13.2 Answers

Answer 1

$$y = \sec x \tan x$$

$$\frac{dy}{dx} = \sec x \sec^2 x + \sec x \tan x \tan x$$

$$= \sec x \left(\sec^2 x + \tan^2 x \right)$$

$$= \sec x \left(\sec^2 x + \sec^2 x - 1 \right) \qquad \text{from identity } 1 + \tan^2 x = \sec^2 x$$

$$= \sec x \left(2 \sec^2 x - 1 \right) \qquad \square$$

[4 marks]

Answer 2

$$y = \csc x \cot x$$

$$\frac{dy}{dx} = \csc x \left(-\csc^2 x \right) + \left(-\csc x \cot x \right) \cot x$$

$$= -\csc x \left(\csc^2 x + \cot^2 x \right)$$

$$= -\csc x \left(\csc^2 x + \csc^2 x - 1 \right) \qquad \text{from identity } \cot^2 x + 1 = \csc^2 x$$

$$= -\csc x \left(2 \csc^2 x - 1 \right)$$

$$= \csc x \left(1 - 2 \csc^2 x \right)$$

[4 marks]

Answer 3

$$f(x) = \frac{8}{(1 - 3x)^3}$$

$$= 8 (1 - 3x)^{-3}$$

$$f'(x) = -24 (1 - 3x)^{-4} \times (-3)$$

$$= \frac{72}{(1 - 3x)^4}$$

$$\therefore f'(1) = \frac{72}{(-2)^4}$$

$$= \frac{9}{2}$$

[4 marks]

Answer 4

$$x = \cos(2y + \pi)$$

$$\frac{dx}{dy} = -2\sin(2y + \pi)$$

When $y = \frac{\pi}{4}$, $\frac{dx}{dy} = -2\sin\left(\frac{3\pi}{2}\right) = -2 \times -1 = 2$

$$\therefore \frac{dy}{dx} = \frac{1}{2} \text{ at } \left(0, \frac{\pi}{4}\right)$$

For the tangent, $y = mx + c$

$$\frac{\pi}{4} = \frac{1}{2} \times 0 + c \Rightarrow c = \frac{\pi}{4}$$

$\therefore$ Tangent is $y = \frac{1}{2}x + \frac{\pi}{4}$

[6 marks]

Answer 5

(i) The curve will cut the x-axis when $y = 0$

$$\ln\left(x^2 - 3\right) = 0 \Leftrightarrow x^2 - 3 = e^0$$

$$x^2 = 4$$

$$x = \pm 2$$

$\therefore$ Crosses the x-axis at $(-2, 0)$, $(2, 0)$

[3 marks]

(ii) $\frac{dy}{dx} = \frac{2x}{x^2 - 3}$

When $x = 2$, $y = 0$ and $\frac{dy}{dx} = 4$ so this normal has gradient $\left(-\frac{1}{4}\right)$

It's equation will be, $y = mx + c$

$$0 = -\frac{1}{4} \times 2 + c \Rightarrow c = \frac{1}{2}$$

$$\therefore y = -\frac{1}{4}x + \frac{1}{2}$$

When $x = -2$, $y = 0$ and $\frac{dy}{dx} = -4$ so this normal has gradient $\left(\frac{1}{4}\right)$

It's equation will be, $y = mx + c$

$$0 = \frac{1}{4} \times -2 + c \Rightarrow c = \frac{1}{2}$$

$$\therefore y = \frac{1}{4}x + \frac{1}{2}$$

Point of intersection is thus $\left(0, \frac{1}{2}\right)$

[5 marks]

Answer 6

$$y = arccot\, x$$
$$\therefore\; x = cot\, y$$
$$\frac{dx}{dy} = -csc^2 y$$
$$= -\left(cot^2 y + 1 \right)$$
$$= -\left(x^2 + 1 \right)$$
$$\frac{dy}{dx} = -\frac{1}{1 + x^2}$$

[6 marks]

Answer 7

Cuts x-axis when $y = 0$, which is when $2x + 1 = 0$ $\quad \therefore\; A\left(-\frac{1}{2},\; 0\right)$

Cuts y-axis when $x = 0$. which is when $y = -1$ $\quad \therefore\; B\;(\,0.\; -\;1\,)$

$$y = \frac{2x + 1}{2x - 1} \;\Rightarrow\; \frac{dy}{dx} = \frac{(2x - 1)\; 2 - 2\,(\,2x + 1\,)}{(\,2x - 1\,)^2}$$
$$= \frac{4x - 2 - 4x - 2}{(\,2x - 1\,)^2}$$
$$= -\frac{4}{(\,2x - 1\,)^2}$$

For A, $\frac{dy}{dx} = -1$ giving a tangent, $\quad y = mx + c$

$$0 = -1 \times -\frac{1}{2} + c \;\Rightarrow\; c = -\frac{1}{2}$$
$$y = -x - \frac{1}{2}$$

For B, $\frac{dy}{dx} = -4$ giving a tangent, $\quad y = mx + c$

$$-\;1 = -4 \times 0 + c \;\Rightarrow\; c = -1$$
$$y = -4x - 1$$

These tangents intersect when, $\quad -\;4x - 1 = -x - \frac{1}{2}$

$$x = -\frac{1}{6} \text{ with } y = -\frac{1}{3} \text{ i.e. } \left(-\frac{1}{6}, -\frac{1}{3}\right)$$

[8 marks]