## 10.4 Answers



### 10.4.1 Solutions to 10.1 Example (First Teaching Video)

**( i )**

$$x = e^y \cos y$$

$$\frac{dx}{dy} = e^y(-\sin y) + e^y \cos y \qquad \text{By The Product Rule}$$

$$\frac{dx}{dy} = e^y(\cos y - \sin y)$$

$$\frac{dy}{dx} = \frac{1}{e^y(\cos y - \sin y)}$$

**( ii )** When $y = 0$

$x = e^0 \cos 0 = 1$ so $(1, 0)$ is a point on the normal

Also, when $y = 0$

$$\frac{dy}{dx} = \frac{1}{e^0(\cos 0 - \sin 0)} = 1$$

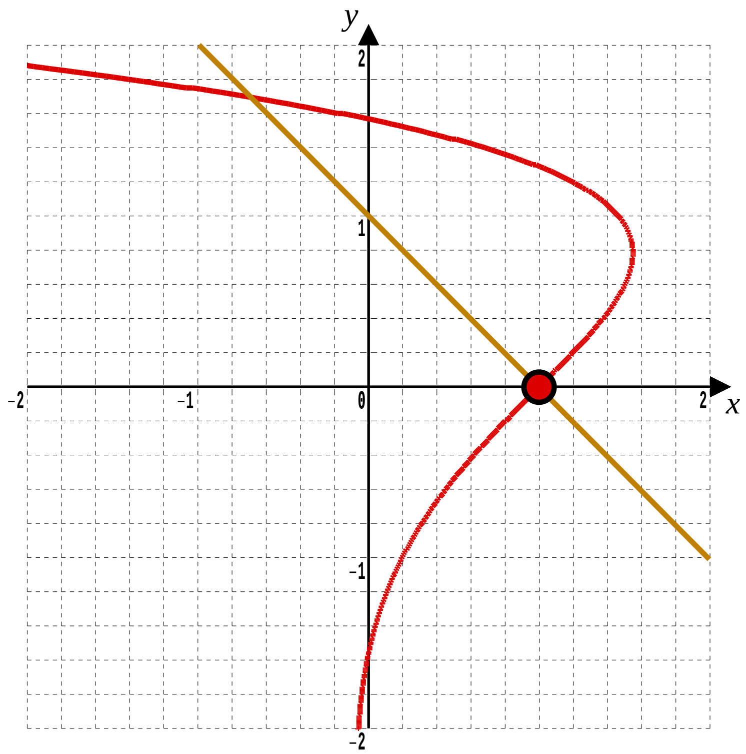
$\therefore$ Gradient of normal will be the "sign changed reciprocal" of $(-1)$

$$y = mx + c$$

$$0 = -1 \times 1 + c \qquad \Rightarrow c = 1$$

Normal has equation $y = -x + 1$ ( That is, $y = 1 - x$ )

**( iii )**




**[2, 2, 2 marks ]**

### 10.4.2 Proof for 10.2 Differentiating *arcsin x*

**Reciprocal Gradients Rule**

$$\frac{dy}{dx} \times \frac{dx}{dy} = 1 \qquad \Leftrightarrow \qquad \frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$$

$$y = arcsin\, x$$

$$\therefore x = sin\, y$$

$$\frac{dx}{dy} = cos\, y$$

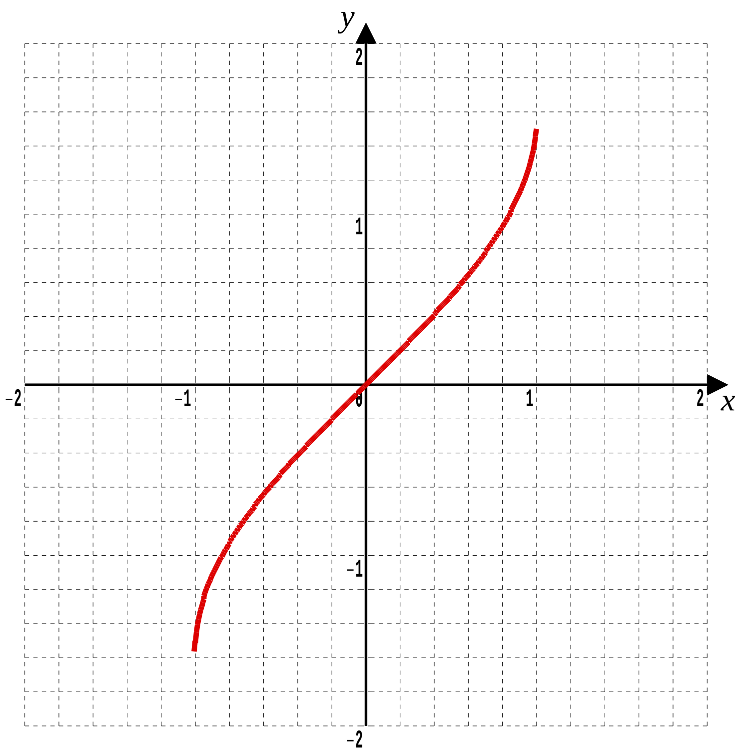
Now use the fact that $cos^2 y + sin^2 y = 1$ to get...

$$\frac{dx}{dy} = \pm\sqrt{1 - sin^2 y}$$

$$\frac{dy}{dx} = \pm\frac{1}{\sqrt{1 - x^2}}$$

From the graph of $y = arcsin\, x$ we can see that the gradient is always positive

$$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}} \qquad \square$$




**[ 6 marks ]**

### 10.4.3 Solutions to 10.3 Exercise

**Answer 1**

**( i )**

$$y = \frac{sin\left(x^2\right)}{x}$$

$$\frac{dy}{dx} = \frac{x\,cos\left(x^2\right)2x - sin\left(x^2\right)}{x^2} \qquad \text{By The Quotient Rule}$$

$$\frac{dy}{dx} = \frac{2\,x^2\,cos\left(x^2\right) - sin\left(x^2\right)}{x^2}$$

**[ 3 marks ]**

**( ii )** When $x = \sqrt{\pi}$

$$y = \frac{sin\ (\pi)}{\sqrt{\pi}} = 0$$ so $\left(\sqrt{\pi},\ 0\right)$ is a point on the tangent

Also, when $x = \sqrt{\pi}$

$$\frac{dy}{dx} = \frac{2\,\pi\,cos\,(\pi) - sin\,(\pi)}{\pi} = -2$$

$\therefore$ Gradient of tangent will be $-2$

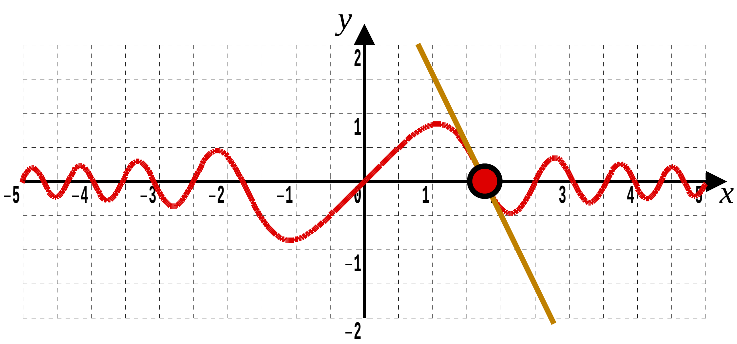
$$y = mx + c$$

$$0 = -2 \times \sqrt{\pi} + c \qquad \Rightarrow\ c = 2\sqrt{\pi}$$

Tangent has equation $y = -2x + 2\sqrt{\pi}$

**[ 5 marks ]**

**( iii )**




**[ 1 mark ]**

**Answer 2**

$$y = arccos\,x$$

$$\therefore x = cos\,y$$

$$\frac{dx}{dy} = -sin\,y$$

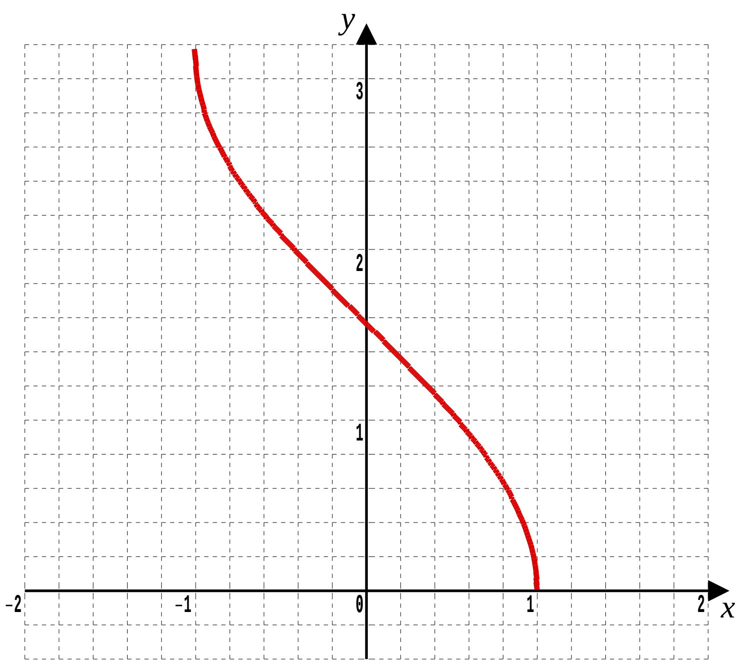
Now use the trigonometry identity $cos^2 y + sin^2 y = 1$ to get...

$$\frac{dx}{dy} = \pm\sqrt{1 - cos^2 y}$$

$$\frac{dy}{dx} = \pm\frac{1}{\sqrt{1 - x^2}}$$

From the graph of $y = arccos\,x$ it can be seen that the gradient is always negative

$$\therefore \frac{dy}{dx} = -\frac{1}{\sqrt{1 - x^2}} \qquad \square$$




**[ 7 marks ]**

**Answer 3**

**( i )**

$$y = x^2 \sin x$$

$$\frac{dy}{dx} = x^2 \cos x + 2x \sin x \qquad \text{by The Product Rule}$$

**[ 3 marks ]**

**( ii )** When $x = \pi$

$$y = \pi^2 \sin \pi = 0 \quad \text{so} \quad (\pi, 0) \text{ is a point on the normal}$$

Also, when $x = \pi$

$$\frac{dy}{dx} = \pi^2 \cos \pi + 2\pi \sin \pi = -\pi^2$$

$\therefore$ Gradient of normal will be the "sign changed reciprocal", $\dfrac{1}{\pi^2}$

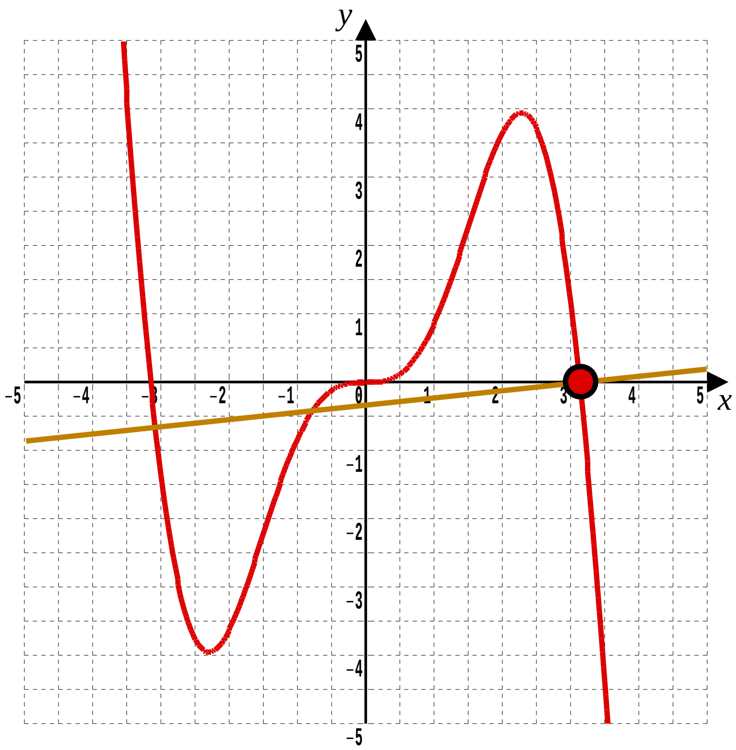
$$y = mx + c$$

$$0 = \frac{1}{\pi^2} \times \pi + c \qquad \Rightarrow c = -\frac{1}{\pi}$$

Normal has equation $y = \dfrac{x}{\pi^2} - \dfrac{1}{\pi}$

**[ 6 marks ]**

**( iii )**




**[ 1 mark ]**

**Answer 4**

**( i )**

$$y = \tan x$$

$$y = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{\cos x \ \cos x \ - \ (-\sin x)\ \sin x}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x \qquad \square$$

**[ 2 marks ]**

**( ii )**

$$y = \arctan x$$

$$\therefore x = \tan y$$

$$\frac{dx}{dy} = \sec^2 y$$

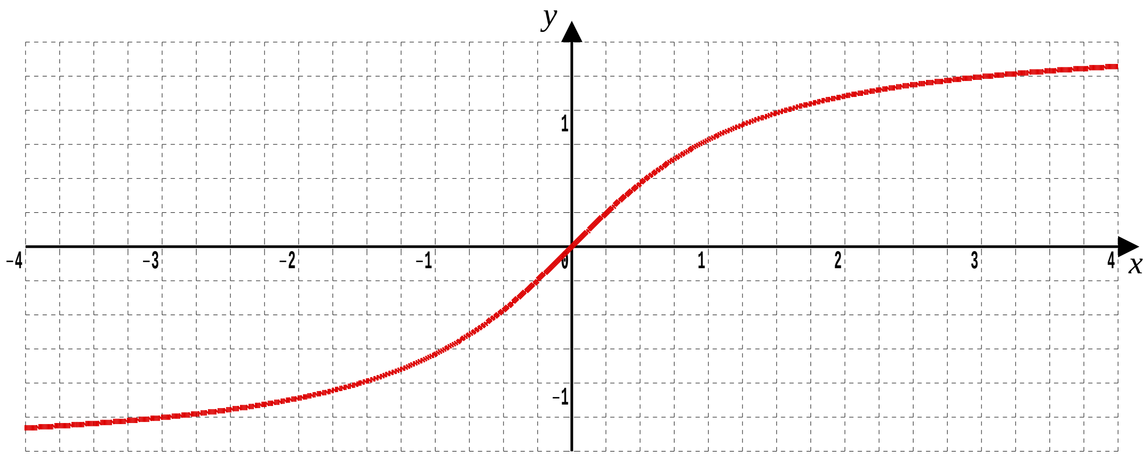
Now use the fact that $1 + \tan^2 y = \sec^2 y$ to get...

$$\frac{dx}{dy} = 1 + \tan^2 y$$

$$\frac{dx}{dy} = 1 + x^2$$

$$\frac{dy}{dx} = \frac{1}{1 + x^2} \qquad \square$$

**[ 6 marks ]**



**Answer 5**

$$y = \cos^2 x + \sin x$$

$$\frac{dy}{dx} = 2\cos x\ (-\sin x) + \cos x$$

$$= \cos x\ (1 - 2\sin x)$$

For stationary points...

Either $\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ (maxima)

or $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$ (minima)

**[ 6 marks ]**