

2.4 Answers

A-Level Pure Mathematics : Year 2

Differentiation III

2.4.1 Solution to 2.2 Example (Teaching Video)

$$\begin{aligned}y &= x^5 x^3 \\ \frac{dy}{dx} &= x^5 3x^2 + 5x^4 x^3 \\ \frac{dy}{dx} &= 3x^7 + 5x^7 \\ \frac{dy}{dx} &= 8x^7\end{aligned}$$

[2 marks]

The sensible way to do this problem is to simplify first then differentiate.
This allows the answer via The Chain Rule to be checked.

$$\begin{aligned}y &= x^5 x^3 \\ y &= x^8 \\ \frac{dy}{dx} &= 8x^7\end{aligned}$$

The first method allows practice of the product rule before having to use it on questions that can't be done using the second method.

2.4.2 Solutions to 2.3 Exercise

Answer 1

(i) Using the Product Rule;

$$\begin{aligned}y &= x^9 \times x^{11} \\ \frac{dy}{dx} &= x^9 \times 11x^{10} + 9x^8 \times x^{11} \\ \frac{dy}{dx} &= 11x^{19} + 9x^{19} \\ \frac{dy}{dx} &= 20x^{19}\end{aligned}$$

[2 marks]

(ii) Using algebra to simplify first;

$$\begin{aligned}y &= x^9 \times x^{11} \\ y &= x^{20} \\ \frac{dy}{dx} &= 20x^{19}\end{aligned}$$

[2 marks]

Answer 2

(i) Using the product rule;

$$y = 3x^5 \times 4x^7$$

$$\frac{dy}{dx} = 3x^5 \times 28x^6 + 15x^4 \times 4x^7$$

$$\frac{dy}{dx} = 84x^{11} + 60x^{11}$$

$$\frac{dy}{dx} = 144x^{11}$$

[2 marks]

(ii) Using algebra to simplify first;

$$y = 3x^5 \times 4x^7$$

$$y = 12x^{12}$$

$$\frac{dy}{dx} = 144x^{11}$$

[2 marks]

Answer 3

(i) Using the product rule;

$$y = x \times x$$

$$\frac{dy}{dx} = x \times 1 + 1 \times x$$

$$\frac{dy}{dx} = 2x$$

[2 marks]

(ii) Using algebra to simplify first;

$$y = x \times x$$

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

[2 marks]

Answer 4

(i) Using the product rule;

$$y = 8x^{\frac{3}{2}} \times 6x^{\frac{5}{2}}$$

$$\frac{dy}{dx} = 8x^{\frac{3}{2}} \times 15x^{\frac{3}{2}} + 12x^{\frac{1}{2}} \times 6x^{\frac{5}{2}}$$

$$\frac{dy}{dx} = 120x^3 + 72x^3$$

$$\frac{dy}{dx} = 192x^3$$

[3 marks]

(ii) Using algebra to simplify first;

$$y = 8x^{\frac{3}{2}} \times 6x^{\frac{5}{2}}$$

$$y = 48x^4$$

$$\frac{dy}{dx} = 192x^3$$

[3 marks]

Answer 5

(i) Using the product rule,

$$y = x^{-3} \times x^8$$

$$\frac{dy}{dx} = x^{-3} \times 8x^7 - 3x^{-4} \times x^8$$

$$\frac{dy}{dx} = 8x^4 - 3x^4$$

$$\frac{dy}{dx} = 5x^4$$

[2 marks]

(ii) Using algebra to simplify first,

$$y = x^{-3} x^8$$

$$y = x^5$$

$$\frac{dy}{dx} = 5x^4$$

[2 marks]

Answer 6

(i) Using the product rule,

$$y = (x^2 - 1)(x^2 + 1)$$

$$\frac{dy}{dx} = (x^2 - 1)2x + 2x(x^2 + 1)$$

$$\frac{dy}{dx} = 2x^3 - 2x + 2x^3 + 2x$$

$$\frac{dy}{dx} = 4x^3$$

[3 marks]

(ii) Using algebra to simplify first,

$$y = (x^2 - 1)(x^2 + 1)$$

$$y = x^4 - 1 \quad \text{Difference of two squares}$$

$$\frac{dy}{dx} = 4x^3$$

[3 marks]

Answer 7

Using the product rule to show that;

$$y = x^4(3x^2 + 1)$$

has a derivative given by;

$$\frac{dy}{dx} = 2x^3(9x^2 + 2)$$

and a second derivative given by;

$$\frac{d^2y}{dx^2} = 6x^2(15x^2 + 2)$$

before determining the third derivative.

$$\frac{d^3y}{dx^3} = 24x(15x^2 + 1) \quad \text{or} \quad \frac{d^3y}{dx^3} = 360x^3 + 24x$$

[6 marks]