11.2 Answers

A-Level Pure Mathematics : Year 2
Differentiation III

Answer 1

(i) $y = 7x^4 \quad \Rightarrow \quad \frac{dy}{dx} = 28x^3$

(ii) $y = 11\sqrt{x}$

$= 11x^{\frac{1}{2}} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{11}{2}x^{-\frac{1}{2}}$

$= \frac{11}{2\sqrt{x}}$

[2 marks]

Answer 2

(i) $y = (x + 6)(2x - 5)$

$= 2x^2 + 7x - 30 \quad \Rightarrow \quad \frac{dy}{dx} = 4x + 7$

(ii) $y = \sqrt{x}\left(\frac{1}{\sqrt{x}} + 3\sqrt{x}\right)$

$= 1 + 3x \quad \Rightarrow \quad \frac{dy}{dx} = 3$

[4 marks]

Answer 3

(i) $y = 4(x^3 + 3)^5 \quad \Rightarrow \quad \frac{dy}{dx} = 20(x^3 + 3)^4(3x^2)$

$= 60x^2(x^3 + 3)^4$

(ii) $y = 4\cos(2x) \quad \Rightarrow \quad \frac{dy}{dx} = -8\sin(2x)$

(iii) $y = \sec^3 x$

$= (\sec x)^3 \quad \Rightarrow \quad \frac{dy}{dx} = 3(\sec x)^2 \sec x \tan x$

$= 3\sec^3 x \tan x$

(iv) $y = e^{\sin x} + e^{\cos x} \quad \Rightarrow \quad \frac{dy}{dx} = e^{\sin x}\cos x + e^{\cos x}(-\sin x)$

$= e^{\sin x}\cos x - e^{\cos x}\sin x$

[8 marks]

Answer 4

(i)

$$y = x \ln x$$

$$\frac{dy}{dx} = x \times \frac{1}{x} + 1 \times \ln x$$

$$= 1 + \ln x$$

[2 marks]

(ii)

$$\frac{d^2y}{dx^2} = \frac{1}{x}$$

[1 marks]

Answer 5

$$f(x) = \tan(3x)$$

$$f'(x) = 3\sec^2(3x)$$

$$f'\left(\frac{\pi}{18}\right) = 3\sec^2\left(\frac{\pi}{6}\right)$$

$$= 3 \times \frac{1}{\left(\cos\left(\frac{\pi}{6}\right)\right)^2}$$

$$= 3 \times \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 3 \times \frac{4}{3}$$

$$= 4$$

[4 marks]

Answer 6

$$y = \frac{x^3 - 1}{x^3 + 1}$$

$$\frac{dy}{dx} = \frac{(x^3 + 1)(3x^2) - (3x^2)(x^3 - 1)}{(x^3 + 1)^2}$$

$$= \frac{(3x^2)\{x^3 + 1 - x^3 + 1\}}{(x^3 + 1)^2}$$

$$= \frac{(3x^2)\{2\}}{(x^3 + 1)^2}$$

$$= \frac{6x^2}{(x^3 + 1)^2} \qquad \square$$

[4 marks]

Answer 7

$$y = \frac{11}{x^2 - 3}$$

$$= 11\left(x^2 - 3\right)^{-1}$$

$$\frac{dy}{dx} = -11\left(x^2 - 3\right)^{-2} \times 2x$$

$$= -\frac{22x}{\left(x^2 - 3\right)^2}$$

When $x = 5$, $y = \frac{11}{5^2 - 3} = \frac{11}{22} = \frac{1}{2}$

$\therefore$ Point on curve where we want the tangent is $(5,\ 0.5)$

When $x = 5$, $\frac{dy}{dx} = -\frac{22 \times 5}{\left(5^2 - 3\right)^2} = -\frac{22 \times 5}{22 \times 22} = -\frac{5}{22}$

For the tangent, $y = mx + c$

$$0.5 = -\frac{5}{22} \times 5 + c$$

$$c = \frac{1}{2} \times \frac{11}{11} + \frac{25}{22}$$

$$= \frac{36}{22}$$

Equation of the tangent, $y = -\frac{5}{22}x + \frac{36}{22}$

$$22y = -5x + 36$$

$$5x + 22y - 36 = 0$$

[6 marks]

Answer 8

(i)

$$f(x) = \frac{1}{9\pi}\left(x^2 \sin(2x)\right)$$

$$f'(x) = \frac{1}{9\pi}\left(2x^2 \cos(2x) + 2x \sin(2x)\right)$$

$$= \frac{2x}{9\pi}\left(x \cos(2x) + \sin(2x)\right)$$

(ii)

$$f'\left(\frac{\pi}{4}\right) = \frac{2\pi}{9\pi \times 4}\left(\frac{\pi}{4}\cos\left(\frac{\pi}{2}\right) + \sin\left(\frac{\pi}{2}\right)\right)$$

$$= \frac{1}{18}\left(\frac{\pi}{4} \times 0 + 1\right)$$

$$= \frac{1}{18} \qquad \square$$

[5 marks]

Answer 9

$$y = ln(ax + b)^{\frac{1}{2}}$$

$$= \frac{1}{2} ln(ax + b)$$

$$\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{ax + b} \times a$$

$$\therefore \frac{a}{2(ax + b)} = \frac{4}{ax + 1}$$

$$\therefore \frac{a}{2} = 4, \qquad b = 1$$

Answer : $a = 8,\ b = 1$

[4 marks]