9.5 Answers

Additional Mathematics
A-Level Pure Mathematics : Year 1
Topics in Algebra

Answer 1

Starting with the equations,

$$y = x - 4$$
$$x^2 + y^2 = 58$$

$$x^2 + (x - 4)^2 = 58$$
$$x^2 + x^2 - 8x + 16 = 58$$
$$2x^2 - 8x - 42 = 0$$

divide through by 2

$$x^2 - 4x - 21 = 0$$
$$(x + 3)(x - 7) = 0$$

Either $x = -3$ or $x = 7$

Using $y = x - 4$ then gives the solutions as,

$$(-3, -7) \text{ or } (7, 3)$$

[5 marks]

Answer 2

Starting with the equations,

$$y = 3x - 1$$
$$x^2 + y^2 = 5$$

$$x^2 + (3x - 1)^2 = 5$$
$$x^2 + 9x^2 - 6x + 1 = 5$$
$$10x^2 - 6x - 4 = 0$$

divide through by 2

$$5x^2 - 3x - 2 = 0$$
$$(5x + 2)(x - 1) = 0$$

Either $x = -\frac{2}{5}$ or $x = 1$

Using $y = 3x - 1$ then gives the solutions as,

$$\left(-\frac{2}{5}, -\frac{11}{5}\right) \text{ or } (1, 2)$$

[5 marks]

Answer 3

METHOD 1

Starting with the equations,

$$x + y = 2$$
$$4y^2 - x^2 = 11$$

$$4y^2 - \left[(2 - y)^2 \right] = 11$$
$$4y^2 - \left[4 - 4y + y^2 \right] = 11$$
$$4y^2 - 4 + 4y - y^2 = 11$$
$$3y^2 + 4y - 15 = 0$$
$$(3y - 5)(y + 3) = 0$$
$$\text{Either } y = \frac{5}{3} \text{ or } y = -3$$

Using $x = 2 - y$ then gives the solutions as,

$$\left(\frac{1}{3}, \frac{5}{3} \right) \text{ or } (5, -3)$$

[5 marks]

METHOD 2

Starting with the equations,

$$x + y = 2$$
$$4y^2 - x^2 = 11$$

$$4\left[(2 - x)^2 \right] - x^2 = 11$$
$$4\left[4 - 4x + x^2 \right] - x^2 = 11$$
$$16 - 16x + 4x^2 - x^2 = 11$$
$$3x^2 - 16x + 5 = 0$$
$$(3x - 1)(x - 5) = 0$$
$$\text{Either } x = \frac{1}{3} \text{ or } x = 5$$

Using $y = 2 - x$ then gives the solutions as,

$$\left(\frac{1}{3}, \frac{5}{3} \right) \text{ or } (5, -3)$$

[5 marks]

Answer 4

Starting with the equations,

$$y = x - 7$$

$$x^2 + y^2 = 109$$

$$x^2 + (x - 7)^2 = 109$$

$$x^2 + x^2 - 14x + 49 = 109$$

$$2x^2 - 14x - 60 = 0$$

divide through by 2

$$x^2 - 7x - 30 = 0$$

$$(x + 3)(x - 10) = 0$$

Either $x = -3$ or $x = 10$

Using $y = x - 7$ then gives the solutions as,

$$(-3, -10) \text{ or } (10, 3)$$

[5 marks]

Answer 5

Starting with the equations,

$$y - 3x + 2 = 0 \Leftrightarrow y = 3x - 2$$

$$y^2 - x - 6x^2 = 0$$

$$(3x - 2)^2 - x - 6x^2 = 0$$

$$9x^2 - 12x + 4 - x - 6x^2 = 0$$

$$3x^2 - 13x + 4 = 0$$

$$(3x - 1)(x - 4) = 0$$

Either $x = \frac{1}{3}$ or $x = 4$

Using $y = 3x - 2$ then gives the solutions as,

$$\left(\frac{1}{3}, -1\right) \text{ or } (4, 10)$$

[5 marks]

Answer 6

Starting with the equations,

$$y = 2x - 3$$
$$x^2 + y^2 = 18$$

$$x^2 + (2x - 3)^2 = 18$$
$$x^2 + 4x^2 - 12x + 9 = 18$$
$$5x^2 - 12x - 9 = 0$$
$$(5x + 3)(x - 3) = 0$$

Either $x = -\frac{3}{5}$ or $x = 3$

Using $y = 2x - 3$ then gives the solutions as,

$$\left(-\frac{3}{5}, -\frac{21}{5}\right) \text{ or } (3, 3)$$

[5 marks]