

3.4 Answers

A-Level Pure Mathematics : Year 2

Differentiation III

3.4.1 Solution to 3.2 Example (Teaching Video)

$$\begin{aligned}y &= x^3 (2x + 5)^3 \\ \frac{dy}{dx} &= x^3 \times 3(2x + 5)^2 \times 2 + 3x^2 (2x + 5)^3 \\ \frac{dy}{dx} &= 6x^3 (2x + 5)^2 + 3x^2 (2x + 5)^3 \\ \frac{dy}{dx} &= 3x^2 (2x + 5)^2 \{ 2x + (2x + 5) \} \\ \frac{dy}{dx} &= 3x^2 (2x + 5)^2 (4x + 5)\end{aligned}$$

[5 marks]

There is a clever way of checking this question via another method which can fill five minutes at the end of the lesson. At the start it's an unhelpful distraction! The idea is to convert the given expression into a form to which the chain rule can be applied.

$$\begin{aligned}y &= x^3 (2x + 5)^3 \\ \frac{dy}{dx} &= \{ x(2x + 5) \}^3 \\ \frac{dy}{dx} &= \{ 2x^2 + 5x \}^3 \\ \frac{dy}{dx} &= 3 \{ 2x^2 + 5x \}^2 (4x + 5) \\ \frac{dy}{dx} &= \{ x(2x + 5) \}^2 (4x + 5) \\ \frac{dy}{dx} &= 3x^2 (2x + 5)^2 (4x + 5)\end{aligned}$$

3.4.2 Solutions to 3.3 Exercise

Answer 1

$$\begin{aligned}y &= x^5 (x - 1)^2 \\ \frac{dy}{dx} &= x^5 \times 2(x - 1)^1 \times 1 + 5x^4(x - 1)^2 \\ &= 2x^5(x - 1) + 5x^4(x - 1)^2 \\ &= x^4(x - 1)\{2x + 5(x - 1)\} \\ &= x^4(x - 1)\{2x + 5x - 5\} \\ &= x^4(x - 1)(7x - 5) \quad \square\end{aligned}$$

[5 marks]

Answer 2

$$\begin{aligned}y &= x^7(6x + 5)^3 \\ \frac{dy}{dx} &= x^7 \times 3(6x + 5)^2 \times 6 + 7x^6(6x + 5)^3 \\ &= 18x^7(6x + 5)^2 + 7x^6(6x + 5)^3 \\ &= x^6(6x + 5)^2 \{18x + 7(6x + 5)\} \\ &= x^6(6x + 5)^2 \{18x + 42x + 35\} \\ &= x^6(6x + 5)^2 \{60x + 35\} \\ &= 5x^6(6x + 5)^2(12x + 7) \quad \square\end{aligned}$$

[5 marks]

Answer 3

$$\begin{aligned}y &= (x^2 - 3)(x + 1)^2 \\ \frac{dy}{dx} &= (x^2 - 3) \times 2(x + 1)^1 \times 1 + 2x(x + 1)^2 \\ &= 2(x^2 - 3)(x + 1)^1 + 2x(x + 1)^2 \\ &= 2(x + 1)\{x^2 - 3 + x(x + 1)\} \\ &= 2(x + 1)\{x^2 - 3 + x^2 + x\} \\ &= 2(x + 1)\{2x^2 + x - 3\} \\ &= 2(x + 1)\{(2x + 3)(x - 1)\} \\ &= 2(x^2 - 1)(2x + 3) \quad \square\end{aligned}$$

[5 marks]

Answer 4

$$\begin{aligned}
y &= (4x + 1)^{\frac{3}{2}} (x^2 + 5) \\
\frac{dy}{dx} &= (4x + 1)^{\frac{3}{2}} \times 2x + \frac{3}{2} (4x + 1)^{\frac{1}{2}} \times 4 \times (x^2 + 5) \\
&= 2x(4x + 1)^{\frac{3}{2}} + 6(4x + 1)^{\frac{1}{2}}(x^2 + 5) \\
&= 2(4x + 1)^{\frac{1}{2}} \{ x(4x + 1) + 3(x^2 + 5) \} \\
&= 2(4x + 1)^{\frac{1}{2}} \{ 4x^2 + x + 3x^2 + 15 \} \\
&= 2\sqrt{4x + 1} \{ 7x^2 + x + 15 \} \quad \square
\end{aligned}$$

[5 marks]**Answer 5**

$$\begin{aligned}
y &= (2x - 3)^3 (x^2 + 1)^2 \\
\frac{dy}{dx} &= (2x - 3)^3 \times 2(x^2 + 1)^1 \times 2x + 3(2x - 3)^2 \times 2 \times (x^2 + 1)^2 \\
&= 4x(2x - 3)^3 (x^2 + 1) + 6(2x - 3)^2 (x^2 + 1)^2 \\
&= 2(2x - 3)^2 (x^2 + 1) \{ 2x(2x - 3) + 3(x^2 + 1) \} \\
&= 2(2x - 3)^2 (x^2 + 1) \{ 4x^2 - 6x + 3x^2 + 3 \} \\
&= 2(2x - 3)^2 (x^2 + 1) (7x^2 - 6x + 3) \quad \square
\end{aligned}$$

[5 marks]**Answer 6**

$$\begin{aligned}
y &= x^3 (5x + 1)^2 \\
\frac{dy}{dx} &= x^3 \times 2(5x + 1)^1 \times 5 + 3x^2 (5x + 1)^2 \\
&= 10x^3 (5x + 1) + 3x^2 (5x + 1)^2 \\
&= x^2 (5x + 1) \{ 10x + 3(5x + 1) \} \\
&= x^2 (5x + 1) \{ 10x + 15x + 3 \} \\
&= x^2 (5x + 1) (25x + 3)
\end{aligned}$$

[5 marks]

Answer 7

$$\begin{aligned}
 y &= (x^2 - 1)\sqrt{1+x} \\
 y &= (x^2 - 1)(x + 1)^{\frac{1}{2}} \\
 \frac{dy}{dx} &= (x^2 - 1) \times \frac{1}{2}(x + 1)^{-\frac{1}{2}} \times 1 + 2x(x + 1)^{\frac{1}{2}} \\
 &= \frac{1}{2}(x - 1)(x + 1) \times \frac{1}{\sqrt{1+x}} + 2x\sqrt{1+x} \\
 &= \frac{1}{2}(x - 1)\sqrt{1+x} + 2x\sqrt{1+x} \\
 &= \frac{1}{2}\sqrt{1+x} \{ (x - 1) + 4x \} \\
 &= \frac{1}{2}\sqrt{1+x} (5x - 1)
 \end{aligned}$$

For a stationary point,

$$\frac{dy}{dx} = 0$$

$$\frac{1}{2}\sqrt{1+x} (5x - 1) = 0$$

$$\text{Either } \sqrt{1+x} = 0 \text{ or } 5x - 1 = 0$$

$$x = -1 \text{ or } x = \frac{1}{5}$$

[5 marks]