20 < 0$

is $-5 \leq x \leq -4$

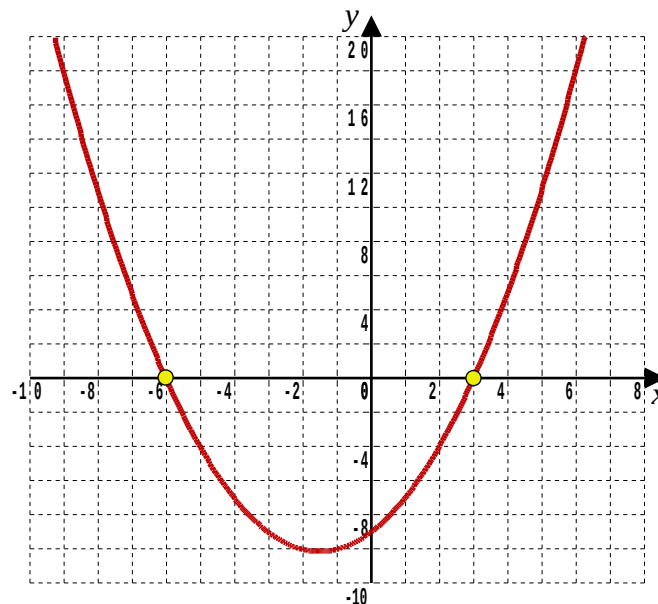
[3 marks]

(ii) $x^2 + 3x - 18 < 0$

Finding the critical values...

$$(x + 6)(x - 3) = 0$$

$$x = -6 \text{ or } x = 3$$



From the graph, the solution to the inequality $x^2 + 3x - 18 < 0$

is $-6 < x < 3$

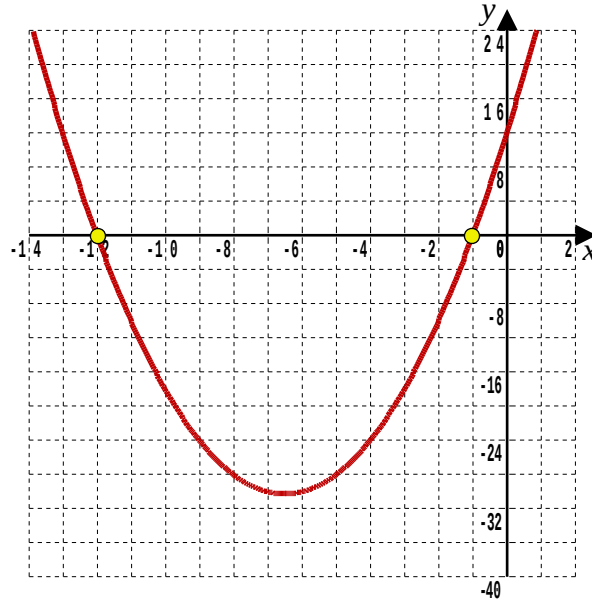
[3 marks]

(iii) $x^2 + 13x + 12 > 0$

Finding the critical values...

$$(x + 12)(x + 1) = 0$$

$$x = -12 \text{ or } x = -1$$



From the graph, the solution to the inequality $x^2 + 13x + 12 > 0$

is $x < -12$ or $x > -1$

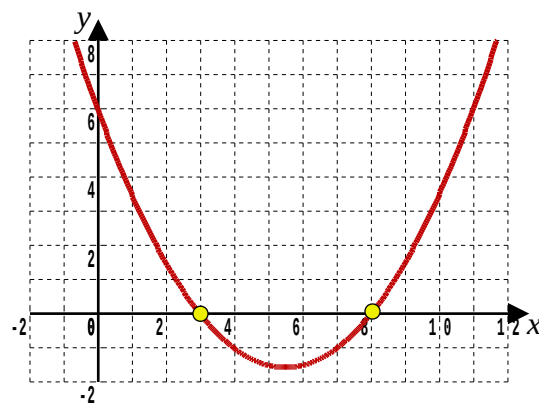
[3 marks]

(iv) $x^2 - 11x + 24 \geq 0$

Finding the critical values...

$$(x - 3)(x - 8) = 0$$

$$x = 3 \text{ or } x = 8$$



From the graph, the solution to the inequality $x^2 - 11x + 24 \geq 0$

is $x \leq 3$ or $x \geq 8$

[3 marks]

Answer 3

(i) $x^2 + 6x + k = 0$

$$D > 0$$

$$36 - 4 \times 1 \times k > 0$$

$$- 4k > -36$$

$$k < 9$$

(ii) $kx^2 + 4x + 2 = 0$

$$D > 0$$

$$16 - 4 \times k \times 2 > 0$$

$$- 8k > -16$$

$$k < 2$$

[4, 4 marks]

(iii) $x^2 - 2x + k = 0$

$$D > 0$$

$$4 - 4 \times 1 \times k > 0$$

$$- 4k > -4$$

$$k < 1$$

(iv) $kx^2 - 6x + 3 = 0$

$$D > 0$$

$$36 - 4 \times k \times 3 > 0$$

$$- 12k > -36$$

$$k < 3$$

[4, 4 marks]**Answer 4**

(i) $kx^2 - 2x + k = 0$

Two distinct real roots implies that $D > 0$

$$\therefore 4 - 4 \times k \times k > 0$$

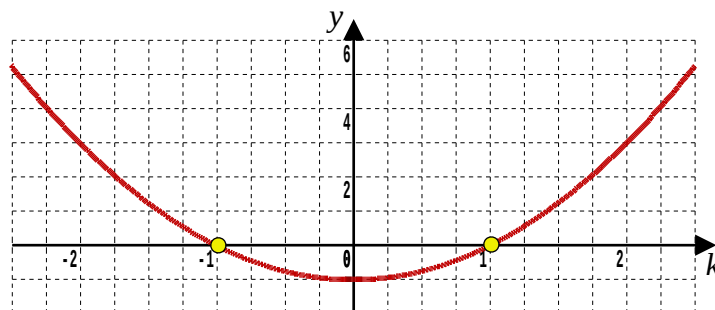
$$4 - 4k^2 > 0$$

$$4k^2 - 4 < 0$$

divide through by 4

$$k^2 - 1 < 0$$

$$(k - 1)(k + 1) < 0 \quad \square$$

[4 marks]**(ii)** Let $y = (k - 1)(k + 1)$ in which case, for $y < 0$, the critical values are $k = -1$ or $k = 1$ and the graph is,From the graph, the solution to the inequality $(k - 1)(k + 1) < 0$ is $-1 < k < 1$ **[4 marks]**

Answer 5

$$\begin{aligned}
 \text{(i)} \quad 2x^2 - 28x + 87 &= 2[x^2 - 14x] + 87 \\
 &= 2[(x - 7)^2 - 49] + 87 \\
 &= 2(x - 7)^2 - 98 + 87 \\
 &= 2(x - 7)^2 - 11 \\
 \therefore a &= 2, b = -7 \text{ and } c = -11
 \end{aligned}$$

[3 marks]

$$\begin{aligned}
 2x^2 - 28x + 87 &= 0 \\
 \therefore 2(x - 7)^2 - 11 &= 0 \quad \text{from part (i)} \\
 2(x - 7)^2 &= 11 \\
 (x - 7)^2 &= \frac{11}{2}
 \end{aligned}$$

square root both sides

$$\begin{aligned}
 x - 7 &= \pm \frac{\sqrt{11}}{\sqrt{2}} \\
 &= 7 \pm \frac{\sqrt{11}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
 &= 7 \pm \frac{1}{2} \sqrt{22}
 \end{aligned}$$

$$\text{That is, } c = 7 \text{ and } d = \frac{1}{2}$$

[3 marks]