9.5 Answers

A-Level Pure Mathematics : Year 2
Differentiation III

9.5.1 Solution to 9.3 Example (Teaching Video)

$$y = \cos x$$

$$y = \left(1 - \sin^2 x\right)^{\frac{1}{2}} \qquad \text{Using} \quad \cos^2 x + \sin^2 x = 1$$

$$\frac{dy}{dx} = \frac{1}{2}\left(1 - \sin^2 x\right)^{-\frac{1}{2}}\left(-2\sin x\right)\left(\cos x\right)$$

$$\frac{dy}{dx} = -\frac{\sin x \cos x}{\sqrt{1 - \sin^2 x}}$$

$$\frac{dy}{dx} = -\frac{\sin x \cos x}{\cos x}$$

$$\frac{dy}{dx} = -\sin x \qquad \square$$

[6 marks]

9.5.2 Solutions to 9.4 Exercise

Answer 1

(i) $y = \cos\left(3x^5 + 2e^{3x}\right)$

$$\Rightarrow \frac{dy}{dx} = -\left(15x^4 + 6e^{3x}\right)\sin\left(3x^5 + 2e^{3x}\right)$$

(ii) $y = 5x^7 + \cos^4(3x)$

$$= 5x^7 + \left(\cos(3x)\right)^4$$

$$\Rightarrow \frac{dy}{dx} = 35x^6 + 4\left(\cos(3x)\right)^3\left(-\sin(3x)\right)3$$

$$= 35x^6 - 12\cos^3(3x)\sin(3x)$$

(iii) $y = \cos\left(\frac{5}{x} + \ln x\right)$

$$\Rightarrow \frac{dy}{dx} = -\sin\left(\frac{5}{x} + \ln x\right)\left(-5x^{-2} + \frac{1}{x}\right)$$

$$= -\sin\left(\frac{5}{x} + \ln x\right)\left(-\frac{5}{x^2} + \frac{1}{x}\right)$$

(iv) $y = \sin(1 - \cos x)$

$$\Rightarrow \frac{dy}{dx} = \cos\left(1 - \cos x\right)\left(\sin x\right)$$

[2, 2, 2, 2 marks]

Answer 2

$$\begin{aligned} y &= \sec x \\ &= \frac{1}{\cos x} \\ &= (\cos x)^{-1} \\ \frac{dy}{dx} &= (-1)\ (\cos x)^{-2}(-\sin x) \\ &= \frac{1}{\cos x} \times \frac{\sin x}{\cos x} \\ &= \sec x \tan x \qquad \square \end{aligned}$$

[3 marks]

Answer 3

$$\begin{aligned} y &= \tan x \\ &= \frac{\sin x}{\cos x} \\ \frac{dy}{dx} &= \frac{\cos x \cos x\ -\ (-\sin x)\ \sin x}{\cos^2 x} \\ &= \frac{\cos^2 x\ +\ \sin^2 x}{\cos^2\ x} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \qquad \square \end{aligned}$$

[3 marks]

Answer 4

$$\begin{aligned} y &= \cot x \\ &= \frac{\cos x}{\sin x} \\ \frac{dy}{dx} &= \frac{\sin x\ (-\sin x)\ -\ \cos x \cos x}{\sin^2 x} \\ &= \frac{(-1)\left(\sin^2 x\ +\ \cos^2 x\right)}{\sin^2 x} \\ &= -\frac{1}{\sin^2 x} \\ &= -\csc^2 x \qquad \square \end{aligned}$$

[3 marks]

Answer 5

(i) $y = tan\ (5x)$

$$\Rightarrow \frac{dy}{dx} = 5\, sec^2 (5x)$$

(ii) $y = sec\left(3x^4 + x^2 \right)$

$$\Rightarrow \frac{dy}{dx} = \left(12x^3 + 2x \right) sec\left(3x^4 + x^2 \right) tan\left(3x^4 + x^2 \right)$$

(iii) $y = cot\left(5 - ln\, x \right)$

$$\Rightarrow \frac{dy}{dx} = -csc^2\ (5 - ln\, x) \times \left(-\frac{1}{x} \right)$$

$$= \frac{csc^2\ (5 - ln\, x)}{x}$$

(iv) $y = e^{tan\, 4x}$

$$\Rightarrow \frac{dy}{dx} = e^{tan\, 4x}\, sec^2 (4x)\ 4$$

$$= 4\, sec^2 (4x)\ e^{tan\, 4x}$$

(v) $y = ln(\, sec\, x + tan\, x\,)$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{sec\, x + tan\, x} \times \left(sec\, x\, tan\, x + sec^2 x \right)$$

$$= \frac{1}{sec\, x + tan\, x} \times sec\, x\ (\, tan\, x + sec\, x\,)$$

$$= sec\, x$$

[10 marks]

Answer 6

(i) Starting from the definition,

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\therefore (\, sin\, x\,)' = \lim_{h \to 0} \frac{cos(x + h) - cos(x)}{h}$$

$$= \lim_{h \to 0} \frac{cos\, x\, cos\, h - sin\, x\, sin\, h - cos\, x}{h}$$

$$= \lim_{h \to 0} \left(\frac{cos\, x\, cos\, h - cos\, x}{h} - \frac{sin\, x\, sin\, h}{h} \right)$$

$$= \lim_{h \to 0} \left(\left(\frac{cos\, h - 1}{h} \right) cos\, x - \frac{sin\, h}{h}\, sin\, x \right)$$

[4 marks]

(ii)

$$= \cos x \lim_{h \to 0} \frac{\cos h - 1}{h} - \sin x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \cos x \times 0 - \sin x \times 1$$

$$= -\sin x \qquad \square$$

[2 marks]

Answer 7

$$f(x) = \csc x$$

$$f'(x) = -\csc x \cot x$$

$$f'\left(\frac{\pi}{6}\right) = -\frac{1}{\sin\left(\frac{\pi}{6}\right)} \times \frac{1}{\tan\left(\frac{\pi}{6}\right)}$$

$$= -2 \times \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= -2\sqrt{3}$$

[3 marks]

Answer 8

$$f(x) = 10\, e^{-0.25x} \sin x$$

$$f'(x) = 10\, e^{-0.25x} \cos x - 2.5\, e^{-0.25x} \sin x$$

For turning points, $f'(x) = 0$

$$\therefore 10\, e^{-0.25x} \cos x - 2.5\, e^{-0.25x} \sin x = 0$$

$$2.5\, e^{-0.25x} \left(4 \cos x - \sin x\right) = 0$$

Either $2.5\, e^{-0.25x} = 0$ which has no solutions,

or $4 \cos x - \sin x = 0$

$$\sin x = 4 \cos x$$

That is, $\tan x = 4 \qquad \square$

[4 marks]