2.3 Answers

2.3.1 Solution to 2.1 Example (Teaching Video)

(i) Setting out of the algebraic long division clearly is key to success,

$$
\begin{array}{r|llll}
 & x^2 & +\ 7x & +\ 12 & \\
\hline
x + 5 & x^3 + 12x^2 & +\ 47x & +\ 60 & \\
 & x^3 + 5x^2 & & & \\
\cline{2-2}
 & 7x^2 & +\ 47x & & \\
 & 7x^2 & +\ 35x & & \\
\cline{2-3}
 & & 12x & +\ 60 & \\
 & & 12x & +\ 60 & \\
\cline{3-4}
 & & & 0 & \leftarrow \text{Remainder}
\end{array}
$$

$\therefore\ f(x)$ is divisible by $x + 5$ $\square$

[4 marks]

(ii) Part (i) reveals that,

$$\frac{x^3 + 12x^2 + 47x + 60}{x + 5} = x^2 + 7x + 12$$

$$\therefore\ x^3 + 12x^2 + 47x + 60 = \left(x^2 + 7x + 12\right)\left(x + 5\right)$$

$$= (x + 3)(x + 4)(x + 5)$$

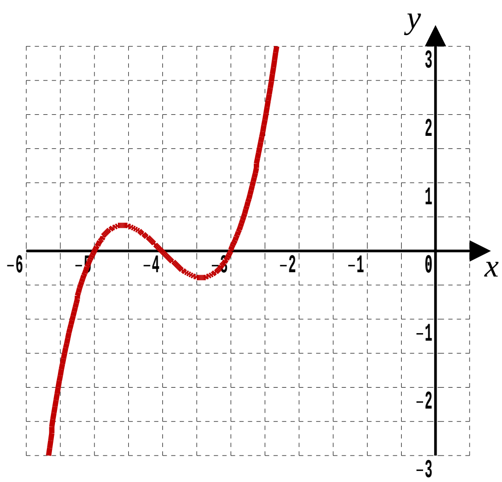
[2 marks]

(iii) To sketch the graph, work out that it crosses the x-axis when $y = 0$

That is, when $(x + 3)(x + 4)(x + 5) = 0$

Either $x = -3$, or $x = -4$, or $x = -5$

It crosses the y-axis when $y = 60$



[2 marks]

2.3.2 Solution to 2.2 Exercise
Answer 1

(i)

$$
\begin{array}{r|l}
 & \quad\; x^2 + 3x + 2 \\
\cline{2-2}
x + 4 & x^3 + 7x^2 + 14x + 8 \\
 & \underline{x^3 + 4x^2} \\
 & \qquad 3x^2 + 14x \\
 & \qquad \underline{3x^2 + 12x} \\
 & \qquad\qquad\quad 2x + 8 \\
 & \qquad\qquad\quad \underline{2x + 8} \\
 & \qquad\qquad\qquad\;\; 0 \leftarrow \text{Remainder}
\end{array}
$$

$\therefore\ f(x)$ is divisible by $x + 4$ ☐

[4 marks]

(ii)

$$\frac{x^3 + 7x^2 + 14x + 8}{x + 4} = x^2 + 3x + 2$$

$$\therefore\ x^3 + 7x^2 + 14x + 8 = \left(x^2 + 3x + 2\right)\left(x + 4\right)$$

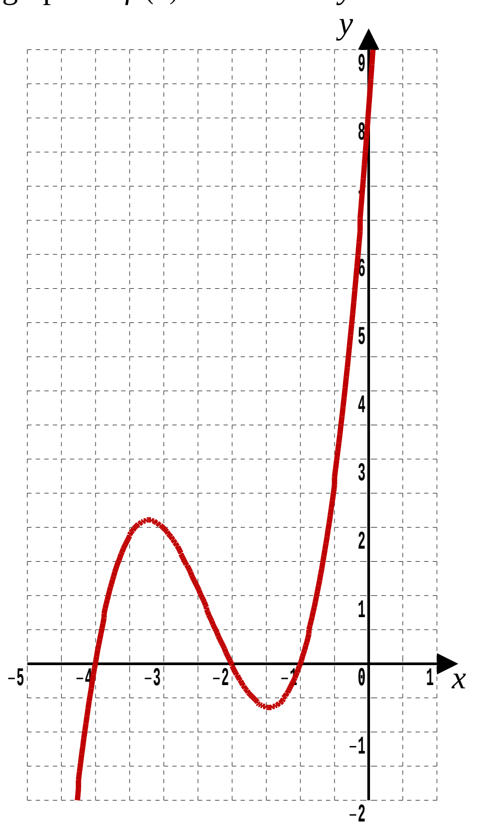
$$= (x + 1)(x + 2)(x + 4)$$

[2 marks]

(iii) When $y = 0,\ (x + 1)(x + 2)(x + 4) = 0$

$\therefore$ Either $x = -1$, or $x = -2$, or $x = -4$

Also the graph of $f(x)$ crosses the y-axis when $y = 8$



[2 marks]

Answer 2

(i)

$$\begin{array}{r|l} & x^2 + 4x - 21 \\ \hline x + 3 & x^3 + 7x^2 - 9x - 63 \\ & \underline{x^3 + 3x^2} \\ & \quad 4x^2 - 9x \\ & \quad \underline{4x^2 + 12x} \\ & \quad\quad -21x - 63 \\ & \quad\quad \underline{-21x - 63} \\ & \quad\quad\quad 0 \leftarrow \text{Remainder} \end{array}$$

$\therefore\ f(x)$ is divisible by $x + 3$ ☐

[4 marks]

(ii)

$$\frac{x^3 + 7x^2 - 9x - 63}{x + 3} = x^2 + 4x - 21$$

$$\therefore\ x^3 + 7x^2 - 9x - 63 = \left(x^2 + 4x - 21\right)\left(x + 3\right)$$

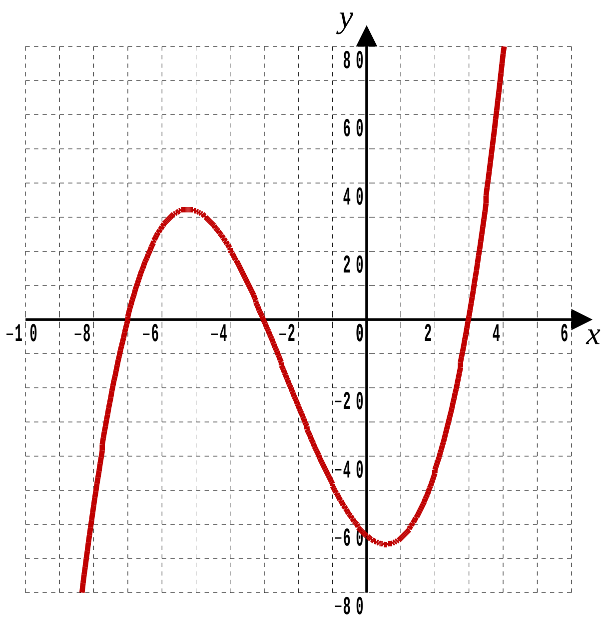
$$= (x - 3)(x + 7)(x + 3)$$

[2 marks]

(iii) When $y = 0$, $(x - 3)(x + 7)(x + 3) = 0$

$\therefore$ Either $x = 3$, or $x = -7$, or $x = -3$

Also the graph of $f(x)$ crosses the y-axis when $y = -63$



[2 marks]

Answer 3

(i)

$$
\begin{array}{r|l}
 & \quad\; x^2 + 5x + 6 \\ \hline
x - 2 & x^3 + 3x^2 - 4x - 12 \\
 & \underline{x^3 - 2x^2} \\
 & \qquad 5x^2 - 4x \\
 & \qquad \underline{5x^2 - 10x} \\
 & \qquad\qquad 6x - 12 \\
 & \qquad\qquad \underline{6x - 12} \\
 & \qquad\qquad\qquad 0 \quad \leftarrow \text{Remainder}
\end{array}
$$

$\therefore\ f(x)$ is divisible by $x - 2$ ☐

[4 marks]

(ii) $$\frac{x^3 + 3x^2 - 4x - 12}{x - 2} = x^2 + 5x + 6$$

$$\therefore\ x^3 + 7x^2 - 9x - 63 = \left(x^2 + 5x + 6\right)\left(x - 2\right)$$

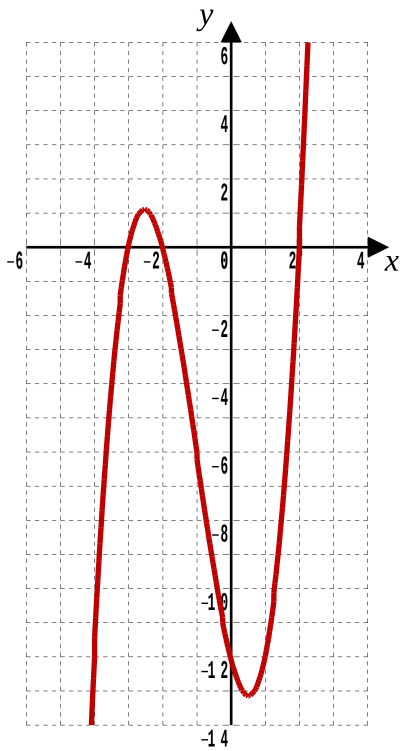
$$= (x + 3)(x + 2)(x - 2)$$

[2 marks]

(iii) When $y = 0$, $(x + 3)(x + 2)(x - 2) = 0$

$\therefore$ Either $x = -3$, or $x = -2$, or $x = 2$

Also the graph of $f(x)$ crosses the y-axis when $y = -12$



[2 marks]

Answer 4

(i)

$$
\begin{array}{r|l}
 & x^2 + 5x - 2 \\
\cline{2-2}
x + 1 & x^3 + 6x^2 + 3x - 10 \\
 & \underline{x^3 + x^2} \\
 & \quad 5x^2 + 3x \\
 & \quad \underline{5x^2 + 5x} \\
 & \qquad - 2x - 10 \\
 & \qquad \underline{- 2x - 2} \\
 & \qquad\qquad - 8 \leftarrow \text{Remainder}
\end{array}
$$

$\therefore\ f(x)$ is not divisible by $x + 1$ □

[4 marks]

(ii) The factors that will give an exact division are, $(x - 1)$, $(x + 2)$, $(x + 5)$
Looking for a successful long division with one of those factors.

[4 marks]

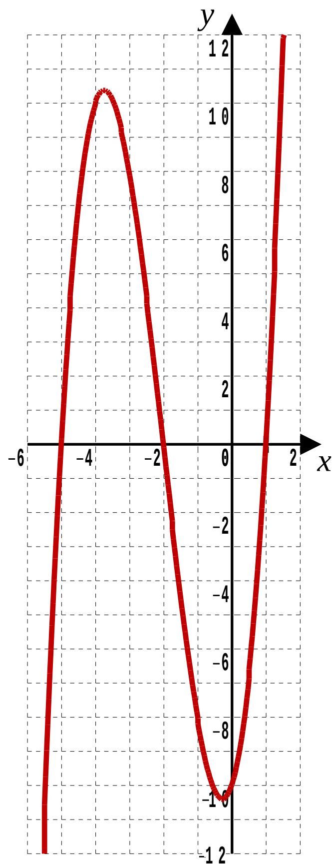
(iii) $\therefore\ x^3 + 6x^2 + 3x - 10 = (x + 5)(x + 2)(x - 1)$

[2 marks]

(iv) When $y = 0$, $(x + 5)(x + 2)(x - 1) = 0$

$\therefore$ Either $x = -5$, or $x = -2$, or $x = 1$

Also the graph of $f(x)$ crosses the y-axis when $y = -10$



[2 marks]

Answer 5

(i) The likely factors are $(x \pm 1)$, $(x \pm 2)$, $(x \pm 3)$, or $(x \pm 6)$

[2 marks]

(ii) The factors that will give an exact division are $(x - 1)$, $(x + 2)$, $(x + 3)$
Look for a successful long division with one of those factors.

[4 marks]

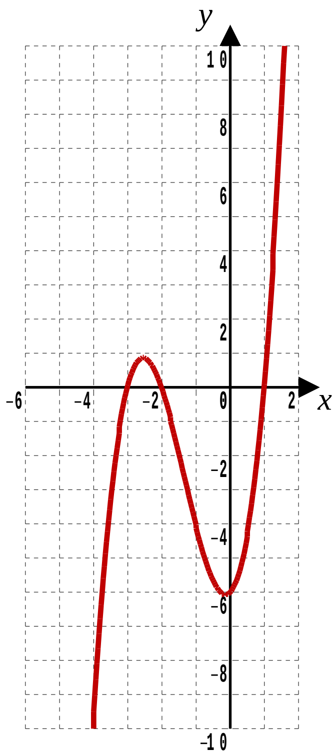
(iii) $\therefore\ x^3 + 4x^2 + x - 6 = (x - 1)(x + 2)(x + 3)$

[2 marks]

(iv) When $y = 0$, $(x + 3)(x + 2)(x - 1) = 0$

$\therefore$ Either $x = -3$, or $x = -2$, or $x = 1$

Also the graph of $f(x)$ crosses the y-axis when $y = -6$



[2 marks]