1.4 Answers

1.4.1 Solution to the 1.1 Example

$$(a + \sqrt{5})(4 + \sqrt{5}) = 29 + b\sqrt{5}$$

$$4a + 4\sqrt{5} + a\sqrt{5} + 5 = 29 + b\sqrt{5}$$

$$4a + 5 + (4 + a)\sqrt{5} = 29 + b\sqrt{5}$$

$$\therefore 4a + 5 = 29 \quad \text{and} \quad 4 + a = b$$

$$4a = 24$$

$$a = 6 \quad \text{and so} \quad b = 10$$

[2 marks]

1.4.2 Solution to the 1.2 Example

$$3^{2x+1} - 28(3^x) + 3^2 = 0$$

$$3^{2x} \times 3^1 - 28(3^x) + 9 = 0$$

$$3(3^x)^2 - 28(3^x) + 9 = 0$$

Let $z = 3^x$ in which case,

$$3z^2 - 28z + 9 = 0$$

$$(3z - 1)(z - 9) = 0$$

Either $3z - 1 = 0$ or $z - 9 = 0$

$$z = \frac{1}{3} \qquad z = 9$$

$$3^x = \frac{1}{3} \qquad 3^x = 9$$

$$x = -1 \qquad x = 2$$

[3 marks]

1.4.3 Solution to the 1.3 Exercise

Answer 1

$$(a + \sqrt{7})(5 + \sqrt{7}) = 47 + b\sqrt{7}$$

$$5a + 5\sqrt{7} + a\sqrt{7} + 7 = 47 + b\sqrt{7}$$

$$5a + 7 + (5 + a)\sqrt{7} = 47 + b\sqrt{7}$$

$$\therefore 5a + 7 = 47 \qquad \text{and} \qquad 5 + a = b$$

$$5a = 40$$

$$a = 8 \quad \text{and so} \quad b = 13$$

[2 marks]

Answer 2

$$2^{2x+2} - 33\left(2^x\right) + 2^3 = 0$$

$$2^{2x} \times 2^2 - 33\left(2^x\right) + 8 = 0$$

$$4\left(2^x\right)^2 - 33\left(2^x\right) + 8 = 0$$

Let $z = 2^x$ in which case,

$$4z^2 - 33z + 8 = 0$$

$$(4z - 1)(z - 8) = 0$$

Either $4z - 1 = 0$ or $z - 8 = 0$

$$z = \frac{1}{4} \qquad z = 8$$

$$2^x = \frac{1}{4} \qquad 2^x = 8$$

$$x = -2 \qquad x = 3$$

[3 marks]

Answer 3

$$21x + 19 > 6x - 26$$

$$15x > -45$$

$$x > -3$$

[2 marks]

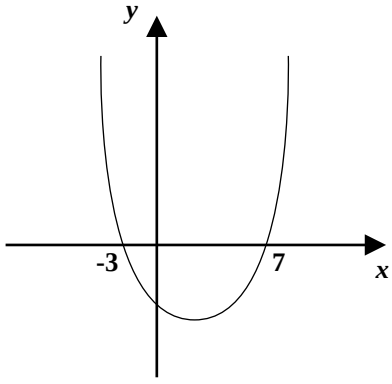
Answer 4

(i) $x^2 - 4x - 21 \geqslant 0$

$$(x + 3)(x - 7) \geqslant 0$$

$\therefore$ The critical values are $x = -3$ or $x = 7$

(ii)



(iii) $x \leqslant -3$ *or* $x \geqslant 7$

(iv) $-3 < x < 7$

[2, 2, 2, 2 marks]

Answer 5

(a) $(6, 11)$

(b) $m_{AB} = \dfrac{\Delta y}{\Delta x} = \dfrac{17 - 5}{9 - 3} = \dfrac{12}{6} = 2$

(c) $y = 2x - 1 \quad \Rightarrow \quad 2x - y - 1 = 0$

(d) $m_{PERP} = -\dfrac{1}{2}$ through $(6, 11)$

$$y = mx + c$$

$$11 = -\frac{1}{2} \times 6 + c \text{ so } c = 14$$

$$y = -\frac{1}{2}x + 14 \Rightarrow x + 2y - 28 = 0$$

[2, 2, 3, 3 marks]

Answer 6

(i)
$$f(x) = x^3 + 4x^2 - 9x - 36$$
$$f(-4) = (-4)^3 + 4 \times (-4)^2 - 9 \times (-4) - 36$$
$$= -64 + 64 + 36 - 36$$
$$= 0 \qquad \square$$

[2 marks]

(ii) The long division is unusual,

$$\begin{array}{r|l} & x^2 \qquad\qquad - 9 \\ \hline x + 4 & x^3 + 4x^2 - 9x - 36 \\ & \underline{x^3 + 4x^2} \\ & 0 \quad - 9x - 36 \\ & \underline{\quad - 9x - 36} \\ & 0 \end{array}$$

$$x^3 + 4x^2 - 9x - 36 = (x + 4)(x^2 - 9)$$
$$= (x + 4)(x + 3)(x - 3)$$

[4 marks]

Answer 7

$$243\sqrt{3} = 3^a$$
$$3^5 \times 3^{0.5} = 3^a \qquad \text{could use } FACT \text{ calculator button}$$
$$3^{5.5} = 3^a$$
$$\therefore \; a = 5.5$$

[3 marks]

Answer 8

(a) $3x - 8 > x + 13$

$2x > 21$

$x > 10.5$

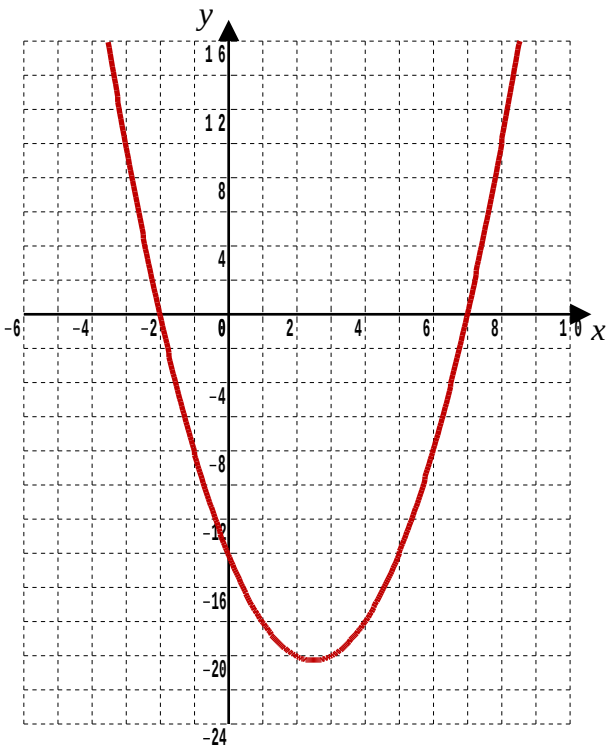
[2 marks]

(b) $x^2 - 5x - 14 > 0$

$(x + 2)(x - 7) > 0$

The critical values are $x = -2$ or $x = 7$

Sketch the graph to get the last bit correct...



$x < -2$ or $x > 7$

[2 marks]

Answer 9

(i)

$$\frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= \frac{2-\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-3}$$

$$= 2-\sqrt{3}$$

[2 marks]

(ii)

$$\frac{1}{(3-\sqrt{2})^2} = \frac{1}{9-6\sqrt{2}+2}$$

$$= \frac{1}{11-6\sqrt{2}} \times \frac{11+6\sqrt{2}}{11+6\sqrt{2}}$$

$$= \frac{11+6\sqrt{2}}{121+66\sqrt{2}-66\sqrt{2}-72}$$

$$= \frac{11+6\sqrt{2}}{49}$$

[3 marks]

Answer 10

$$Midpoint = \left(\frac{4+16}{2}, \frac{11+3}{2}\right) = (10,\ 7)$$

$$m_{AB} = \frac{\Delta y}{\Delta x} = \frac{3-11}{16-4} = -\frac{8}{12} = -\frac{2}{3}$$

$$m_{PERP} = \frac{3}{2}$$

$$y = mx + c$$

$$7 = \frac{3}{2} \times 10 + c$$

$$c = -8$$

$y = \frac{3}{2}x - 8$ is the perpendicular bisector

[4 marks]

Answer 11

(a) $6x - 7 < 2x + 3$

$4x < 10$

$x < 2.5$

[2 marks]

(b) $2x^2 - 11x + 5 < 0$

$(2x-1)(x-5) < 0$

The critical values are $x = \frac{1}{2}$ or $x = 5$

$\frac{1}{2} < x < 5$ (Draw a graph to see why)

[3 marks]

(c) $\frac{1}{2} < x < 2.5$

[1 mark]

Answer 12

(i)

$$g(x) = x^3 - 3x^2 - 10x + 24$$

$$g(4) = 4^3 - 3 \times 4^2 - 10 \times 4 + 24$$

$$= 64 - 48 - 40 + 24$$

$$= 0 \qquad \square$$

[1 mark]

(ii)

$$\begin{array}{rl|l} & & x^2 + x - 6 \\ \hline x - 4 & x^3 - 3x^2 - 10x + 24 & \\ & x^3 - 4x^2 & \\ \hline & + x^2 - 10x & \\ & x^2 - 4x & \\ \hline & - 6x + 24 & \\ & - 6x + 24 & \\ \hline & 0 & \end{array}$$

$$x^3 - 3x^2 - 10x + 24 = (x - 4)(x^2 + x - 6)$$

$$= (x - 4)(x + 3)(x - 2)$$

[4 marks]

Answer 13

$$8 + x\sqrt{12} = \frac{8x}{\sqrt{3}}$$

Multiply through by $\sqrt{3}$

$$8\sqrt{3} + x\sqrt{36} = 8x$$

$$8\sqrt{3} + 6x = 8x$$

$$2x = 8\sqrt{3}$$

$$x = 4\sqrt{3}$$

[4 marks]

Answer 14

(i)

$$(x + 1)^2 = x^2 + 2x + 1$$

[1 mark]

(ii)

$$x^2 + y^2 = 25$$

$$x^2 + (x + 1)^2 = 25$$

$$x^2 + x^2 + 2x + 1 = 25$$

$$2x^2 + 2x - 24 = 0$$

Divide through by 2

$$x^2 + x - 12 = 0$$

$$(x + 4)(x - 3) = 0$$

Either $x + 4 = 0$ or $x - 3 = 0$

$$x = -4 \qquad x = 3$$

The solutions are $(-4, -3)$ or $(3, 4)$

[4 marks]

Answer 15

Told, effectively, that,

$$f(-2) = 0$$

$$x^3 - 2x^2 + ax + 6 = 0$$

$$(-2)^3 - 2 \times (-2)^2 + a(-2) + 6 = 0$$

$$-8 - 8 - 2a + 6 = 0$$

$$2a = -10$$

$$a = -5$$

[3 marks]