5.4 Answers

A-Level Pure Mathematics : Year 2
Differentiation III

5.4.1 Solution to 5.2 Example (Teaching Video)

$$y = \frac{x+4}{x+5}$$

$$\frac{dy}{dx} = \frac{(x+5)\times 1 - 1\times(x+4)}{(x+5)^2}$$

$$= \frac{x+5-x-4}{(x+5)^2}$$

$$= \frac{1}{(x+5)^2}$$

[3 marks]

There is a clever way of checking this question via an algebraic method.

$$y = \frac{x+4}{x+5}$$

$$= \frac{x+4+1-1}{x+5}$$

$$= \frac{x+5}{x+5} - \frac{1}{x+5}$$

$$= 1 - (x+5)^{-1}$$

$$\frac{dy}{dx} = -(-1)(x+5)^{-2}\times 1$$

$$= (x+5)^{-2}$$

$$= \frac{1}{(x+5)^2}$$

5.4.2 Solutions to 5.3 Exercise

Answer 1

$$y = \frac{4x}{x+3}$$

$$\frac{dy}{dx} = \frac{(x+3)\times 4 - 1\times 4x}{(x+3)^2}$$

$$= \frac{4x+12-4x}{(x+3)^2}$$

$$= \frac{12}{(x+3)^2} \qquad \square$$

[3 marks]

Answer 2

$$y = \frac{x^2}{(x+5)}$$

$$\frac{dy}{dx} = \frac{(x+5)\times 2x - 1\times x^2}{(x+5)^2}$$

$$= \frac{2x^2+10x-x^2}{(x+5)^2}$$

$$= \frac{x^2+10x}{(x+5)^2}$$

$$= \frac{x(x+10)}{(x+5)^2} \qquad \square$$

[3 marks]

Answer 3

$$y = \frac{5x-2}{3x+1}$$

$$\frac{dy}{dx} = \frac{(3x+1)\times 5 - 3\times(5x-2)}{(3x+1)^2}$$

$$= \frac{15x+5-15x+6}{(3x+1)^2}$$

$$= \frac{11}{(3x+1)^2} \qquad \square$$

[3 marks]

Answer 4

$$y = \frac{x^2 + 1}{x^2 + 4}$$

$$\frac{dy}{dx} = \frac{\left(x^2 + 4\right) \times 2x - 2x \times \left(x^2 + 1\right)}{\left(x^2 + 4\right)^2}$$

$$= \frac{2x^3 + 8x - 2x^3 - 2x}{\left(x^2 + 4\right)^2}$$

$$= \frac{6x}{\left(x^2 + 4\right)^2} \qquad \square$$

[3 marks]

Answer 5

$$y = \frac{x^5}{(2x + 1)^3}$$

$$\frac{dy}{dx} = \frac{(2x + 1)^3 \times 5x^4 - 3(2x + 1)^2 \times 2 \times x^5}{(2x + 1)^6}$$

$$= \frac{5x^4(2x + 1)^3 - 6x^5(2x + 1)^2}{(2x + 1)^6}$$

$$= \frac{x^4(2x + 1)^2 \{5(2x + 1) - 6x\}}{(2x + 1)^6}$$

$$= \frac{x^4\{10x + 5 - 6x\}}{(2x + 1)^4}$$

$$= \frac{x^4(4x + 5)}{(2x + 1)^4} \qquad \square$$

[4 marks]

Answer 6

$$y = \frac{x^7}{(3x+2)^5}$$

$$\frac{dy}{dx} = \frac{(3x+2)^5 \times 7x^6 - 5(3x+2)^4 \times 3 \times x^7}{(3x+2)^{10}}$$

$$= \frac{7x^6(3x+2)^5 - 15x^7(3x+2)^4}{(3x+2)^{10}}$$

$$= \frac{x^6(3x+2)^4\ \{7(3x+2) - 15x\}}{(3x+2)^{10}}$$

$$= \frac{x^6\{21x + 14 - 15x\}}{(3x+2)^6}$$

$$= \frac{2x^6(3x+7)}{(3x+2)^6} \qquad \square$$

[5 marks]

Answer 7

$$y = \frac{2(x+3)^3}{\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{x^{\frac{1}{2}} \times 6(x+3)^2 \times 1 - \frac{1}{2}x^{-\frac{1}{2}} \times 2(x+3)^3}{x} \times \frac{x^{\frac{1}{2}}}{x^{\frac{1}{2}}}$$

$$= \frac{6x(x+3)^2 - (x+3)^3}{x^{\frac{3}{2}}}$$

$$= \frac{(x+3)^2\ \{6x - (x+3)\}}{x^{\frac{3}{2}}}$$

$$= \frac{(x+3)^2(5x-3)}{x^{\frac{3}{2}}} \qquad \square$$

[4 marks]

Answer 8

$$y = x^2\sqrt{x+5} = x^2(x+5)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = x^2 \times \frac{1}{2}(x+5)^{-\frac{1}{2}} + 2x(x+5)^{\frac{1}{2}}$$

$$= \frac{x^2}{2\sqrt{x+5}} + 2x\sqrt{x+5} \times \frac{2\sqrt{x+5}}{2\sqrt{x+5}}$$

$$= \frac{x^2 + 4x(x+5)}{2\sqrt{x+5}}$$

$$= \frac{x^2 + 4x^2 + 20x}{2\sqrt{x+5}}$$

$$= \frac{5x(x+4)}{2\sqrt{x+5}} \qquad \square$$

[5 marks]

Answer 9

(a)

$$f(x) = \frac{2x}{x+5} + \frac{6x}{x^2+7x+10}$$

$$= \frac{2x}{x+5} + \frac{6x}{(x+5)(x+2)}$$

$$= \frac{2x}{x+5} \times \frac{(x+2)}{(x+2)} + \frac{6x}{(x+5)(x+2)}$$

$$= \frac{2x^2 + 4x + 6x}{(x+5)(x+2)}$$

$$= \frac{2x(x+5)}{(x+5)(x+2)}$$

$$= \frac{2x}{x+2} \qquad \square$$

(b)

$$f'(x) = \frac{(x+2) \times 2 - 1 \times 2x}{(x+2)^2}$$

$$= \frac{2x + 4 - 2x}{(x+2)^2}$$

$$= \frac{4}{(x+2)^2}$$

$$f'(3) = \frac{4}{(3+2)^2}$$

$$= \frac{4}{25}$$

[5 marks]

Answer 10

$$f(x) = \frac{x}{x^2 + 2}$$

$$f'(x) = \frac{(x^2 + 2) \times 1 - 2x \times x}{(x^2 + 2)^2}$$

$$= \frac{x^2 + 2 - 2x^2}{(x^2 + 2)^2}$$

$$= \frac{2 - x^2}{(x^2 + 2)^2}$$

We require $f'(x) \geqslant 0$

$$\frac{2 - x^2}{(x^2 + 2)^2} \geqslant 0$$

$$2 - x^2 \geqslant 0$$

$$-\sqrt{2} \leqslant x \leqslant \sqrt{2}$$

That is, $k = \sqrt{2}$

[5 marks]