3.4 Answers

3.4.1 Solution to the 3.1 Example #1

$$9^{5x-3} = 27^{x+5}$$
$$\left(3^2\right)^{5x-3} = \left(3^3\right)^{x+5}$$
$$3^{10x-6} = 3^{3x+15}$$
$$10x - 6 = 3x + 15$$
$$7x = 21$$
$$x = 3$$

[3 marks]

3.4.2 Solution to the 3.1 Example #2

$$x^2 + 3mx + (4m + 1) = 0$$

(i) For "equal roots" the discriminant must equal zero,

$$b^2 - 4ac = 0 \qquad \text{For } ax^2 + bx + c = 0$$
$$(3m)^2 - 4 \times 1 \times (4m + 1) = 0$$
$$9m^2 - 16m - 4 = 0$$
$$(9m + 2)(m - 2) = 0$$
$$\text{Either } 9m + 2 = 0 \text{ or } m - 2 = 0$$
$$m = -\frac{2}{9} \qquad m = 2$$

[4 marks]

(ii) When $m = 2$, $x^2 + 3mx + (4m + 1) = 0$

$$x^2 + 6x + 9 = 0$$
$$(x + 3)^2 = 0$$
$$x + 3 = 0$$
$$x = -3$$

When $m = -\frac{2}{9}$, $x^2 + 3mx + (4m + 1) = 0$

$$x^2 - \frac{2}{3}x + \frac{1}{9} = 0$$
$$\left(x - \frac{1}{3}\right)^2 = 0$$
$$x = \frac{1}{3}$$

[4 marks]

3.4.3 Solution to the 3.3 Revision Exercise

Answer 1

$$8^{7x-6} = 16^{8-x}$$
$$\left(2^3\right)^{7x-6} = \left(2^4\right)^{8-x}$$
$$2^{21x-18} = 2^{32-4x}$$
$$21x - 18 = 32 - 4x$$
$$25x = 50$$
$$x = 2$$

[3 marks]

Answer 2

$$x^2 + 2mx + (3m + 4) = 0$$

(i) For "equal roots" the discriminant must equal zero,

$$b^2 - 4ac = 0 \qquad \text{for } ax^2 + bx + c = 0$$
$$(2m)^2 - 4 \times 1 \times (3m + 4) = 0$$
$$4m^2 - 12m - 16 = 0$$

divide through by 4

$$m^2 - 3m - 4 = 0$$
$$(m + 1)(m - 4) = 0$$

Either $m + 1 = 0$ or $m - 4 = 0$

$$m = -1 \qquad m = 4$$

[4 marks]

(ii)

When $m = -1$,
$$x^2 + 2mx + (3m + 4) = 0$$
$$x^2 - 2x + 1 = 0$$
$$(x - 1)^2 = 0$$
$$x = 1$$

When $m = 4$,
$$x^2 + 2mx + (3m + 4) = 0$$
$$x^2 + 8x + 16 = 0$$
$$(x + 4)^2 = 0$$
$$x = -4$$

[4 marks]

Answer 3

$$\frac{x^2 + 2x - 24}{x^2 - 3x - 54} = \frac{(x + 6)(x - 4)}{(x - 9)(x + 6)}$$

$$= \frac{x - 4}{x - 9}$$

[2 marks]

Answer 4

$$5x^2 - 4xy + y^2 = 13$$

$$5x^2 - 4x(x + 1) + (x + 1)^2 = 13$$

$$5x^2 - 4x^2 - 4x + x^2 + 2x + 1 = 13$$

$$2x^2 - 2x - 12 = 0$$

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x = 3 \text{ or } x = -2$$

The (x, y) solutions are $(3, 4)$ or $(-2, -1)$

[5 marks]

Answer 5

(i)

$$m(x) = x^3 + 13x^2 + ax + 63$$

Told that, $m(-7) = 0$

$$(-7)^3 + 13(-7)^2 + a(-7) + 63 = 0$$

$$-343 + 637 - 7a + 63 = 0$$

$$357 - 7a = 0$$

$$a = 51$$

[2 marks]

(ii)

$$\begin{array}{rl|l} & & x^2 + 6x + 9 \\ \hline x + 7 & x^3 + 13x^2 + 51x + 63 & \\ & x^3 + 7x^2 & \\ \hline & + 6x^2 + 51x & \\ & 6x^2 + 42x & \\ \hline & 9x + 63 & \\ & 9x + 63 & \\ \hline & 0 & \end{array}$$

$$x^3 + 13x^2 + 51x + 63 = (x + 7)(x^2 + 6x + 9)$$

$$= (x + 7)(x + 3)(x + 3)$$

[4 marks]

Answer 6

$$x^2 + y^2 + 96x - 110y = 0$$
$$[x^2 + 96x] + [y^2 - 110y] = 0$$
$$[(x + 48)^2 - 2304] + [(y - 55)^2 - 3025] = 0$$
$$(x + 48)^2 + (y - 55)^2 = 5329$$
$$(x + 48)^2 + (y - 55)^2 = 73^2$$

Circle has centre $(-48,\ 55)$ and radius 73

[4 marks]

Answer 7

$$\frac{\sqrt{3}}{81} = 3^a$$
$$\frac{3^{0.5}}{3^4} = 3^a$$
$$3^{-3.5} = 3^a$$
$$\therefore\ a = -3.5$$

[3 marks]

Answer 8

(i) $X\,(3, 15)$

[2 marks]

(ii) Diameter = 34

[1 mark]

(iii) Crosses the x-axis when $y = 0$

$$(x - 3)^2 + (-15)^2 = 17^2$$
$$x^2 - 6x + 9 + 225 - 289 = 0$$
$$x^2 - 6x - 55 = 0$$
$$(x - 11)(x + 5) = 0$$

Either $x - 11 = 0$ or $x + 5 = 0$

$x = 11$ $\qquad$ $x = -5$

[3 marks]

(iv) $Perimeter = 17 + 17 + 16$

$= 50$

[2 marks]

Answer 9

(i) If x were allowed to equal 3 a division by zero would occur which has no meaning in mathematics.

[1 mark]

(ii) Multiplying both sides by $(x - 3)$ is not a good move because it is not known if $(x - 3)$ is positive or negative so it's then not known if the inequality sign should be reversed or not.

[1 mark]

(iii)

$$5 (x - 3) < 2 (x - 3)^2$$

$$5x - 15 < 2x^2 - 12x + 18$$

$$2x^2 - 12x + 18 > 5x - 15$$

$$2x^2 - 17x + 33 > 0$$

$$(2x - 11)(x - 3) > 0$$

$$x < 3 \quad \text{or} \quad x > \frac{11}{2} \qquad \text{(Sketch a graph to see why)}$$

[3 marks]

(iv) It's valid to multiply both sides by a quantity that is certain to be greater than or equal to zero, as it's then known to leave the inequality sign as it is.

$$(x - 3)^2 \geqslant 0 \quad \text{for all real values of } x$$

[1 mark]

Answer 10

To have no real roots the discriminant must be negative.

$$D < 0$$

$$(-2k)^2 - 4 \times k \times 3 < 0$$

$$4k^2 - 12k < 0$$

divide through by 4

$$k^2 - 3k < 0$$

$$k (k - 3) < 0$$

Critical values are $k = 0$ and $k = 3$

$$0 < k < 3$$

Observe that $k = 0$ does not give a solvable equation either,

$$\therefore \; 0 \leqslant k < 3$$

[4 marks]

Answer 11

$$L_1 : 4y - 3x = 10$$

$$y = \frac{3}{4}x + \frac{10}{4} \text{ which has gradient } \frac{3}{4}$$

$$\begin{aligned} L_2 : m &= \frac{\Delta y}{\Delta x} \\ &= \frac{8 - (-1)}{-1 - 5} \\ &= \frac{9}{-6} \\ &= -\frac{3}{2} \end{aligned}$$

If the two gradients were the same the two lines would be parallel, but they are not.
If each gradient were the sign changed reciprocal of the other they would be mutually perpendicular, but they are not.

$\therefore$ The two lines are neither parallel nor perpendicular.

[3 marks]