## 6.3 Answers to 6.2 Exercise



**Answer 1**

$p(x)$ has degree 3 (It's a cubic)

**[ 1 mark ]**

**Answer 2**

**( i )** A degree 2 polynomial is called a quadratic
(and its graph is termed a parabola)

**[ 1 mark ]**

**( ii )** A degree 4 polynomial is called a quartic

**[ 1 mark ]**

**Answer 3**

$$f(x) = 2x^3 - 7x^2 - 5x + 4$$

**( a )** The remainder when $f(x)$ is divided by $(x - 1)$ is given by $f(1)$ by the remainder theorem.

$$\begin{aligned} f(1) &= 2(1)^3 - 7(1)^2 - 5(1) + 4 \\ &= 2 - 7 - 5 + 4 \\ &= -6 \end{aligned}$$

**[ 2 marks ]**

**( b )**

$$\begin{aligned} f(-1) &= 2(-1)^3 - 7(-1)^2 - 5(-1) + 4 \\ &= -2 - 7 + 5 + 4 \\ &= 0 \quad \therefore\ (x + 1) \text{ is a factor of } f(x) \text{ by the factor theorem} \end{aligned}$$

**[ 2 marks ]**

**( c )**

$$\begin{array}{r|rrrr} & & 2x^2 & -9x & +4 \\ \hline x + 1 & 2x^3 & -7x^2 & -5x & +4 \\ & 2x^3 & +2x^2 & & \\ \hline & & -9x^2 & -5x & \\ & & -9x^2 & -9x & \\ \hline & & & 4x & +4 \\ & & & 4x & +4 \\ \hline & & & & 0 \end{array} \leftarrow \text{As expected}$$

$$\begin{aligned} f(x) &= 0 \\ 2x^3 - 7x^2 - 5x + 4 &= 0 \\ (x + 1)(2x^2 - 9x + 4) &= 0 \\ (x + 1)(2x - 1)(x - 4) &= 0 \end{aligned}$$

**[ 4 marks ]**

**Answer 4**

$$f(x) = x^4 + x^3 + 2x^2 + ax + b$$

**( a )** $f(1) = 7$ by the remainder theorem

$$(1)^4 + (1)^3 + 2(1)^2 + a(1) + b = 7$$

$$1 + 1 + 2 + a + b = 7$$

$$a + b = 3 \qquad \square$$

**[ 2 marks ]**

**( b )** $f(-2) = -8$ by the remainder theorem

$$(-2)^4 + (-2)^3 + 2(-2)^2 + a(-2) + b = -8$$

$$16 - 8 + 8 - 2a + b = -8$$

$$-2a + b = -24$$

Solving simultaneous equations,

$$a + b = 3$$

SUBTRACT $\quad -2a + b = -24$

$$3a = 27$$

$$a = 9 \qquad \Rightarrow \qquad b = -6$$

**[ 5 marks ]**

**Answer 5**

**( a )**

**( i )** With $x = 3$, $x^3 - 2x^2 - 4x + 8 = (3)^3 - 2(3)^2 - 4(3) + 8$

$$= 27 - 18 - 12 + 8$$

$$= 5$$

**( ii )** With $x = -2$, $x^3 - 2x^2 - 4x + 8 = (-2)^3 - 2(-2)^2 - 4(-2) + 8$

$$= -8 - 8 + 8 + 8$$

$$= 0$$

**[ 3 marks ]**

**( b )** From part (a), $(x + 2)$ is a factor, by the factor theorem

$$\begin{array}{r|l}
 & x^2 - 4x + 4 \\ \hline
x + 2 & x^3 - 2x^2 - 4x + 8 \\
 & \underline{x^3 + 2x^2} \\
 & \quad -4x^2 - 4x \\
 & \quad \underline{-4x^2 - 8x} \\
 & \qquad\quad 4x + 8 \\
 & \qquad\quad \underline{4x + 8} \\
 & \qquad\qquad 0 \leftarrow \text{ As expected}
\end{array}$$

$$x^3 - 2x^2 - 4x + 8 = 0$$

$$(x + 2)(x^2 - 4x + 4) = 0$$

$$(x + 2)(x - 2)(x - 2) = 0$$

Either $x = -2$ or $x = 2$

The solution set is $x \in \{\pm 2\}$

**[ 4 marks ]**

**Answer 6**

$$f(x) = x^3 + ax + 6$$

**( i )** By the remainder theorem, $f(3) = 12$

$$(3)^3 + a(3) + 6 = 12$$

$$27 + 3a + 6 = 12$$

$$3a = -21$$

$$a = -7 \qquad \square$$

**[ 2 marks ]**

**( ii )** The function can now be written with $-7$ in place of $a$

$$f(x) = x^3 - 7x + 6$$

Looking at this, it's obvious that $x = 1$ is a solution

That is, $(x - 1)$ is a factor

$$\begin{array}{r|rrrr}
 & & x^2 & +\; x & -\; 6 \\ \hline
x - 1 & x^3 & +\; 0x^2 & -\; 7x & +\; 6 \\
 & x^3 & -\; x^2 & & \\ \cline{2-3}
 & & x^2 & -\; 7x & \\
 & & x^2 & -\; x & \\ \cline{3-4}
 & & & -\; 6x & +\; 6 \\
 & & & -\; 6x & +\; 6 \\ \cline{4-5}
 & & & & 0
\end{array} \leftarrow \text{ As expected}$$

$$x^3 - 7x + 6 = 0$$

$$\left(x - 1\right)\left(x^2 + x - 6\right) = 0$$

$$(x - 1)(x + 3)(x - 2) = 0$$

**[ 3 marks ]**

**Answer 7**

$$f(x) = x^3 + a x^2 + bx + 3$$

**( a )** By the remainder theorem, $f(-2) = 7$

$$(-2)^3 + a(-2)^2 + b(-2) + 3 = 7$$

$$-8 + 4a - 2b + 3 = 7$$

$$4a - 2b = 12$$

Divide through by 2

$$2a - b = 6 \qquad \square$$

**[ 2 marks ]**

**( b )** By the remainder theorem, $f(1) = 4$

$$(1)^3 + a(1)^2 + b(1) + 3 = 4$$

$$1 + a + b + 3 = 4$$

$$a + b = 0$$

Combining the two results as simultaneous equations,

$$\begin{array}{lrcl} & 2a - b & = & 6 \\ \text{ADD} & a + b & = & 0 \\ \hline & 3a \quad & = & 6 \end{array}$$

$$a = 2 \qquad \Rightarrow \qquad b = -2$$

**[ 4 marks ]**

**Answer 8**

$$f(x) = 2x^3 - 3x^2 - 39x + 20$$

**( a )**

$$\begin{aligned} f(-4) &= 2(-4)^3 - 3(-4)^2 - 39(-4) + 20 \\ &= -128 - 48 + 156 + 20 \\ &= 0 \end{aligned}$$

$\therefore$ $(x + 4)$ is a factor of $f(x)$ by the factor theorem

**[ 2 marks ]**

**( b )**

$$\begin{array}{r|llll} & 2x^2 & -\ 11x & +\ 5 & \\ \hline x + 4 & 2x^3 - 3x^2 & -\ 39x & +\ 20 & \\ & 2x^3 + 8x^2 & & & \\ \cline{2-2} & -\ 11x^2 & -\ 39x & & \\ & -\ 11x^2 & -\ 44x & & \\ \cline{2-3} & & 5x & +\ 20 & \\ & & 5x & +\ 20 & \\ \cline{3-4} & & & 0 & \leftarrow \text{ As expected} \end{array}$$

$$\begin{aligned} 2x^3 - 3x^2 - 39x + 20 &= 0 \\ (x + 4)(2x^2 - 11x + 5) &= 0 \\ (x + 4)(2x - 1)(x - 5) &= 0 \end{aligned}$$

**[ 4 marks ]**

**Answer 9**

$$f(x) = x^3 - 2x^2 + ax + b$$

**( a )** By the remainder theorem, $f(2) = 1$

$$(2)^3 - 2(2)^2 + a(2) + b = 1$$

$$8 - 8 + 2a + b = 1$$

$$2a + b = 1$$

By the remainder theorem, $f(-1) = 28$

$$(-1)^3 - 2(-1)^2 + a(-1) + b = 28$$

$$-1 - 2 - a + b = 28$$

$$-a + b = 31$$

Combining the two results as simultaneous equations,

$$2a + b = 1$$

SUBTRACT $\quad -a + b = 31$

$$3a = -30$$

$$a = -10 \quad \Rightarrow \quad b = 21$$

**[ 6 marks ]**

**( b )** The function is now known to be,

$$f(x) = x^3 - 2x^2 - 10x + 21$$

$$f(3) = (3)^3 - 2(3)^2 - 10(3) + 21$$

$$= 27 - 18 - 30 + 21$$

$$= 0$$

$\therefore \; (x - 3)$ is a factor of $f(x)$, by the factor theorem

**[ 2 marks ]**