12.2 Answers

Answer 1

(i) $y = 13x^5 \quad \Rightarrow \quad \dfrac{dy}{dx} = 65x^4$

(ii) $y = \dfrac{24}{x}$

$= 24x^{-1} \quad \Rightarrow \quad \dfrac{dy}{dx} = -24x^{-2}$

$= -\dfrac{24}{x^2}$

[2 marks]

Answer 2

(i) $y = (x + 7)(3x - 2)$

$= 3x^2 + 19x - 14 \quad \Rightarrow \quad \dfrac{dy}{dx} = 6x + 19$

(ii) $y = \dfrac{1}{\sqrt{x}}\left(3\sqrt{x} + x\right)$

$= 3 + x^{\frac{1}{2}} \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$

$= \dfrac{1}{2\sqrt{x}}$

[4 marks]

Answer 3

(i) $y = \left(5x^3 + 1\right)^4 \quad \Rightarrow \quad \dfrac{dy}{dx} = 4\left(5x^3 + 1\right)^3\left(15x^2\right)$

$= 60x^2\left(5x^3 + 1\right)^3$

(ii) $y = tan\,(5x) \quad \Rightarrow \quad \dfrac{dy}{dx} = 5\,sec^2\,(5x)$

(iii) $y = csc^4\,x \quad \Rightarrow \quad \dfrac{dy}{dx} = 4\,csc^3\,x\,(-\,csc\,x\,cot\,x\,)$

$= -4\,csc^4\,x\,cot\,x$

$= -\dfrac{4\,cos\,x}{sin^5\,x}$

(iv) $y = ln\,(2x) + e^{-3x}$

$= ln\,2 + ln\,x + e^{-3x} \quad \Rightarrow \quad \dfrac{dy}{dx} = \dfrac{1}{x} - 3e^{-3x}$

[8 marks]

Answer 4

(i) $$y = x e^x \Rightarrow \frac{dy}{dx} = x e^x + e^x$$

[2 marks]

(ii) $$\frac{d^2y}{dx^2} = x e^x + e^x + e^x$$
$$= x e^x + 2 e^x$$

[1 marks]

Out of interest : $$\frac{d^n y}{dx^n} = e^x (x + n)$$

(Proof by induction) $$\frac{d^{n+1} y}{dx^{n+1}} = e^x + e^x (x + n) = e^x (x + n + 1)$$

Answer 5

$$f(x) = cot\,(6x)$$
$$f'(x) = -6\, csc^2\,(6x)$$
$$f'\left(\frac{\pi}{18} \right) = -6\, csc^2 \left(\frac{\pi}{3} \right)$$
$$= -6 \times \frac{1}{\left(sin\left(\frac{\pi}{3}\right)\right)^2}$$
$$= -6 \times \left(\frac{2}{\sqrt{3}} \right)^2$$
$$= -6 \times \frac{4}{3}$$
$$= -8$$

[4 marks]

Answer 6

$$y = \frac{x^5 - 2}{x^5 + 2}$$
$$\frac{dy}{dx} = \frac{(x^5 + 2)(5 x^4) - (5 x^4)(x^5 - 2)}{(x^5 + 2)^2}$$
$$= \frac{(5 x^4)\{ x^5 + 2 - x^5 + 2 \}}{(x^5 + 2)^2}$$
$$= \frac{(5 x^4)\{ 4 \}}{(x^5 + 2)^2}$$
$$= \frac{20\, x^4}{(x^5 + 2)^2} \qquad \square$$

[4 marks]

Answer 7

$$y = \frac{28}{x^2 - 2}$$

$$= 28\left(x^2 - 2\right)^{-1}$$

$$\frac{dy}{dx} = -28\left(x^2 - 2\right)^{-2} \times 2x$$

$$= -\frac{56x}{(x^2 - 2)^2}$$

When $x = 3, \quad y = \dfrac{28}{3^2 - 2} = \dfrac{28}{7} = 4$

$\therefore$ Point on curve where we want the tangent is $(3,\ 4)$

When $x = 3, \quad \dfrac{dy}{dx} = -\dfrac{56 \times 3}{(3^2 - 2)^2} = -\dfrac{56 \times 3}{49} = -\dfrac{24}{7}$

For the tangent, $y = mx + c$

$$4 = -\frac{24}{7} \times 3 + c$$

$$c = 4 \times \frac{7}{7} + \frac{72}{7}$$

$$= \frac{100}{7}$$

Equation of the tangent, $y = -\dfrac{24}{7}x + \dfrac{100}{7}$

$$7y = -24x + 100$$

$$24x + 7y - 100 = 0$$

[6 marks]

Answer 8

(i)

$$f(x) = \frac{1}{9\pi}\left(x^2 \sin(3x)\right)$$

$$f'(x) = \frac{1}{9\pi}\left(3x^2 \cos(3x) + 2x \sin(3x)\right)$$

$$= \frac{x}{9\pi}\left(3x \cos(3x) + 2 \sin(3x)\right)$$

(ii)

$$f'\left(\frac{\pi}{3}\right) = \frac{\pi}{9\pi \times 3}\left(\frac{3\pi}{3}\cos(\pi) + 2\sin(\pi)\right)$$

$$= \frac{1}{27}\left(\pi \times (-1) + 2 \times 0\right)$$

$$= -\frac{\pi}{27} \qquad \square$$

[5 marks]

Answer 9

$$f(x) = x - ln(2x - 1)$$

$$f'(x) = 1 - \frac{1}{2x - 1} \times 2$$

$$= \frac{(2x - 1) - 2}{2x - 1}$$

$$= \frac{2x - 3}{2x - 1}$$

For a turning point, $f'(x) = 0$

$$\frac{2x - 3}{2x - 1} = 0$$

$$2x - 3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

Thus, $f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right) - ln\left(2 \times \frac{3}{2} - 1\right)$

$$= 1.5 - ln\,2$$

$\therefore$ The turning point is at $\left(\frac{3}{2}, \frac{3}{2} - ln\,2\right)$ ☐

[4 marks]