

2.4 Answers

A-Level Pure Mathematics : Year 1 Progress Test Revision

2.4.1 Solutions to 2.2 Examples

$$(i) \quad \left(\frac{3a^6}{5a^2\sqrt{b}} \right)^{-2} = \left(\frac{5a^2\sqrt{b}}{3a^6} \right)^2 = \left(\frac{5\sqrt{b}}{3a^4} \right)^2 = \frac{25b}{9a^8}$$

[3 marks]

$$(ii) \quad \left(\frac{4x^2}{9\sqrt[3]{y}} \right)^{-\frac{3}{2}} = \left(\frac{9\sqrt[3]{y}}{4x^2} \right)^{\frac{3}{2}} = \frac{27\sqrt{y}}{8x^3}$$

[3 marks]

2.4.2 Solutions to 2.3 Exercise

Answer 1

$$(i) \quad \left(\frac{36x^6}{25} \right)^{-\frac{1}{2}} = \left(\frac{25}{36x^6} \right)^{\frac{1}{2}} = \frac{5}{6x^3}$$

[3 marks]

$$(ii) \quad \left(\frac{4xy^2}{3x^2y^3} \right)^{-1} = \frac{3x^2y^3}{4xy^2} = \frac{3xy}{4}$$

[3 marks]

Answer 2

$$\begin{aligned} (x+3)(x-3)(x+4) &= (x^2-9)(x+4) \quad \text{Diff of two squares} \\ &= x^3 + 4x^2 - 9x - 36 \end{aligned}$$

[3 marks]

Answer 3

$$\begin{aligned} 4x^2 - x^4 &= x^2(4 - x^2) \\ &= x^2(2+x)(2-x) \end{aligned}$$

[3 marks]

Answer 4

$$\begin{aligned} \frac{8}{3-\sqrt{5}} &= \frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} \\ &= \frac{8(3+\sqrt{5})}{9-5} \\ &= 2(3+\sqrt{5}) \\ &= 6+2\sqrt{5} \end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned}
 \text{(i)} \quad \sqrt{75} + \sqrt{12} &= \sqrt{25 \times 3} + \sqrt{4 \times 3} \\
 &= 5\sqrt{3} + 2\sqrt{3} \\
 &= 7\sqrt{3} \quad \text{That is, } a = 7
 \end{aligned}$$

[2 marks]

$$\begin{aligned}
 \text{(ii)} \quad \frac{\sqrt{75} + \sqrt{12}}{2 - \sqrt{3}} &= \frac{7\sqrt{3}}{2 - \sqrt{3}} \quad \text{from part (i)} \\
 &= \frac{7\sqrt{3}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \quad \text{rationalising the denominator} \\
 &= \frac{7\sqrt{3}(2 + \sqrt{3})}{4 - 3} \\
 &= 14\sqrt{3} + 21 \\
 &= 21 + 14\sqrt{3} \quad \text{that is, } b = 21 \text{ and } c = 14
 \end{aligned}$$

[4 marks]**Answer 6**

$$\begin{aligned}
 \text{LHS} &= \frac{a + b}{a - \sqrt{b}} \\
 &= \frac{a + b}{a - \sqrt{b}} \times \frac{a + \sqrt{b}}{a + \sqrt{b}} \\
 &= \frac{(a + b)(a + \sqrt{b})}{a^2 - b^2} \quad \Leftarrow \text{ This denominator is wrong} \\
 &= \frac{(a + b)(a + \sqrt{b})}{(a + b)(a - b)} \quad \text{(Difference of two squares)} \\
 &= \frac{a + \sqrt{b}}{a - b} \\
 &= \text{RHS} \quad \square
 \end{aligned}$$

[3 marks]**Answer 7**

$$\begin{aligned}
 x^2 - x &> 20 \\
 x^2 - x - 20 &> 0 \\
 (x + 4)(x - 5) &> 0
 \end{aligned}$$

The critical values are, $x = -4$ or $x = 5$ $x < -4$ or $x > 5$ (Draw a graph to see why)

$$\{x : x < -4\} \cup \{x : x > 5\}$$

[3 marks]

Answer 8

$$\begin{aligned}
 \text{(i)} \quad 9^x &= (3^2)^x \\
 &= (3^x)^2 \\
 &= y^2
 \end{aligned}$$

[1 mark]

$$\begin{aligned}
 \text{(ii)} \quad &9(9^x) - 10(3^x) + 1 = 0 \\
 &9y^2 - 10y + 1 = 0 \quad \text{with } y = 3^x \\
 &(9y - 1)(y - 1) = 0 \\
 &\text{Either } 9y - 1 = 0 \text{ in which case } y = \frac{1}{9} \\
 &\text{Or } y - 1 = 0 \text{ in which case } y = 1 \\
 &\text{But } y = 3^x \text{ so either } 3^x = \frac{1}{9} \text{ giving } x = -2 \\
 &\text{or } 3^x = 1 \text{ giving } x = 0 \\
 &\therefore x \in \{-2, 0\}
 \end{aligned}$$

[5 marks]**Answer 9**

$$\begin{aligned}
 \text{(i)} \quad &x\sqrt{2} - \sqrt{18} = x \\
 &x(\sqrt{2} - 1) = \sqrt{18} \\
 &x = \frac{\sqrt{18}}{(\sqrt{2} - 1)} \times \frac{(\sqrt{2} + 1)}{(\sqrt{2} + 1)} \\
 &= 6 + 3\sqrt{2}
 \end{aligned}$$

[3 marks]

$$\begin{aligned}
 \text{(ii)} \quad &4^{3x-2} = \frac{1}{2\sqrt{2}} \\
 &(2^2)^{3x-2} = 2^{-\frac{3}{2}} \\
 &6x - 4 = -\frac{3}{2} \\
 &6x = \frac{5}{2} \\
 &x = \frac{5}{12}
 \end{aligned}$$

[3 marks]

This document is a part of a **Mathematics Community Outreach Project** initiated by Shrewsbury School

It may be freely duplicated and distributed, unaltered, for non-profit educational use

In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

© 2025 Number Wonder

Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com