

7.1 Test : Provided Materials

The Factor Theorem

If, for a given polynomial function $p(x)$, $p(a) = 0$ (for some constant, a)
then $(x - a)$ is a factor of $p(x)$

The Remainder Theorem

When a polynomial $p(x)$ is divided by $(x - a)$, where a is a constant,
the remainder is $p(a)$

7.2 The Test

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 50

Question 1

$$p(x) = 3x^4 + 5x^2 - 23x + 31$$

What is the degree of polynomial $p(x)$?

[1 mark]

Question 2

A polynomial of degree seven is called a Septic.

(i) What is a polynomial of degree 2 called ?

[1 mark]

(ii) What is a polynomial of degree 5 called ?

[1 mark]

Question 3

A-Level Examination Question from May 2007, Paper C2, Q2 (Edexcel)

$$f(x) = 3x^3 - 5x^2 - 16x + 12$$

- (a) Find the remainder when $f(x)$ is divided by $(x - 2)$

[2 marks]

Given that $(x + 2)$ is a factor of $f(x)$

- (b) factorise $f(x)$ completely

[4 marks]

Question 4

A-Level Examination Question from January 2007, Paper C2, Q5 (Edexcel)

$$f(x) = x^3 + 4x^2 + x - 6$$

- (a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

[2 marks]

- (b) Factorise $f(x)$ completely.

[4 marks]

- (c) Write down all the solutions to the equation

$$x^3 + 4x^2 + x - 6 = 0$$

[1 mark]

Question 5

A-Level Examination Question from January 2013, Paper C2, Q2 (Edexcel)

$$f(x) = ax^3 + bx^2 - 4x - 3, \quad \text{where } a \text{ and } b \text{ are constants}$$

Given that $(x - 1)$ is a factor of $f(x)$

(a) Show that $a + b = 7$

[2 marks]

Given also that, when $f(x)$ is divided by $(x + 2)$, the remainder is 9,

(b) find the value of a and the value of b , showing each step in your working.

[4 marks]

Question 6

$$f(x) = x^3 - x^2 - 7x + c$$

Given that $f(4) = 0$

(a) find the value of c

[2 marks]

(b) factorise $f(x)$ as the product of a linear factor and a quadratic factor.

[3 marks]

(c) Explain why, apart from at $x = 4$, there are no other real values of x for which $f(x) = 0$

[2 marks]

Question 7

$$p(x) = 4x^3 + ax^2 + bx + 15$$

- When $p(x)$ is divided by $(x - 1)$ the remainder is 90
- When $p(x)$ is divided by $(x + 1)$ the remainder is -4

Find the values of a and b

[7 marks]

Question 8

$$f(x) = x^4 - 3x^2 - ax + b$$

$$g(x) = x^3 - 3ax + 5b$$

Both $f(x)$ and $g(x)$ are divisible by $(x - 2)$

Prove that $3b = 2a$

[5 marks]

Question 9

A-Level Examination Question from May 2013, Paper C2, Q3 (Edexcel)

$$f(x) = 2x^3 - 5x^2 + ax + 18 \quad \text{where } a \text{ is a constant}$$

Given that $(x - 3)$ is a factor of $f(x)$

(a) Show that $a = -9$

[2 marks]

(b) Factorise $f(x)$ completely

[4 marks]

Given that $g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$

(c) By considering your earlier answers, find the values of y that satisfy $g(y) = 0$, giving your answers to 2 decimal places where appropriate. You may use "trial and improvement" if you wish.

[3 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk