

Lesson 12

A-Level Pure Mathematics : Year 2 Differentiation III

12.1 Test

Marks Available : 40

Table of Standard Derivatives

$f(x)$	$f'(x)$	In Formula Book ?
x^n	$n x^{n-1}$	No
e^x	e^x	No
$\ln x$	$\frac{1}{x}$	No
$\sin x$	$\cos x$	No
$\cos x$	$-\sin x$	No
$\tan x$	$\sec^2 x$	Yes
$\csc x$	$-\csc x \cot x$	Yes
$\sec x$	$\sec x \tan x$	Yes
$\cot x$	$-\csc^2 x$	Yes
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	Yes
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$	Yes
$\arctan x$	$\frac{1}{1+x^2}$	Yes

Question 1

Differentiate each of the following with respect to x ,

(i) $y = 13x^5$

(ii) $y = \frac{24}{x}$

[2 marks]

Question 2

By first expanding the brackets, differentiate the following with respect to x ,

(i) $y = (x + 7)(3x - 2)$

(ii) $y = \frac{1}{\sqrt{x}}(3\sqrt{x} + x)$

[4 marks]

Question 3

Use The Chain Rule to find the derivative of each of the following with respect to x ,

(i) $y = (5x^3 + 1)^4$

(ii) $y = \tan(5x)$

(iii) $y = \csc^4 x$

(iv) $y = \ln(2x) + e^{-3x}$

[8 marks]

Question 4

(i) Use The Product Rule to find $\frac{dy}{dx}$ if $y = x e^x$

[2 marks]

(ii) Find also $\frac{d^2y}{dx^2}$

[1 marks]

Question 5

Consider the function

$$f(x) = \cot(6x)$$

Determine the value of

$$f'\left(\frac{\pi}{18}\right)$$

[4 marks]

Question 6

Use The Quotient Rule to show that if $y = \frac{x^5 - 2}{x^5 + 2}$

$$\text{then } \frac{dy}{dx} = \frac{20x^4}{(x^5 + 2)^2}$$

[4 marks]

Question 7

Find the equation of the tangent to the curve $y = \frac{28}{x^2 - 2}$ when $x = 3$

Give the answer in the form $ax + by + c = 0$
where a , b and c are integers to be found.

[6 marks]

Question 8

The function $f(x)$ is given below

$$f(x) = \frac{x^2 \sin(3x)}{9\pi}$$

(i) Find $f'(x)$

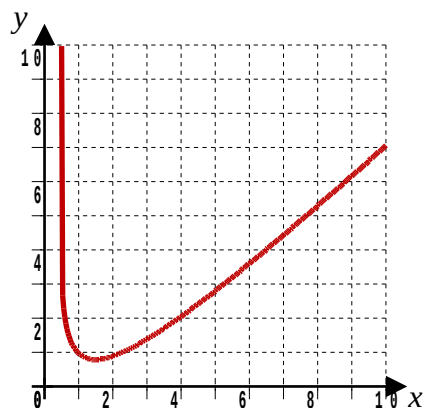
(ii) Show that

$$f' \left(\frac{\pi}{3} \right) = - \frac{\pi}{27}$$

[5 marks]

Question 9

$$f(x) = x - \ln(2x - 1)$$



Show that the exact coordinates of the turning point are $\left(\frac{3}{2}, \frac{3}{2} - \ln 2\right)$

[4 marks]