4.3 Answers

A-Level Pure Mathematics : Year 2
Differentiation III

Answer 1

$$\begin{aligned} f(x) &= \left(2x^4 - 3x + 6\right)^5 \\ f'(x) &= 5\left(2x^4 - 3x + 6\right)^4 \left(8x^3 - 3\right) \\ f'(1) &= 5\,(2 - 3 + 6)^4\,(8 - 3) \\ &= 5 \times 5^4 \times 5 \\ &= 5^6 \qquad \square \end{aligned}$$

[3 marks]

Answer 2

$$\begin{aligned} g(x) &= \sqrt{5x^2 - 4} \\ &= \left(5x^2 - 4\right)^{\frac{1}{2}} \\ g'(x) &= \frac{1}{2}\left(5x^2 - 4\right)^{-\frac{1}{2}} \times 10x \\ &= \frac{5x}{\sqrt{5x^2 - 4}} \\ g'(2) &= \frac{5 \times 2}{\sqrt{5 \times 2^2 - 4}} \\ &= \frac{10}{\sqrt{16}} \\ &= 2.5 \qquad \square \end{aligned}$$

[3 marks]

Answer 3

$$\begin{aligned} y &= (5 - 2x)^3 \\ \frac{dy}{dx} &= 3(5 - 2x)^2 \times (-2) \\ &= -6\,(5 - 2x)^2 \end{aligned}$$

When $x = 2$, $\dfrac{dy}{dx} = -6$ and $y = 1$

$$\begin{aligned} y &= mx + c \\ 1 &= -6 \times 2 + c \\ c &= 13 \\ \therefore\ y &= -6x + 13 \end{aligned}$$

[4 marks]

Answer 4

$$y = (2x-1)^3(3x+1)^2$$

$$\frac{dy}{dx} = (2x-1)^3 \times 2(3x+1)^1 \times 3 + 3(2x-1)^2 \times 2 \times (3x+1)^2$$

$$= 6(2x-1)^3(3x+1) + 6(2x-1)^2(3x+1)^2$$

$$= 6(2x-1)^2(3x+1)\{2x-1+3x+1\}$$

$$= 6(2x-1)^2(3x+1)(5x)$$

$$= 30x(2x-1)^2(3x+1)$$

Stationary points occur when the gradient is zero

$$30x(2x-1)^2(3x+1) = 0$$

$$\text{Either } x = 0, \text{ or } x = \frac{1}{2} \text{ or } x = -\frac{1}{3}$$

$$\text{Points are thus, } (0,-1), \left(\frac{1}{2}, 0\right), \left(-\frac{1}{3}, 0\right)$$

[6 marks]

Answer 5

(i) $\Delta V = (7 + 2) \times (2 + 1) \times (4 + 3) - 7 \times 2 \times 4$

$= 9 \times 3 \times 7 - 7 \times 2 \times 4$

$= 189 - 56$

$= 133$

[2 marks]

(ii) $V_{RED} = 4 \times 2 \times 2 = 16$

$V_{BROWN} = 7 \times 4 \times 1 = 28$

$V_{BLUE} = 7 \times 2 \times 3 = 42$ Total : 86

$$\text{Percentage} = \frac{86}{133} \times 100 = 64.7\%$$

[4 marks]

(iii) $\Delta V = (7 + 0.2) \times (2 + 0.1) \times (4 + 0.3) - 7 \times 2 \times 4$

$= 7.2 \times 2.1 \times 4.3 - 7 \times 2 \times 4$

$= 65.016 - 56$

$= 9.016$

$V_{RED} = 4 \times 2 \times 0.2 = 1.6$

$V_{BROWN} = 7 \times 4 \times 0.1 = 2.8$

$V_{BLUE} = 7 \times 2 \times 0.3 = 4.2$ Total : 8.6

$$\text{Percentage} = \frac{8.6}{9.016} \times 100 = 95.4\%$$

[5 marks]

(iv) It can be argued that the significant part of the extra volume for small extensions is given by the sum of the volumes of the red, brown and blue cuboids shown in the centre of the photograph, and that the white 'filler' parts can be ignored. This argument is strengthened by the calculation part of the question. Firstly, in part (ii) the volume from just the red plus brown plus blue volume answer is inaccurate. This is because the extensions are relatively large. But then, in part (iii), where the extensions are much smaller, a far more accurate answer is obtained.

[2 marks]

(v)

The Triple Product Rule

If $V = uvw$ then $V' = u\,v\,w' + u\,v'\,w + u'\,v\,w$

[2 marks]

Answer 6

$$f(x) = x^2\sqrt{3x-1}$$

$$= x^2(3x-1)^{\frac{1}{2}}$$

$$f'(x) = x^2 \times \frac{1}{2}(3x-1)^{-\frac{1}{2}} \times 3 + 2x(3x-1)^{\frac{1}{2}}$$

$$= \frac{3x^2}{2\sqrt{3x-1}} + 2x\sqrt{3x-1} \times \frac{2\sqrt{3x-1}}{2\sqrt{3x-1}}$$

$$= \frac{3x^2}{2\sqrt{3x-1}} + \frac{4x(3x-1)}{2\sqrt{3x-1}}$$

$$= \frac{3x^2 + 12x^2 - 4x}{2\sqrt{3x-1}}$$

$$= \frac{15x^2 - 4x}{2\sqrt{3x-1}}$$

$$= \frac{x(15x-4)}{2\sqrt{3x-1}} \qquad \square$$

[5 marks]

Answer 7

$$y = \frac{1}{(1-2x)^2}$$

$$= (1-2x)^{-2}$$

$$\frac{dy}{dx} = -2(1-2x)^{-3} \times (-2)$$

$$= \frac{4}{(1-2x)^3}$$

When $x = \frac{1}{4}$, $y = 4$, $\frac{dy}{dx} = 32$

$$y = mx + c$$

$$4 = 32 \times \frac{1}{4} + c$$

$$4 = 8 + c$$

$$c = -4$$

$\therefore$ Tangent is $y = 32x - 4$

[4 marks]