8.5 Answers

8.5.1 Solutions to 8.3 Examples (Teaching Video)

(i) $y = sin^4 3x$ (Chain Rule Example)

$$= (sin\, 3x)^4$$

$$\frac{dy}{dx} = 4 (sin\, 3x)^3 \times \left(cos\, 3x \right) \times 3$$

$$= 12\, sin^3 3x\;\; cos\, 3x$$

[3 marks]

(ii) $y = x\, sin \left(x^2 \right)$ (Product Rule Example)

$$\frac{dy}{dx} = x\, cos \left(x^2 \right) \times 2x + 1 \times sin \left(x^2 \right)$$

$$= 2\, x^2\, cos \left(x^2 \right) + sin \left(x^2 \right)$$

[3 marks]

(iii) $y = \dfrac{sin\,(2x)}{e^{2x}}$ (Quotient Rule Example)

$$\frac{dy}{dx} = \frac{e^{2x} \times cos\,(2x) \times 2 - 2e^{2x} \times sin\,(2x)}{e^{2x} \times e^{2x}}$$

$$= \frac{2\, e^{2x} (\, cos\;\; (2x) - sin\,(2x))}{e^{2x} \times e^{2x}}$$

$$= \frac{2\, (\, cos\,(2x) - sin\,(2x))}{e^{2x}}$$

[3 marks]

8.5.2 Solutions to 8.4 Exercise

Answer 1

(i) $y = 7\,sin\ (5x) \quad \Rightarrow \quad \frac{dy}{dx} = 35\,cos\ (5x)$

(ii) $y = 6\,sin\left(3\,x^3 \right) \quad \Rightarrow \quad \frac{dy}{dx} = 54\,x^2\,cos\left(3\,x^3 \right)$

(iii) $y = 2\,sin^3 x$

$= 2\,(\,sin\,x\,)^3 \quad \Rightarrow \quad \frac{dy}{dx} = 6\ (\,sin\,x\,)^2\ cos\,x$

$= 6\,sin^2 x\,cos\,x$

(iv) $y = \sqrt{sin\,x}$

$= (\,sin\,x\,)^{\frac{1}{2}} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{2}(\,sin\,x\,)^{-\frac{1}{2}}\ cos\,x$

$= \frac{cos\,x}{2\sqrt{sin\,x}}$

(v) $y = 11\,e^{sin\,x} \quad \Rightarrow \quad \frac{dy}{dx} = 11\,e^{sin\,x}\,cos\,x$

(vi) $y = 3\,e^{2 sin\,(5x)} \quad \Rightarrow \quad \frac{dy}{dx} = 3\,e^{2\,sin\,(5x)} \times 2\,cos\ (5x) \times 5$

$= 30\,e^{2\,sin\,(5x)}\,cos\ (5x)$

(vii) $y = ln\,(sin\,x) \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{sin\,x} \times cos\,x$

$= cot\,x$

(viii) $y = sin\,(ln\,x\,) \quad \Rightarrow \quad \frac{dy}{dx} = cos\,(\,ln\,x\,) \times \frac{1}{x}$

$= \frac{cos\,(\,ln\,x\,)}{x}$

[16 marks]

Answer 2

$$y = csc\,x$$

$$= \frac{1}{sin\,x}$$

$$= (\,sin\,x\,)^{-1}$$

$$\frac{dy}{dx} = (-1)\ (\,sin\,x\,)^{-2}\ cos\,x$$

$$= -\frac{1}{sin\,x} \times \frac{cos\,x}{sin\,x}$$

$$= -\,csc\,x\,cot\,x \qquad \square$$

[4 marks]

Answer 3

(i)

$$f(x) = x \sin x$$

$$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} \times \sin\left(\frac{\pi}{6}\right)$$

$$= \frac{\pi}{6} \times \frac{1}{2}$$

$$= \frac{\pi}{12}$$

[2 marks]

(ii)

$$f'(x) = x \cos x + \sin x$$

[2 marks]

(iii)

$$f'\left(\frac{\pi}{6}\right) = \frac{\pi}{6} \cos\left(\frac{\pi}{6}\right) + \sin\left(\frac{\pi}{6}\right)$$

$$= \frac{\pi}{6} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{6}{6}$$

$$= \frac{6 + \sqrt{3}\,\pi}{12} \qquad \square$$

[2 marks]

Answer 4

(i) Look at the graph as it approaches the *y*-axis from both the left and the right. In both cases it's smoothly heading towards the number 1 on the *y*-axis.

[2 marks]

(ii)

$$f(x) = \frac{\sin x}{x}$$

$$f'(x) = \frac{x \cos x - \sin x}{x^2}$$

[2 marks]

(iii)

$$f'(\pi) = \frac{\pi \cos \pi - \sin \pi}{\pi^2}$$

$$= \frac{\pi \times (-1) - 0}{\pi^2}$$

$$= \frac{-\pi}{\pi^2}$$

$$= -\frac{1}{\pi}$$

[2 marks]

Answer 5

Look at the graph as it approaches the *y*-axis from both the left and the right.
In both cases it's smoothly heading towards the number 0 on the *y*-axis.

[2 marks]

Answer 6

By definition,

$$f'(x) = \underset{h \to 0}{limit} \frac{f(x + h) - f(x)}{h}$$

$$\therefore (sin\, x)' = \underset{h \to 0}{limit} \frac{sin(x + h) - sin(x)}{h}$$

$$= \underset{h \to 0}{limit} \frac{sin\, x\, cos\, h + cos\, x\, sin\, h - sin\, x}{h}$$

$$= \underset{h \to 0}{limit} \frac{sinx\, cos\, h - sin\, x}{h} + \underset{h \to 0}{limit} \frac{cos\, x\, sin\, h}{h}$$

$$= sin\, x \underset{h \to 0}{limit} \frac{cos\, h - 1}{h} + cos\, x \underset{h \to 0}{limit} \frac{sin\, h}{h}$$

$$= sin\, x \times 0 + cos\, x \times 1$$

$$= cos\, x \qquad \square$$

[6 marks]