

6.6 Answers

A-Level Pure Mathematics : Year 2 Differentiation III

6.6.1 Solutions to 6.4 Examples (Teaching Video)

(i) $y = e^{8x^3+5x^2}$ (Chain Rule Example)

$$\frac{dy}{dx} = e^{8x^3+5x^2} (24x^2 + 10x)$$

[3 marks]

(ii) $y = x e^{3x}$ (Product Rule Example)

$$\begin{aligned}\frac{dy}{dx} &= x \times e^{3x} \times 3 + 1 \times e^{3x} \\ &= 3x e^{3x} + e^{3x} \\ &= e^{3x} (3x + 1)\end{aligned}$$

[3 marks]

(iii) $y = \frac{x}{e^x}$ (Quotient Rule Example)

$$\begin{aligned}\frac{dy}{dx} &= \frac{e^x \times 1 - e^x \times x}{e^x \times e^x} \\ &= \frac{e^x (1 - e^x)}{e^x \times e^x} \\ &= \frac{1 - e^x}{e^x}\end{aligned}$$

[3 marks]

6.6.2 Solutions to 6.5 Exercise

Answer 1

(i) $y = e^{5x} \Rightarrow \frac{dy}{dx} = 5 e^{5x}$

(ii) $y = e^{3x^5} \Rightarrow \frac{dy}{dx} = 15x^4 e^{3x^5}$

(iii) $y = 8 e^{6x} + 6 e^{5x} \Rightarrow \frac{dy}{dx} = 48 e^{6x} + 30 e^{5x}$

(iv) $y = 14 e^x \Rightarrow \frac{dy}{dx} = 14 e^x$

(v) $y = e^{-7x} \Rightarrow \frac{dy}{dx} = -7 e^{-7x}$

(vi) $y = 4 e^{-5x} \Rightarrow \frac{dy}{dx} = -20 e^{-5x}$

[12 marks]

Answer 2

$$\begin{aligned}y &= \frac{e^{7x}}{e^{3x}} \\&= e^{4x} \\ \frac{dy}{dx} &= 4e^{4x}\end{aligned}$$

[2 marks]

Answer 3

$$\begin{aligned}y &= e^{3x}(e^{4x} + 7e^{-9x}) \\&= e^{7x} + 7e^{-6x} \\ \frac{dy}{dx} &= 7e^{7x} - 42e^{-6x}\end{aligned}$$

[3 marks]

Answer 4

(i)

$$\begin{aligned}y &= x^3 e^x \\ \frac{dy}{dx} &= x^3 e^x + 3x^2 e^x \\&= x^2 e^x (x + 3)\end{aligned}$$

[3 marks]

(ii)

$$\begin{aligned}y &= x e^{-x} \\ \frac{dy}{dx} &= x(-e^{-x}) + 1 \times e^{-x} \\&= e^{-x}(-x + 1) \\&= \frac{1 - x}{e^x}\end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned}
 \text{(i)} \quad y &= \frac{e^x}{x} \\
 \frac{dy}{dx} &= \frac{x e^x - 1 \times e^x}{x^2} \\
 &= \frac{e^x (x - 1)}{x^2}
 \end{aligned}$$

[3 marks]

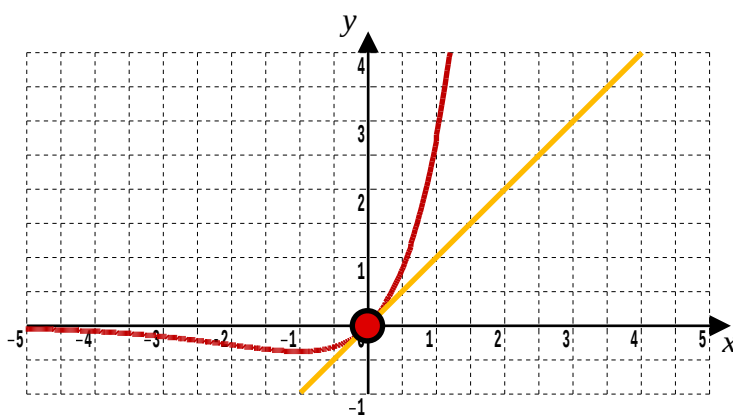
$$\begin{aligned}
 \text{(ii)} \quad y &= \frac{2 - x^2}{e^{2x}} \\
 \frac{dy}{dx} &= \frac{e^{2x} \times (-2x) - e^{2x} \times 2 \times (2 - x^2)}{e^{2x} \times e^{2x}} \\
 &= \frac{e^{2x}(-2x - 4 + 2x^2)}{e^{2x} \times e^{2x}} \\
 &= \frac{2(x^2 - x - 2)}{e^{2x}} \\
 &= \frac{2(x - 2)(x + 1)}{e^{2x}}
 \end{aligned}$$

[3 marks]**Answer 6**

$$\begin{aligned}
 \text{(i)} \quad y &= x e^x \\
 \frac{dy}{dx} &= x e^x + 1 \times e^x \\
 &= e^x (x + 1)
 \end{aligned}$$

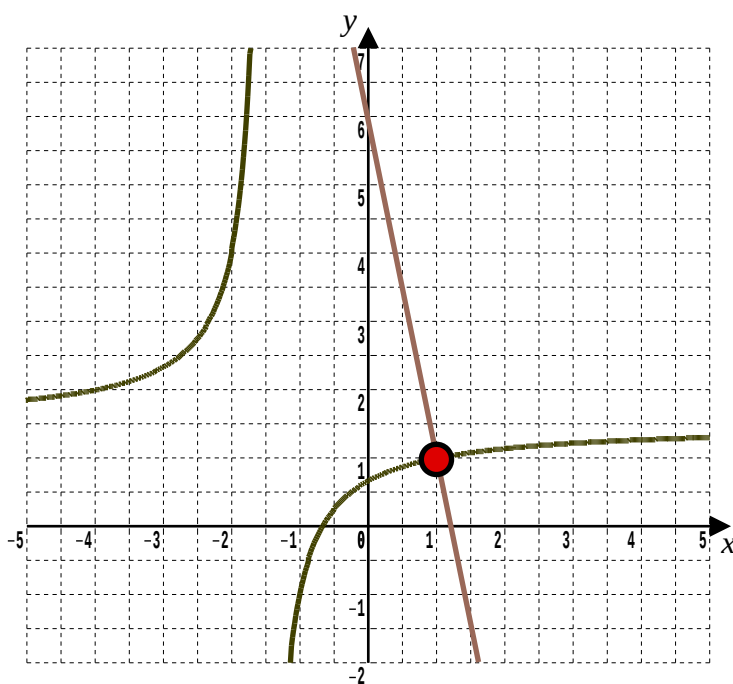
[3 marks]

(ii) When $x = 0$, $y = 0$ and $\frac{dy}{dx} = 1 \Rightarrow y = x$ is the tangent

**[4 marks]**

Answer 7**(i)**

$$\begin{aligned}y &= \frac{3x + 2}{2x + 3} \\ \frac{dy}{dx} &= \frac{(2x + 3) \times 3 - 2 \times (3x + 2)}{(2x + 3)^2} \\ &= \frac{6x + 9 - 6x - 4}{(2x + 3)^2} \\ &= \frac{5}{(2x + 3)^2}\end{aligned}$$

[3 marks]**(ii)** When $x = 1$, $y = 1$ and $\frac{dy}{dx} = \frac{1}{5}$ Thus normal's gradient is $- 5$ $\therefore y = -5x + 6$ is the normal's equation.**[4 marks]**

Answer 8

$$\begin{aligned}
 \text{(i)} \quad y &= \frac{x^2}{(4x - 3)} \\
 \frac{dy}{dx} &= \frac{(4x - 3) \times 2x - 4x^2}{(4x - 3)^2} \\
 &= \frac{8x^2 - 6x - 4x^2}{(4x - 3)^2} \\
 &= \frac{4x^2 - 6x}{(4x - 3)^2} \\
 &= \frac{2x(2x - 3)}{(4x - 3)^2}
 \end{aligned}$$

[3 marks]

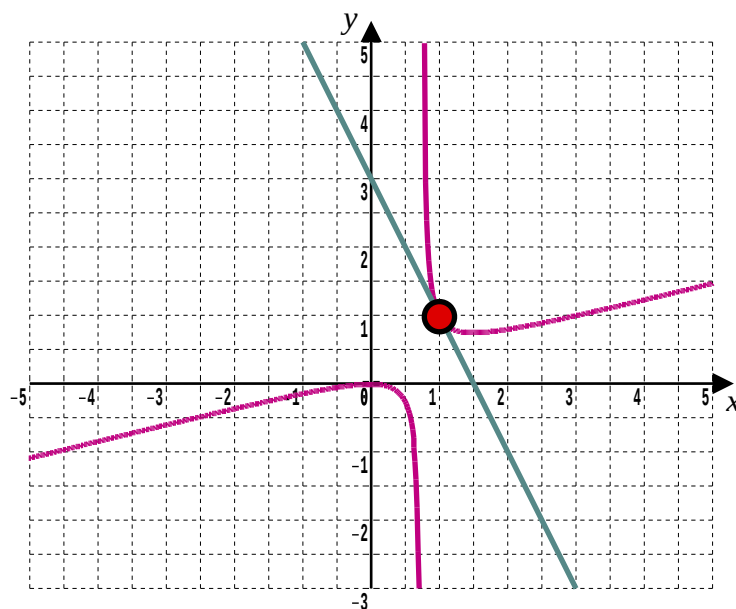
$$\text{(ii)} \quad \text{When } x = 1, y = 1 \text{ and } \frac{dy}{dx} = -2$$

$$y = mx + c$$

$$1 = -2 \times 1 + c$$

$$c = 3$$

$$\therefore y = -2x + 3 \text{ is the tangent}$$

**[4 marks]**

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